Questions

Q1.

The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $f(x) = \frac{x-3}{x-2}$

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate g(x) to show that g '(x) = $\frac{e^x}{(e^x - 2)^2}$

(3)

(c) Find the exact values of x for which g '(x) = 1

(4)

(Total 12 marks)

Q2.

$$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3, x \neq -\frac{1}{2}$$

(a) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation y = f (x). The point $P\left(-1, -\frac{5}{2}\right)$ lies on$C$.

(b) Find an equation of the normal to C at P.

(8)

(Total 13 marks)

Q3.

$$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express f (x) as a single fraction in its simplest form.

(4)

(b) Hence show that f '(x) = $\frac{2}{(x-3)^2}$

(3)

(Total 7 marks)

Q4.

Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

(Total 6 marks)

Q5.

(a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate f (x) and find f '(2)

(3)

(Total 9 marks)

Q6.

$$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geq 0$$

(a) Show that $h(x) = \dfrac{2x}{x^2+5}$

(4)

(b) Hence, or otherwise, find h´(x) in its simplest form.

(3)

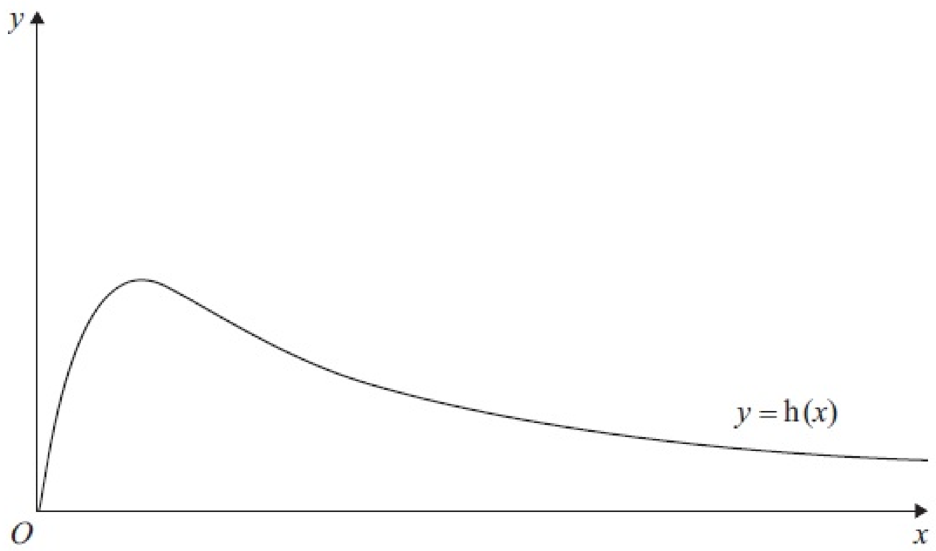


Figure 2

Figure 2 shows a graph of the curve with equation y = h(x) .

(c) Calculate the range of h(x) .

(5)

(Total 12 marks)

Q7.

The functions f and g are defined by

$$f : x \; {}^{x^2} \; 3x + \ln x, \qquad x > 0, \qquad x \in \mathbb{R}$$

$$g : x \; {}^{x^2} \; e\frac{d}{dx}, \qquad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$fg : x \; {}^{x^2} x2 + 3\frac{d}{dx}, \qquad x \in \mathbb{R}.$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\frac{d}{dx}$ [fg(x)] = $x(xe\frac{d}{dx} + 2)$.

(6)

(Total 10 marks)

Q8.

Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2\ln(3x)$

(4)

(b) $\frac{\sin 4x}{x^3}$

(5)

(Total 9 marks)

Q9.

The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

(Total 7 marks)

Q10.

(a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{(5x - 1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

(Total 10 marks)

Q11.

Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

(Total 6 marks)

Q12.

The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to *C* at the point on *C* where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where *a* and *b* are exact constants.

(4)

(Total 8 marks)

Q13.

A curve *C* has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point *P* on *C* has *x*-coordinate 2. Find an equation of the normal to *C* at *P* in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(7)

(Total 7 marks)

Q14.

(i) Given that $y = \frac{\ln(x^2+1)}{x}$, find $\frac{dy}{dx}$.

(4)

(ii) Given that $x = \tan y$, show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

(5)

(Total 9 marks)

Q15.

(a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{d(\sec x)}{dx} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

(b) find $\frac{dy}{dx}$.

(4)

The curve with equation $y = e^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b).

(c) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)

(Total 11 marks)

Q16.

(i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \; y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

(Total 13 marks)

Q17.

The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

(Total 7 marks)

Q18.

(i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\frac{dy}{dx} = \frac{-1}{1+x^2}$

(5)

(Total 11 marks)

Q19.

(a) Differentiate

$$\frac{\cos 2x}{\sqrt{x}}$$

with respect to x.

(3)

(b) Show that $\frac{d}{dx}(\sec^2 3x)$ can be written in the form

$$\mu(\tan 3x + \tan^3 3x)$$

where μ is a constant.

(3)

(c) Given $x = 2\sin\left(\frac{y}{3}\right)$, find $\frac{dy}{dx}$ in terms of x, simplifying your answer.

(4)

(Total 10 marks)

Q20.

The curve C has equation $y = f(x)$ where

$$f(x) = \frac{4x+1}{x-2}, \quad x > 2$$

(a) Show that

$$f'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $f'(x) = -1$,

(b) find the coordinates of P.

(3)

(Total 6 marks)

Q21.

The curve C has equation $x = 8y \tan 2y$

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$

(a) Verify that P lies on C.

(1)

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

(Total 8 marks)

Q22.

A curve C has equation $y = e^{4x} + x^4 + 8x + 5$

(a) Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x}$$

(3)

(b) On the axes given on page 5, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,

(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y-axis and state the equation of any asymptotes.

(4)

(c) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root.

(1)

The iteration formula

$$x_{n+1} = (-2 - e^{4x_n})^{\frac{1}{3}}, \qquad x_0 = -1$$

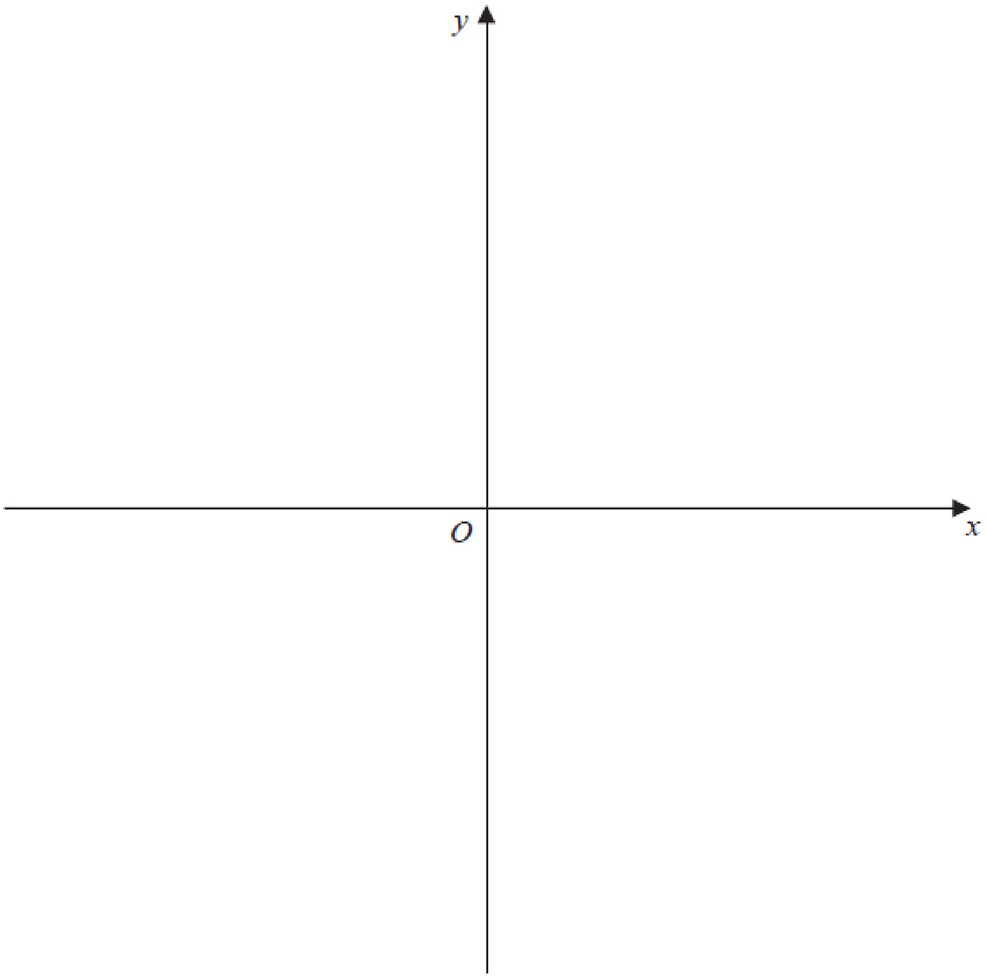
can be used to find an approximate value for this root.

(d) Calculate the values of x_1 and x_2, giving your answers to 5 decimal places.

(2)

(e) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve C.

(2)



(Total 12 marks)

Q23.

(i) (a) Show that $2 \tan x - \cot x = 5 \operatorname{cosec} x$ may be written in the form

$$a \cos^2 x + b \cos x + c = 0$$

stating the values of the constants a, b and c.

(4)

(b) Hence solve, for $0 \leq x < 2\pi$, the equation

$$2 \tan x - \cot x = 5 \operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan \theta + \cot \theta \equiv \lambda \operatorname{cosec} 2\theta, \quad \theta \pm \frac{n\pi}{2}, \; n \in \mathbb{Z}$$

stating the value of the constant λ.

(4)

(Total 12 marks)

Q24.

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p.

(1)

(b) Show that $k = \frac{1}{4}\ln 3$.

(4)

(c) Find the value of t when $\frac{\mathrm{d}m}{\mathrm{d}t} = -0.6\ln 3$.

(6)

(Total 11 marks)

Q25.

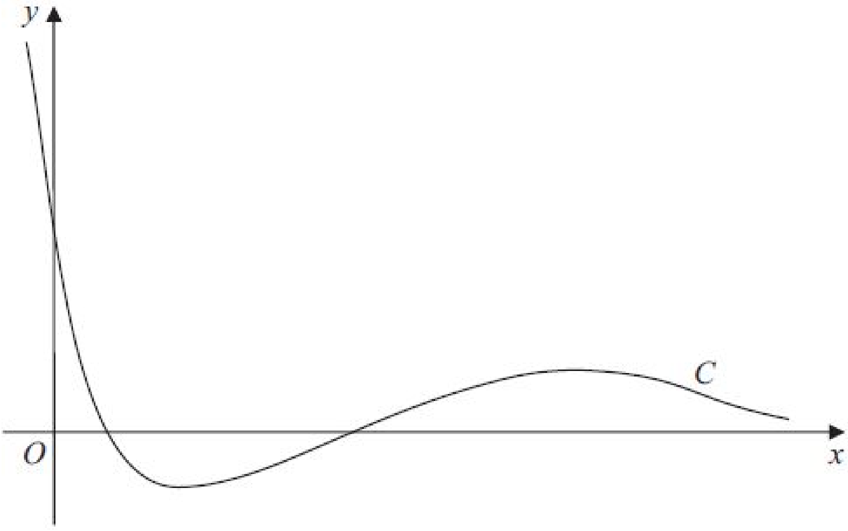


Figure 1

Figure 1 shows a sketch of the curve *C* with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where *C* crosses the *y*-axis.

(1)

(b) Show that *C* crosses the *x*-axis at *x* = 2 and find the *x*-coordinate of the other point where *C* crosses the *x*-axis.

(3)

(c) Find $\frac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of *C*.

(5)

(Total 12 marks)

Q26.

A rare species of primrose is being studied. The population, *P*, of primroses at time *t* years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \quad t \geq 0, t \in \mathbb{R}.$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when P = 250, giving your answer in the form a ln(b) where a and b are integers.

(4)

(c) Find the exact value of $\frac{dP}{dt}$ when t = 10. Give your answer in its simplest form.

(4)

(d) Explain why the population of primroses can never be 270

(1)

(Total 11 marks)

Q27.

$$f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}$$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equationy = f(x).

(5)

(b) Show that the equation f(x) = 0 can be written as $x = \pm\frac{4}{5}e^{-x}$

(1)

The equation f(x) = 0 has a root α, where α = 0.5 to 1 decimal place.

(c) Starting with x_0 = 0.5, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

(Total 11 marks)

Q28.

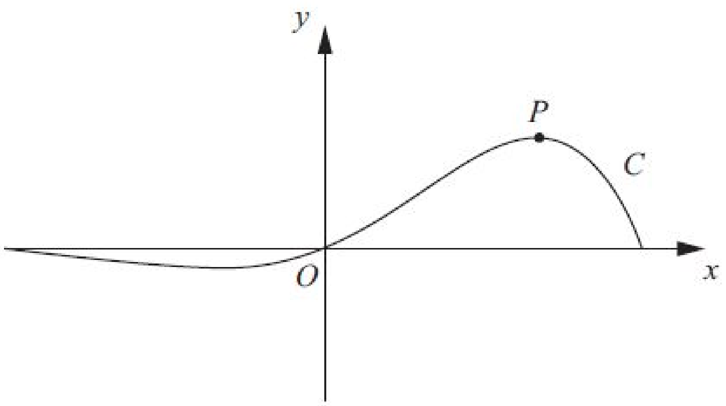


Figure 1

Figure 1 shows a sketch of the curve *C* which has equation

$$y = e^{x\sqrt{3}} \sin 3x , \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Find the *x* coordinate of the turning point *P* on *C*, for which *x* > 0
Give your answer as a multiple of π.

(6)

(b) Find an equation of the normal to *C* at the point where *x* = 0

(3)

(Total 9 marks)

Q29.

(a) Differentiate with respect to *x*,

(i) $x^{\frac{1}{2}} \ln(3x)$

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3\tan 2y$ find $^{dy}/_{dx}$ in terms of *x*.

(5)

(Total 11 marks)

Q30.

Given that

$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$

(a) find $\frac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form.

(4)

(Total 10 marks)

Q31.

Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ°C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90°C,

(a) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

(b) Show that $k = \frac{1}{5}\ln 2$.

(3)

(c) Find the rate at which the temperature of the tea is decreasing at the instant when t = 10. Give your answer, in °C per minute, to 3 decimal places.

(3)

(Total 8 marks)

Q32.

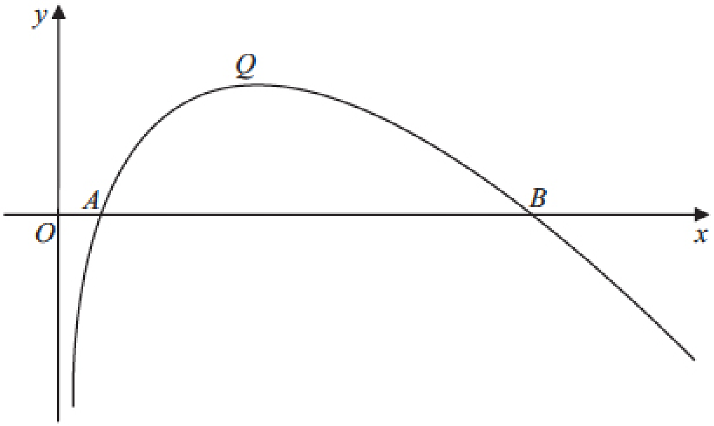


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \qquad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B.

(2)

(b) Find $\mathrm{f}'(x)$.

(3)

(c) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(d) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}$$

(3)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.
Give your answers to 3 decimal places.

(3)

(Total 13 marks)

Q33.

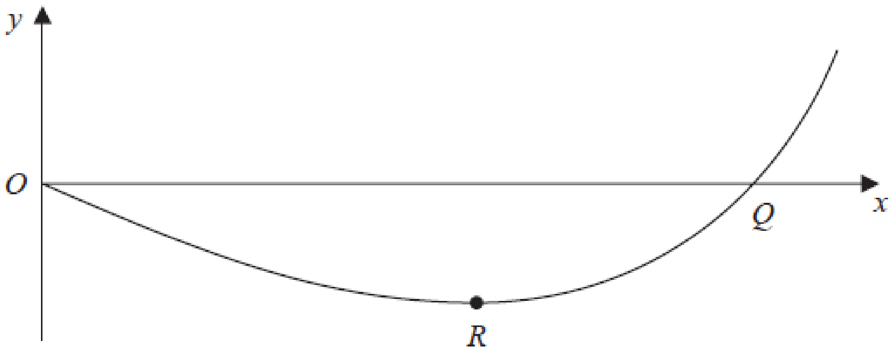


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(a) Show that the x coordinate of Q lies between 2.1 and 2.2

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n^{\ 2}\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

(Total 8 marks)

Q34.

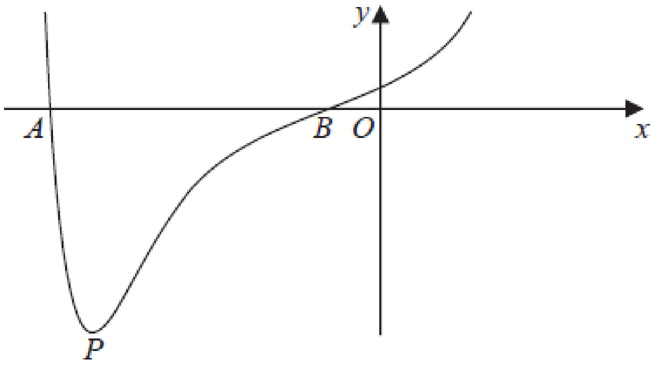


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(a) Calculate the x coordinate of A and the x coordinate of B, giving your answers to 3 decimal places.

(2)

(b) Find $f'(x)$.

(3)

The curve has a minimum turning point at the point P as shown in Figure 2.

(c) Show that the x coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$

(3)

(d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \qquad \text{with } x_0 = -2.4,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

The x coordinate of P is α.

(e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places.

(2)

(Total 13 marks)

Q35.

(a) Express 2cos 3*x* − 3sin 3*x* in the form *R* cos (3*x* + α), where *R* and α are constants, *R* > 0 and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$f(x) = e^{2x}\cos 3x$$

(b) Show that f '(*x*) can be written in the form

$$f'(x) = Re^{2x}\cos(3x + \alpha)$$

where *R* and α are the constants found in part (a).

(5)

(c) Hence, or otherwise, find the smallest positive value of *x* for which the curve with equation *y* = f(*x*) has a turning point.

(3)

(Total 12 marks)

Q36.

(a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that $\frac{d}{dx}$ (sec *x*) sec *x* tan *x*.

(3)

Given that

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of *y*.

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of *x*.

(4)

(Total 9 marks)