## Questions

Q1.

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants *A*, *B* and *C*.

**(4)**

b) Hence find $\int f(x)\,dx$.

(ii) Find $\int_1^2 f(x)\,dx$, leaving your answer in the form *a* + ln *b*, where *a* and *b* are constants.

**(6)**

**(Total 10 marks)**

Q2.

$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants *A*, *B* and *C*.

**(4)**

(b) (i) Hence find $\int f(x)\,dx$.

**(3)**

(ii) Find $\int_0^2 f(x)\,dx$ in the form 1n*k*, where *k* is a constant.

**(3)**

**(Total 10 marks)**

Q3.

(a) Express

$\frac{1}{P(5-P)}$ in partial fractions.

**(3)**

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5-P), \quad t \geqslant 0$$

where *P*, in thousands, is the population of meerkats and *t* is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + ce^{-\frac{1}{3}t}}$$

where *a*, *b* and *c* are integers.

**(8)**

(c) Hence show that the population cannot exceed 5000

**(1)**

**(Total 12 marks)**

Q4.

(a) Express $\frac{5}{(x-1)(3x+2)}$ in partial fractions.

**(3)**

(b) Hence find $\int \frac{5}{(x-1)(3x+2)}\,dx$, where $x > 1$.

**(3)**

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$ . Give your answer in the form $y = f(x)$.

**(6)**

**(Total 12 marks)**

Q5.

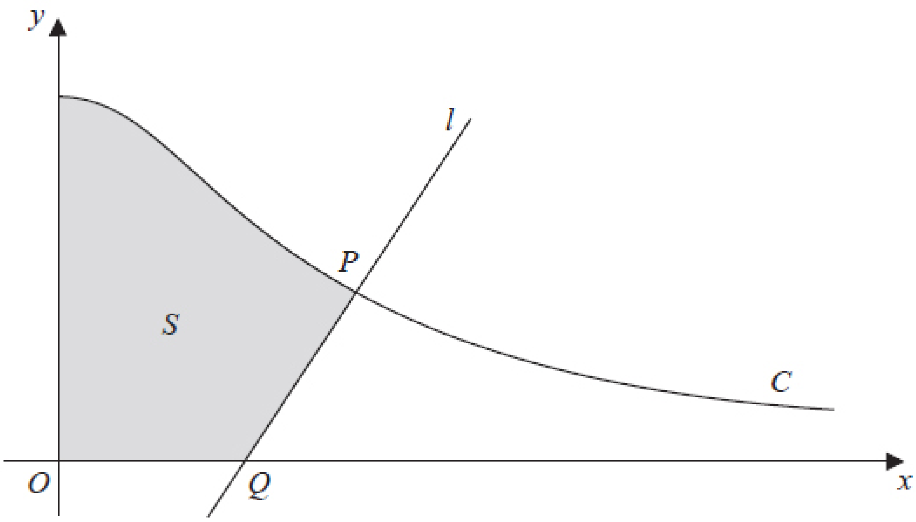


**Figure 4**

Figure 4 shows a sketch of part of the curve $C$ with parametric equations

$$x = 3\tan\theta, \quad y = 4\cos^2\theta, \quad 0 \leq \theta < \tfrac{\pi}{2}$$

The point $P$ lies on $C$ and has coordinates (3, 2).

The line $l$ is the normal to $C$ at $P$. The normal cuts the $x$-axis at the point $Q$.

(a) Find the $x$ coordinate of the point $Q$.

**(6)**

The finite region $S$, shown shaded in Figure 4, is bounded by the curve $C$, the $x$-axis, the $y$-axis and the line $l$. This shaded region is rotated $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Find the exact value of the volume of the solid of revolution, giving your answer in the form $p\pi + q\pi^2$, where $p$ and $q$ are rational numbers to be determined.

[*You may use the formula* $V = \tfrac{1}{3}\pi r^2 h$ *for the volume of a cone*.]

**(9)**

**(Total 15 marks)**

Q6.

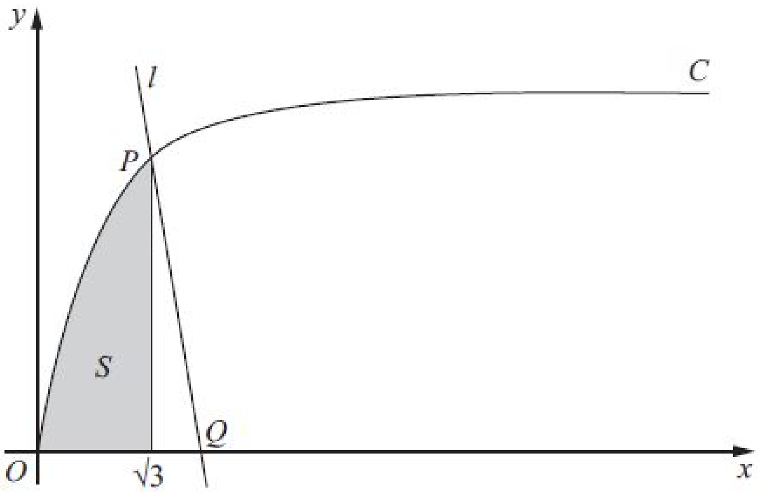


**Figure 3**

Figure 3 shows part of the curve *C* with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point *P* lies on *C* and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point *P*.

**(2)**

The line *l* is a normal to *C* at *P*. The normal cuts the *x*-axis at the point *Q*.

(b) Show that *Q* has coordinates ($k\sqrt{3}$, 0), giving the value of the constant *k*.

**(6)**

The finite shaded region *S* shown in Figure 3 is bounded by the curve *C*, the line $x = \sqrt{3}$ and the *x*-axis. This shaded region is rotated through 2π radians about the *x*-axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where *p* and *q* are constants.

**(7)**

**(Total 15 marks)**

Q7.

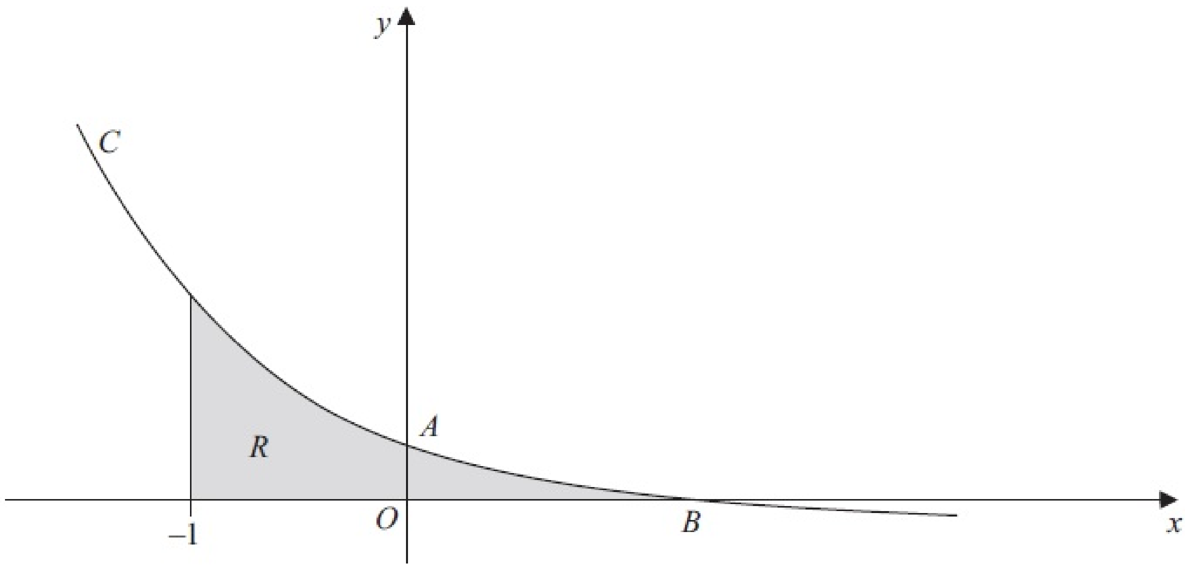


**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the $y$-axis at the point $A$ and crosses the $x$-axis at the point $B$.

(a) Show that $A$ has coordinates (0, 3).

**(2)**

(b) Find the $x$ coordinate of the point $B$.

**(2)**

(c) Find an equation of the normal to $C$ at the point $A$.

**(5)**

The region $R$, as shown shaded in Figure 2, is bounded by the curve $C$, the line $x = -1$ and the $x$-axis.

(d) Use integration to find the exact area of $R$.

**(6)**

**(Total 15 marks)**

Q8.

(a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \, \mathrm{d}\theta$.

**(2)**

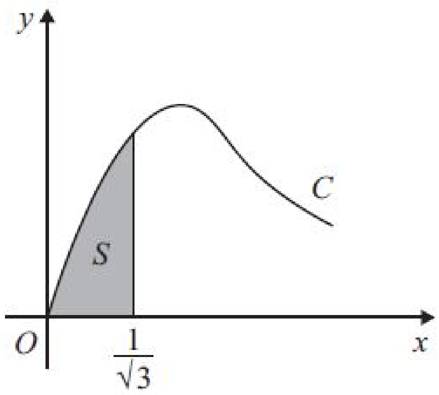


**Figure 4**

Figure 4 shows part of the curve *C* with parametric equations

$$x = \tan\theta, \qquad y = 2\sin 2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The finite shaded region *S* shown in Figure 4 is bounded by *C*, the line $x = \frac{1}{\sqrt{3}}$ and the *x*-axis.

This shaded region is rotated through $2\pi$ radians about the *x*-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$$

where *k* is a constant.

**(5)**

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where *p* and *q* are constants.

**(3)**

**(Total 10 marks)**

Q9.

Water is being heated in a kettle. At time *t* seconds, the temperature of the water is $\theta$ °C.

The rate of increase of the temperature of the water at any time *t* is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \qquad \theta \leqslant 100$$

where $\lambda$ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

**(8)**

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

**(3)**

**(Total 11 marks)**

Q10.

Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}$$

**(5)**

**(Total 5 marks)**

Q11.

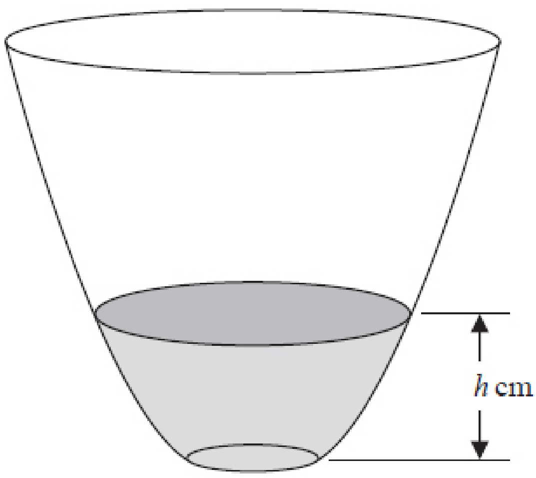


**Figure 2**

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is $h$ cm, the volume of water $V$ cm$^3$ is given by

$$V = 4\pi h(h + 4), \quad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of $80\pi$ cm$^3$s$^{-1}$

Find the rate of change of the depth of the water, in cm s$^{-1}$, when $h = 6$

**(5)**

**(Total 5 marks)**

Q12.

At time $t$ seconds the radius of a sphere is $r$ cm, its volume is $V$ cm$^3$ and its surface area is $S$ cm$^2$.

[*You are given that* $V = \frac{4}{3}\pi r^3$ *and that* $S = 4\pi r^2$]

The volume of the sphere is increasing uniformly at a constant rate of 3 cm$^3$ s$^{-1}$.

(a) Find $\frac{dr}{dt}$ when the radius of the sphere is 4 cm, giving your answer to 3 significant figures.

**(4)**

(b) Find the rate at which the surface area of the sphere is increasing when the radius is 4 cm.

**(2)**

**(Total 6 marks)**

Q13.

A circular cylinder has a perpendicular height equal to the radius of the base $r$ cm.

Given that the volume of this cylinder is $V$ cm$^3$ ,

(a) show that $V = \pi r^3$

**(1)**

Given that $r$ varies with time,

(b) find $\frac{\mathrm{d}V}{\mathrm{d}r}$ in terms of $r$.

**(1)**

The rate of change of the volume of the cylinder at time $t$ seconds, $\frac{\mathrm{d}V}{\mathrm{d}t}$ cm$^3$s$^{-1}$, is given by

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{2t}{2+t^2}$$

Given that $V = 3$ when $t = 0$,

(c) find $V$ in terms of $t$.

**(4)**

(d) When $t = 1$, find $r$. Give your answer to 3 significant figures.

**(4)**

(e) Using the Chain Rule, or otherwise, find $\frac{\mathrm{d}r}{\mathrm{d}t}$ in terms of $r$ and $t$.

**(2)**

(f) When $t = 1$, find the value of $\frac{\mathrm{d}r}{\mathrm{d}t}$. Give your answer to 3 significant figures.

**(2)**

**(Total 14 marks)**

Q14.

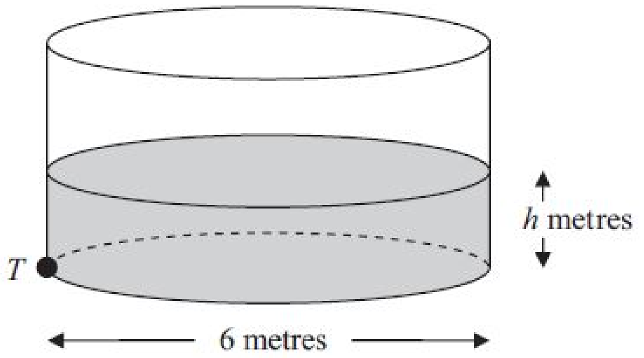


**Figure 2**

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m$^3$ min$^{-1}$. At time $t$ minutes, the depth of the water in the tank is $h$ metres. There is a tap at a point $T$ at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m$^3$ min$^{-1}$.

(a) Show that $t$ minutes after the tap has been opened

$$75\frac{\mathrm{d}h}{\mathrm{d}t} = (4 - 5h)$$

**(5)**

When $t = 0$, $h = 0.2$

(b) Find the value of $t$ when $h = 0.5$

**(6)**

**(Total 11 marks)**

Q15.

(a) Use integration by parts to find ∫ $x$ sin 3$x$ $dx$.

**(3)**

(b) Using your answer to part (a), find ∫ $x^2$cos3$x$ $dx$.

**(3)**

**(Total 6 marks)**

Q16.

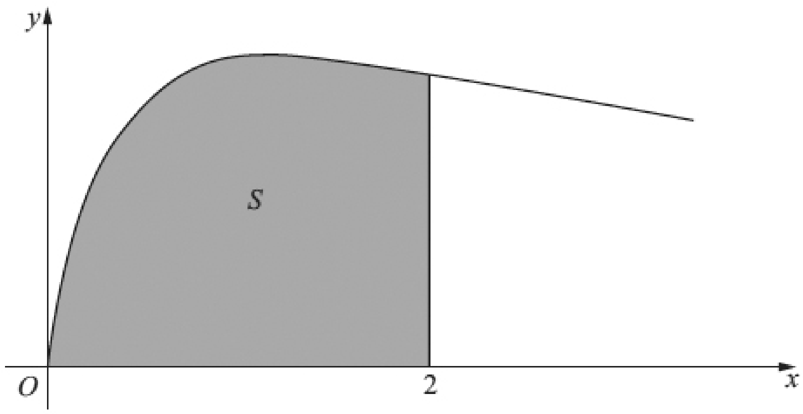


**Figure 1**

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \ x \geqslant 0$$

The finite region $S$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the line $x$ = 2

The region $S$ is rotated 360° about the $x$-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k$ ln $a$, where $k$ and $a$ are constants.

**(5)**

**(Total 5 marks)**

Q17.

(a) Find $\frac{1}{x^3}$ tan$^2 x$ d$x$.

**(2)**

(b) Use integration by parts to find $\frac{1}{x^3}\frac{1}{x^3}$ ln $x$ d$x$.

**(4)**

(c) Use the substitution $u = 1 + e^x$ to show that

$\frac{1}{x^3}\frac{e^{3x}}{1+e^x}$ d$x$ = $\frac{1}{2}$ $e^{2x} - e^x + \ln(1 + e^x) + k$,

where $k$ is a constant.

**(7)**

**(Total 13 marks)**

Q18.

Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

**(6)**

**(Total 6 marks)**

Q19.

$$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}} dx$$

(a) Given that $y = \frac{1}{4+\sqrt{(x-1)}}$, complete the table below with values of $y$ corresponding to $x$ = 3 and $x$ = 5 . Give your values
to 4 decimal places.

| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $y$ | 0.2 | | 0.1745 | |

**(2)**

(b) Use the trapezium rule, with all of the values of *y* in the completed table, to obtain an estimate of *I*, giving your answer

to 3 decimal places.

**(4)**

(c) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of *I*.

**(8)**

**(Total 14 marks)**

Q20.

Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e - 1)$$

**(6)**

**(Total 6 marks)**

Q21.

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta.$

**(3)**

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta) \, d\theta.$

**(7)**

**(Total 10 marks)**

Q22.

(a) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_{1}^{\sqrt{2}} \frac{1}{x^2 \sqrt{(4-x^2)}}\, dx$$

**(7)**

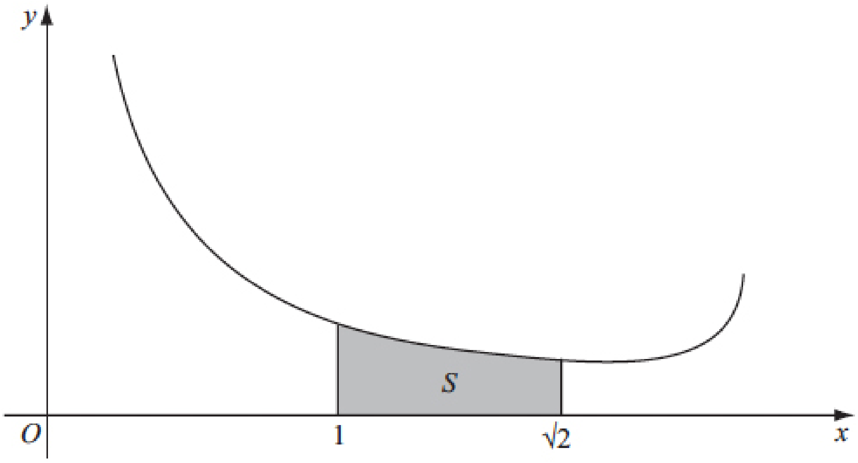


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = \frac{4}{x(4-x^2)^{\frac{1}{4}}}$, $0 < x < 2$.

The shaded region $S$, shown in Figure 3, is bounded by the curve, the $x$-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region $S$ is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

**(3)**

**(Total 10 marks)**

Q23.

(a) Find $\int \sqrt{(5-x)}\, dx.$

**(2)**

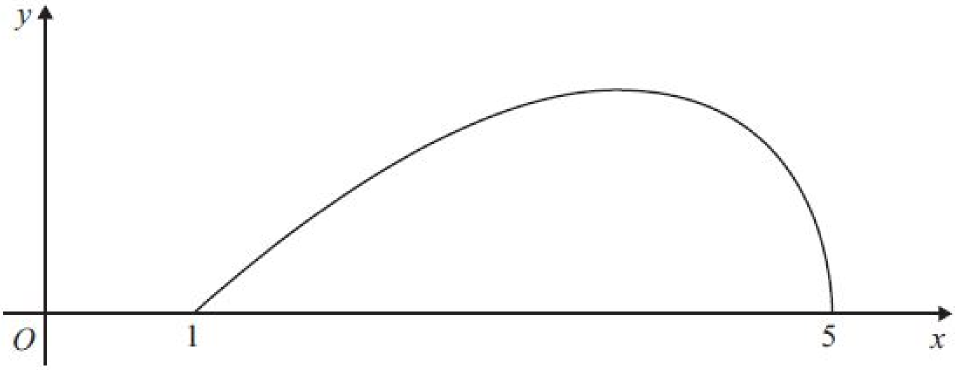


**Figure 3**

Figure 3 shows a sketch of the curve with equation

$$y = (x - 1)\sqrt{(5 - x)}, \qquad 1 \leqslant x \leqslant 5$$

(b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x$$

**(4)**

(b) (ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\,\mathrm{d}x$ .

**(2)**

**(Total 8 marks)**

Q24.

(a) Use integration to find

$$\int \frac{1}{x^3} \ln x \,\mathrm{d}x$$

**(5)**

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \,\mathrm{d}x$$

**(2)**

**(Total 7 marks)**

Q25.

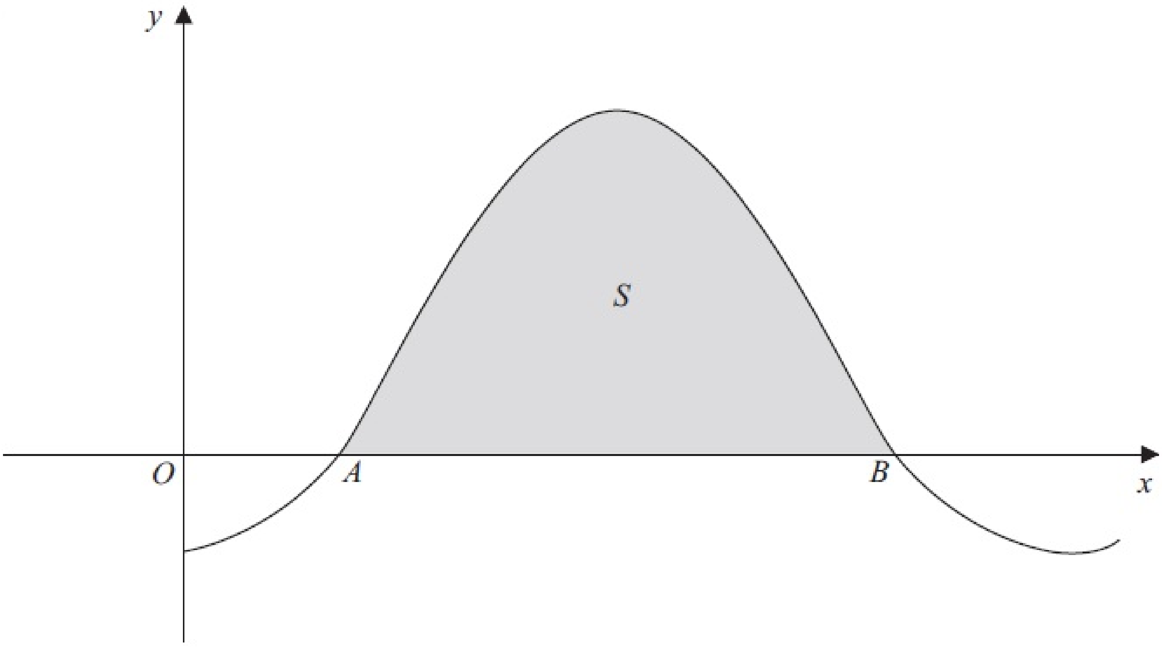


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where $x$ is measured in radians. The curve crosses the $x$-axis at the point $A$ and at the point $B$.

(a) Find, in terms of $\pi$, the $x$ coordinate of the point $A$ and the $x$ coordinate of the point $B$.

**(3)**

The finite region $S$ enclosed by the curve and the $x$-axis is shown shaded in Figure 3. The region $S$ is rotated through $2\pi$ radians about the $x$-axis.

(b) Find, by integration, the exact value of the volume of the solid generated.

**(6)**

**(Total 9 marks)**

Q26.

(a) Find $\int x^2 e^x \, dx$.

**(5)**

(b) Hence find the exact value of $\int_0^1 x^2 e^x \, dx$.

**(2)**

**(Total 7 marks)**

Q27.

Using the substitution $u = 2 + \sqrt{(2x + 1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{(2x+1)}}\,dx$$

giving your answer in the form $A + 2\ln B$, where $A$ is an integer and $B$ is a positive constant.

**(8)**

**(Total 8 marks)**

Q28.

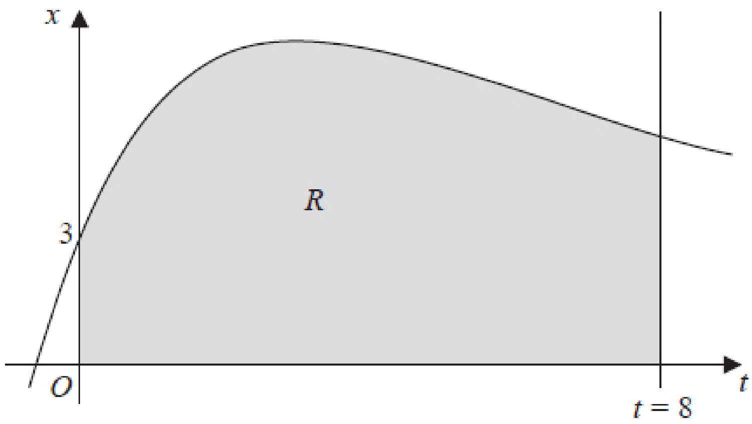


**Figure 1**

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region $R$ shown shaded in Figure 1 is bounded by the curve, the $x$-axis, the $t$-axis and the line $t = 8$.

(a) Complete the table with the value of $x$ corresponding to $t = 6$, giving your answer to 3 decimal places.

| $t$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $x$ | 3 | 7.107 | 7.218 | | 5.223 |

**(1)**

(b) Use the trapezium rule with all the values of $x$ in the completed table to obtain an estimate for the area of the region $R$, giving your answer to 2 decimal places.

**(3)**

(c) Use calculus to find the exact value for the area of $R$.

**(6)**

(d) Find the difference between the values obtained in part (b) and part (c), giving your answer to 2 decimal places.

**(1)**

**(Total 11 marks)**

Q29.

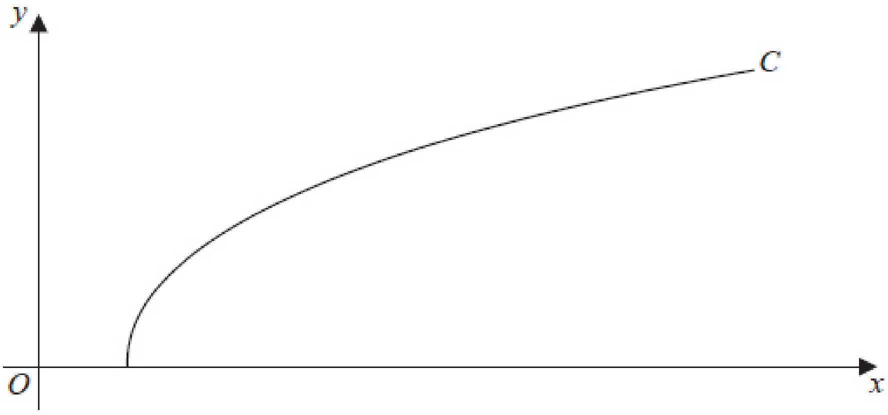


**Figure 2**

Figure 2 shows a sketch of the curve *C* with parametric equations

$$x = 27\sec^3 t,\ \ y = 3\tan t, \qquad 0 \leqslant t \leqslant \frac{\pi}{3}$$

(a) Find the gradient of the curve *C* at the point where $t = \dfrac{\pi}{6}$

**(4)**

(b) Show that the cartesian equation of *C* may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \ a \leq x \leq b$$

stating the values of *a* and *b*.

**(3)**

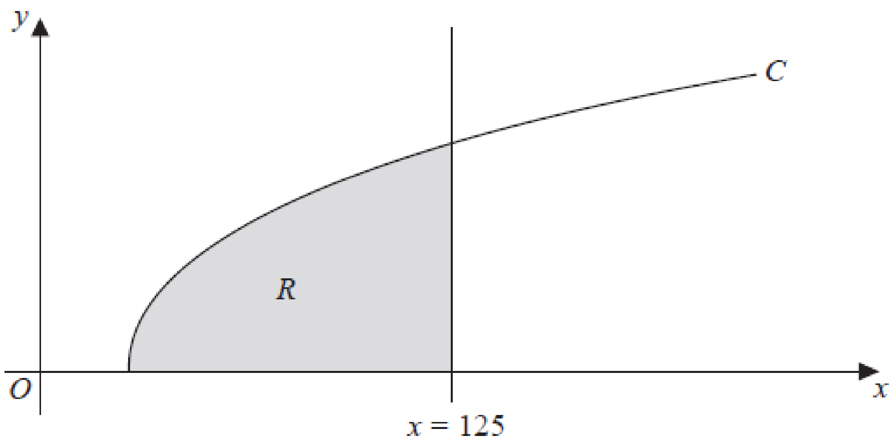


**Figure 3**

The finite region *R* which is bounded by the curve *C*, the *x*-axis and the line *x* = 125 is shown shaded in Figure 3. This region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution.

**(5)**

**(Total 12 marks)**

Q30.

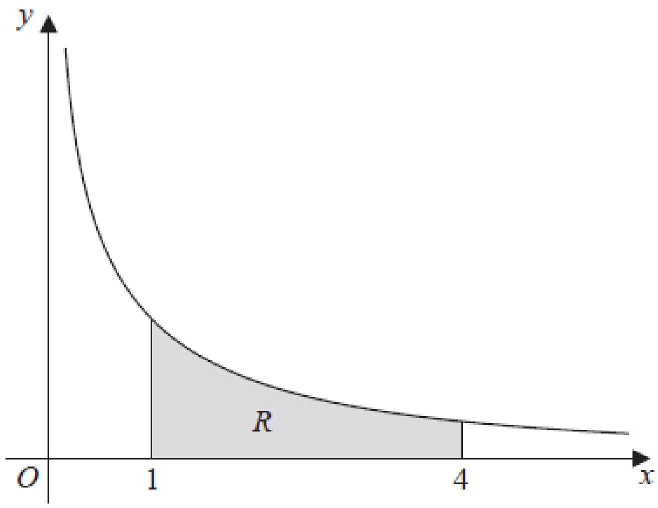


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}, \quad x > 0$

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis, and the lines with equations $x = 1$ and $x = 4$

The table below shows corresponding values of $x$ and $y$ for $y = \dfrac{10}{2x + 5\sqrt{x}}$

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 1.42857 | 0.90326 | | 0.55556 |

(a) Complete the table above by giving the missing value of $y$ to 5 decimal places.

**(1)**

(b) Use the trapezium rule, with all the values of $y$ in the completed table, to find an estimate for the area of $R$, giving your answer to 4 decimal places.

**(3)**

(c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part

(b) is an overestimate or an underestimate for the area of $R$.

**(1)**

(d) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\, dx$$

**(6)**

**(Total 11 marks)**

Q31.

(i) Find

$$\int x\, e^{4x}\, dx$$

**(3)**

(ii) Find

$$\int \frac{8}{(2x-1)^3}\, dx, \quad x > \tfrac{1}{2}$$

**(2)**

Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec}2y \operatorname{cosec}y$$

**(7)**

**(Total 12 marks)**

Q32.

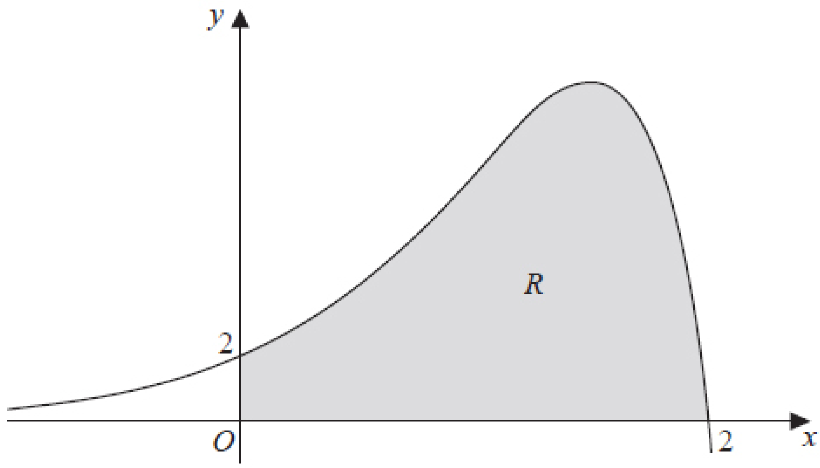


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \quad x \in \mathbb{R}$$

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the $y$-axis.

The table below shows corresponding values of $x$ and $y$ for $y = (2 - x)e^{2x}$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
| $y$ | 2 | 4.077 | 7.389 | 10.043 | 0 |

(a) Use the trapezium rule with all the values of $y$ in the table, to obtain an approximation for the area of $R$, giving your answer to 2 decimal places.

**(3)**

(b) Explain how the trapezium rule can be used to give a more accurate approximation for the area of $R$.

**(1)**

(c) Use calculus, showing each step in your working, to obtain an exact value for the area of $R$. Give your answer in its simplest form.

**(5)**

**(Total 9 marks)**

Q33.

Integrate the following with respect to $x$.

(a) $x^2 \ln x$

**(4)**

(b) $\sec 2x \tan 2x + \sec^2 x$

**(3)**

Using the substitution $u = 2 + \cos\theta$ , or otherwise,

(c) find the exact value of

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{2+\cos\theta}\,\mathrm{d}\theta$$

giving your answer in the form $a \ln b + c$ , where $a$, $b$ and $c$ are constants to be found.

**(8)**

**(Total 15 marks)**

Q34.

(a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x = \int \frac{2}{u(2u-1)}\,\mathrm{d}u$$

**(3)**

(b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x = 2\ln\left(\frac{a}{b}\right)$$

where $a$ and $b$ are integers to be determined.

**(7)**

**(Total 10 marks)**

Q35.

The rate of increase of the number, $N$, of fish in a lake is modelled by the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{(kt-1)(5000-N)}{t} \qquad t > 0, \quad 0 < N < 5000$$

In the given equation, the time $t$ is measured in years from the start of January 2000 and $k$ is a positive constant.

(a) By solving the differential equation, show that

$$N = 5000 - Ate^{-kt}$$

where $A$ is a positive constant.

**(5)**

After one year, at the start of January 2001, there are 1200 fish in the lake.

After two years, at the start of January 2002, there are 1800 fish in the lake.

(b) Find the exact value of the constant $A$ and the exact value of the constant $k$.

**(4)**

(c) Hence find the number of fish in the lake after five years. Give your answer to the nearest hundred fish.

**(1)**

**(Total 10 marks)**

Q36.

(a) Express $\dfrac{25}{x^2(2x+1)}$ in partial fractions.

**(4)**

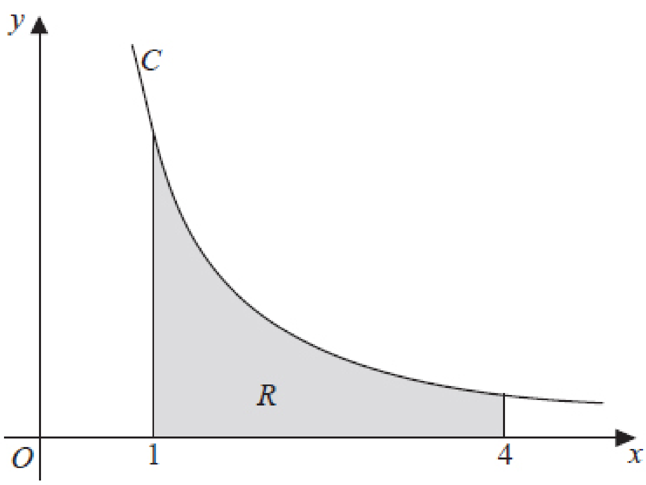


**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation $y = \dfrac{5}{x\sqrt{(2x+1)}}$, $x > 0$

The finite region $R$ is bounded by the curve $C$, the $x$-axis, the line with equation $x = 1$ and the line with equation $x = 4$

This region is shown shaded in Figure 2

The region *R* is rotated through 360° about the *x*-axis.

(b) Use calculus to find the exact volume of the solid of revolution generated, giving your answer in the form $a + b\ln c$, where *a*, *b* and *c* are constants.

**(6)**

**(Total 10 marks)**

Q37.

(a) Find $\int (4y+3)^{-\frac{1}{2}}\,dy$

**(2)**

(b) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{(4y+3)}}{x^2}$$

giving your answer in the form $y = f(x)$.

**(6)**

**(Total 8 marks)**

Q38.

(a) Find $\int \frac{9x+6}{x}\,dx$, $\quad x > 0$

**(2)**

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)\,y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

**(6)**

**(Total 8 marks)**

Q39.

A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3ºC and $t$ minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is *θºC.*

The rate of change of the temperature of the water in the bottle is modelled by the differential equation,

$$\frac{d\theta}{dt} = \frac{(3 - \theta)}{125}$$

(a) By solving the differential equation, show that,

$\theta = Ae^{-0.008t} + 3$

where $A$ is a constant.

**(4)**

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16ºC,

(b) find the time taken for the temperature of the water in the bottle to fall to 10ºC, giving your answer to the nearest minute.

**(5)**

**(Total 9 marks)**

Q40.

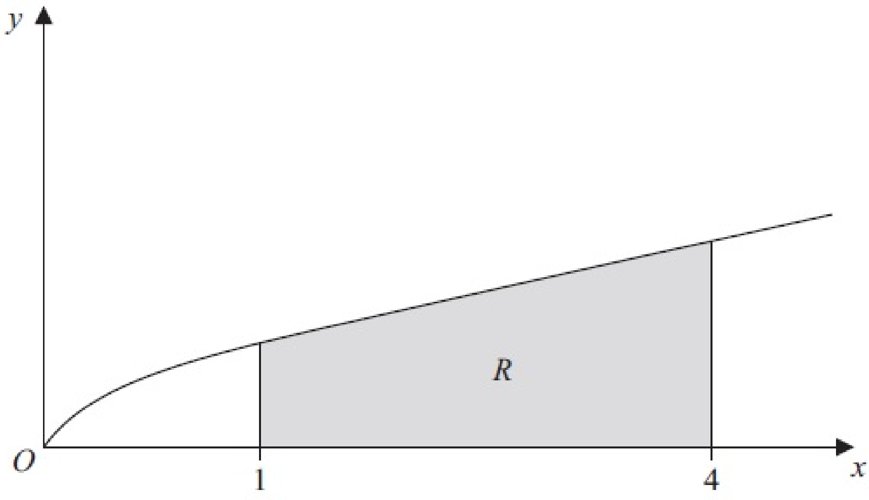


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1 + \sqrt{x}}$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of $y$ corresponding to $x = 3$, giving your answer to 4 decimal places.

**(1)**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 0.5 | 0.8284 | | 1.3333 |

(b) Use the trapezium rule, with all the values of *y* in the completed table, to obtain an estimate of the area of the region *R*, giving your answer to 3 decimal places.

**(3)**

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of *R*.

**(8)**

**(Total 12 marks)**

Q41.

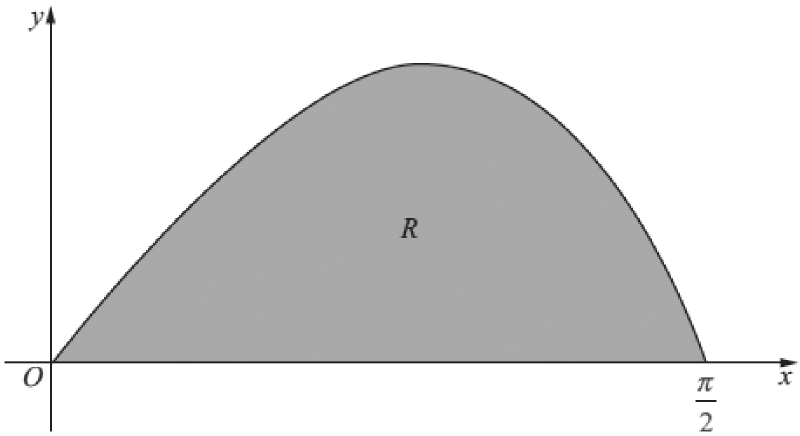


**Figure 3**

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}, \ 0 \leqslant x \leqslant \frac{\pi}{2}$.

The finite region *R*, shown shaded in Figure 3, is bounded by the curve and the *x*-axis.

The table below shows corresponding values of *x* and *y* for $y = \frac{2\sin 2x}{(1+\cos x)}$

| $x$ | 0 | $\frac{\pi}{8}$ | $\frac{\pi}{4}$ | $\frac{3\pi}{8}$ | $\frac{\pi}{2}$ |
|---|---|---|---|---|---|
| $y$ | 0 | | 1.17157 | 1.02280 | 0 |

(a) Complete the table above giving the missing value of *y* to 5 decimal places.

**(1)**

(b) Use the trapezium rule, with all the values of *y* in the completed table, to obtain an estimate for the area of *R*, giving
your answer to 4 decimal places.

**(3)**

(c) Using the substitution $u = 1 + \cos x$ , or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where *k* is a constant.

**(5)**

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

**(3)**

**(Total 12 marks)**

Q42.

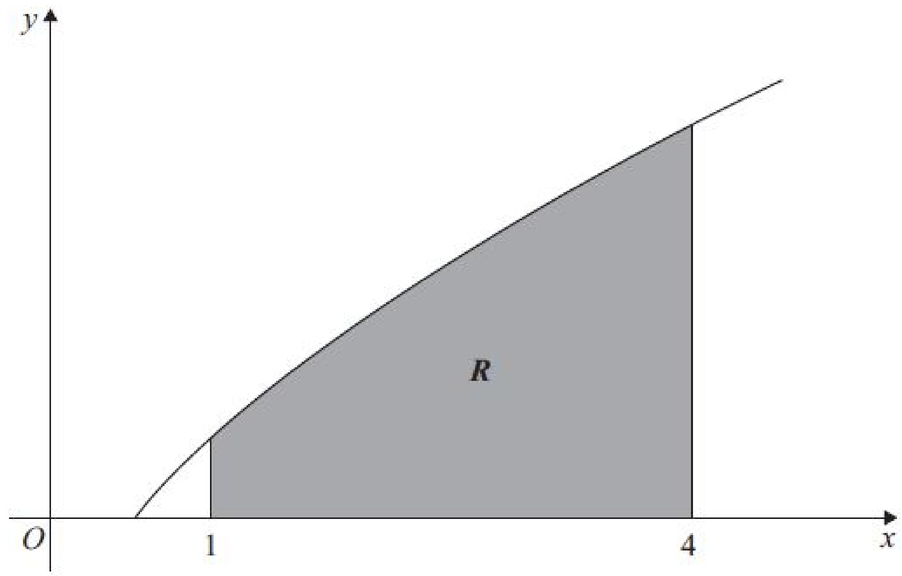


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$

.

The finite region *R*, shown shaded in Figure 3, is bounded by the curve,
the *x*-axis and the lines $x = 1$ and $x = 4$

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of *R*, giving your answer to 2 decimal places.

**(4)**

(b) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$

.

**(4)**

(c) Hence find the exact area of *R*, giving your answer in the form *a*ln 2 + *b*, where *a* and *b* are exact constants.

**(3)**

**(Total 11 marks)**

Q43.

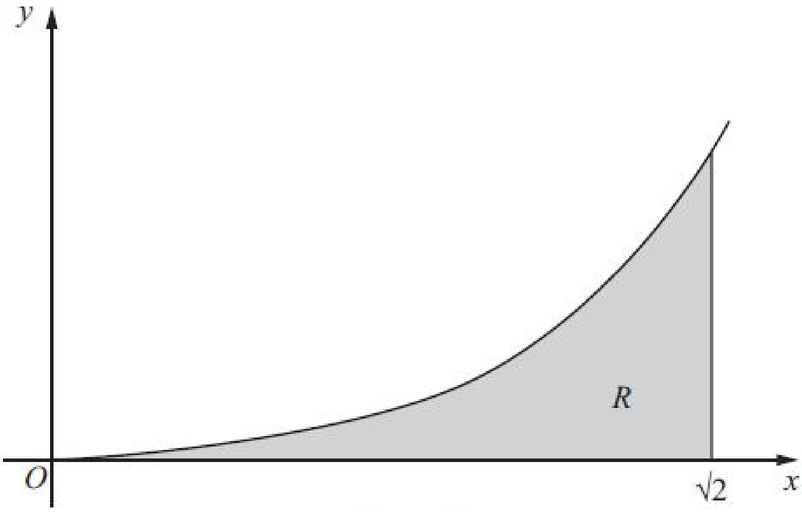


**Figure 2**

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2), \quad x \geqslant 0.$ The finite region *R*, shown shaded in Figure 2, is bounded by the curve, the *x*-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of *x* and *y* for $y = x^3 \ln(x^2 + 2)$.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | $\frac{\sqrt{2}}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{3\sqrt{2}}{4}$ | $\sqrt{2}$ |
| $y$ | 0 | | 0.3240 | | 3.9210 |

(a) Complete the table above giving the missing values of *y* to 4 decimal places.

**(2)**

(b) Use the trapezium rule, with all the values of *y* in the completed table, to obtain an estimate for the area of *R*, giving your answer to 2 decimal places.

**(3)**

(c) Use the substitution $u = x^2 + 2$ to show that the area of *R* is

$$\frac{1}{2}\int_{2}^{4}(u-2)\ln u\ \mathrm{d}u$$

**(4)**

(d) Hence, or otherwise, find the exact area of *R*.

**(6)**

**(Total 15 marks)**

Q44.

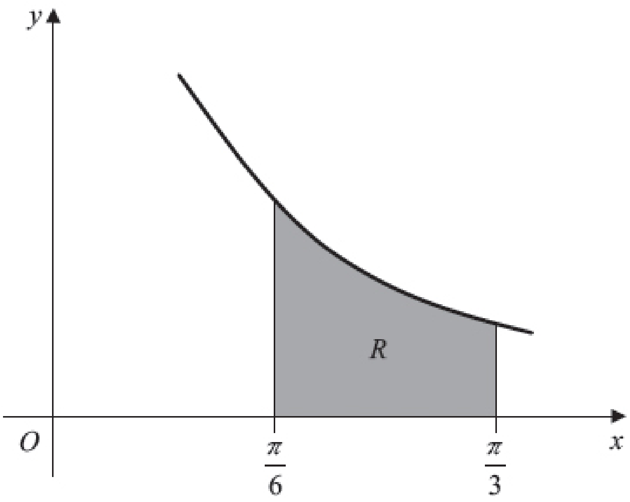


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = \operatorname{cosec} x$ .

The finite region *R* is bounded by the curve, the x-axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ as shown shaded in Figure 1.

(a) Use calculus to find the value of the area of *R* to 3 decimal places.

**(3)**

(b) Use the trapezium rule, with 2 strips of equal width, to estimate the area of *R*. Give your answer to 3 decimal places.

**(5)**

(c) Find the value of the error of your estimate in part (b).

**(1)**

(d) Find the exact value of the volume of the solid formed when the region *R* is rotated through $2\pi$ radians about the *x*-axis.
Give your answer in the form $a\pi\sqrt{3}$ where a is a constant.

**(4)**

**(Total 13 marks)**

Q45.

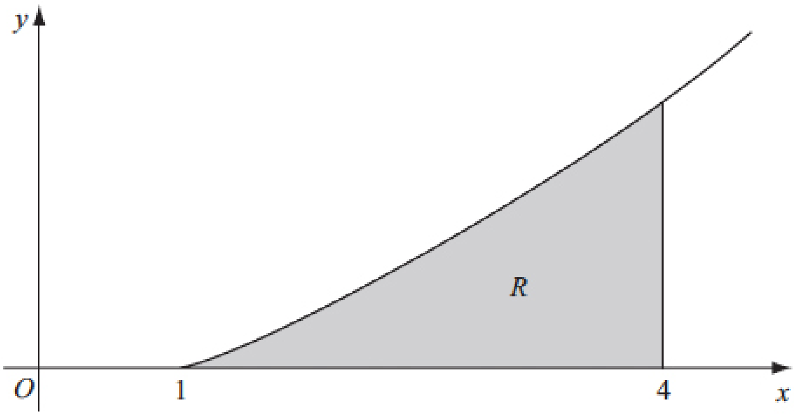


**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = x \text{ 1n } x$, $x \geqslant 1$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the line $x = 4$.

The table shows corresponding values of $x$ and $y$ for $y = x \text{ 1n } x$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| $y$ | 0 | 0.608 | | | 3.296 | 4.385 | 5.545 |

(a) Complete the table with the values of $y$ corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

**(2)**

(b) Use the trapezium rule, with all the values of $y$ in the completed table, to obtain an estimate for the area of $R$, giving your
answer to 2 decimal places.

**(4)**

(c)(i) Use integration by parts to find $\int x \text{ In } x \, dx$.

(ii) Hence find the exact area of $R$, giving your answer in the form $\frac{1}{4}(a \text{ In } 2 + b)$ where $a$ and $b$ are integers.

**(7)**

**(Total 13 marks)**