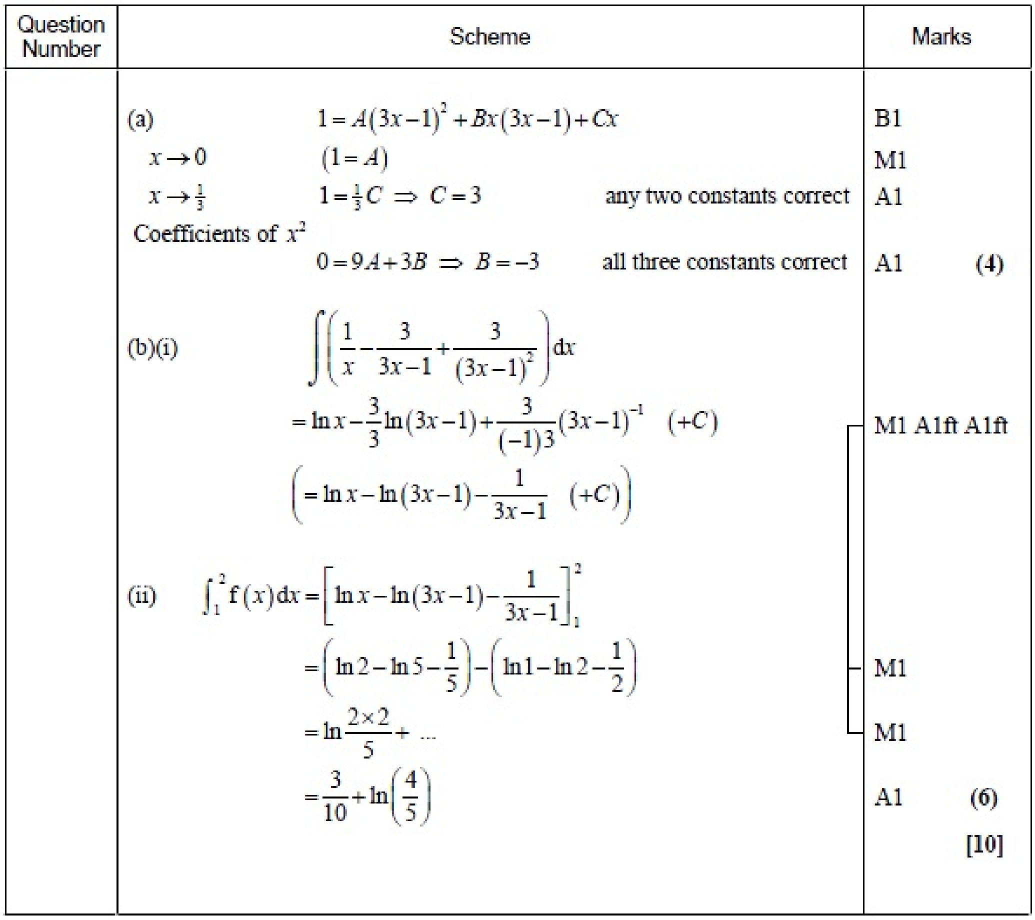
Mark Scheme

Q1.



Question Number	Scheme	Marks
	(a) $1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0$ $(1 = A)$	M1
	$x \to \frac{1}{3}$ $1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct	A1
	Coefficients of x^2	
	$0 = 9A + 3B \Rightarrow B = -3$ all three constants correct	A1 **(4)**
	(b)(i) $\int\left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2}\right)\mathrm{d}x$	
	$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C)\right)$	
	(ii) $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1}\right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5}\right) - \left(\ln 1 - \ln 2 - \frac{1}{2}\right)$	M1
	$= \ln\frac{2\times 2}{5} + \ldots$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1 **(6)**
		[10]

Q2.

Question Number	Scheme	Marks
Q (a)	$\mathrm{f}(x)=\dfrac{4-2x}{(2x+1)(x+1)(x+3)}=\dfrac{A}{2x+1}+\dfrac{B}{x+1}+\dfrac{C}{x+3}$	
	$4-2x=A(x+1)(x+3)+B(2x+1)(x+3)+C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x\to-\frac{1}{2},\quad 5=A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)\Rightarrow A=4$ any one correct constant	A1
	$x\to-1,\quad 6=B(-1)(2)\Rightarrow B=-3$	
	$x\to-3,\quad 10=C(-5)(-2)\Rightarrow C=1$ all three constants correct	A1 (4)
(b)	(i) $\displaystyle\int\left(\frac{4}{2x+1}-\frac{3}{x+1}+\frac{1}{x+3}\right)\mathrm{d}x$	
	$=\frac{4}{2}\ln(2x+1)-3\ln(x+1)+\ln(x+3)+C$ A1 two ln terms correct	M1 A1ft
	All three ln terms correct and "+C" ; ft constants	A1ft (3)
	(ii) $\left[2\ln(2x+1)-3\ln(x+1)+\ln(x+3)\right]_0^2$	
	$=(2\ln5-3\ln3+\ln5)-(2\ln1-3\ln1+\ln3)$	M1
	$=3\ln5-4\ln3$	
	$=\ln\left(\dfrac{5^3}{3^4}\right)$	M1
	$=\ln\left(\dfrac{125}{81}\right)$	A1 (3)
		[10]

Q3.

Question Number	Scheme	Marks
(a)	$1 = A(5-P) + BP$ Can be implied. $A = \frac{1}{5}, B = \frac{1}{5}$ Either one. giving $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$ See notes.	M1 A1 A1 **cao, aef** **[3]**
(b)	$\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{1}{15}\,\mathrm{d}t$ $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \;\; (+c)$ $\{t=0, P=1 \Rightarrow\}\;\; \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \quad \left\{\Rightarrow \;\; c = -\frac{1}{5}\ln 4\right\}$ eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$ Using any of the subtraction (or addition) laws for logarithms CORRECTLY $\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$ eg: $\frac{4P}{5-P} = \mathrm{e}^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = \mathrm{e}^{-\frac{1}{3}t}$ Eliminate ln's correctly. gives $4P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \;\Rightarrow\; P(4 + \mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t}$ $P = \frac{5\mathrm{e}^{\frac{1}{3}t}}{(4 + \mathrm{e}^{\frac{1}{3}t})} \quad \left\{\frac{(\div\, \mathrm{e}^{\frac{1}{3}t})}{(\div\, \mathrm{e}^{\frac{1}{3}t})}\right\}$ Make P the subject. $P = \frac{5}{(1 + 4\mathrm{e}^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5 + 20\mathrm{e}^{-\frac{1}{3}t})}$ etc.	B1 M1* A1ft dM1* dM1* dM1* dM1* A1 **[8]**
(c)	$1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1 \;\Rightarrow\; P < 5$. So population cannot exceed 5000.	B1 **[1]** **12**
(a)	**M1:** Forming a correct identity. For example, $1 = A(5-P) + BP$. **Note** A and B not referred to in question. **A1:** Either one of $A = \frac{1}{5}$ or $B = \frac{1}{5}$. **A1:** $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$ or any equivalent form, eg: $\frac{1}{5P} + \frac{1}{25-5P}$, etc. Ignore subsequent working. This answer must be stated in part (a) only. A1 can also be given for a candidate who finds both $A = \frac{1}{5}$ and $B = \frac{1}{5}$ and $\frac{A}{P} + \frac{B}{5-P}$ is seen in their working. Candidate can use 'cover-up' rule to write down $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$, as so gain all three marks. Candidate cannot gain the marks for part (a) in part (b).	
(b)	**B1:** Separates variables as shown. dP and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1*: Both** $\pm\lambda\ln P$ **and** $\pm\mu\ln(\pm 5 \pm P)$, where λ and μ are constants. **Or** $\pm\lambda\ln mP$ and $\pm\mu\ln(n(\pm 5 \pm P))$, where λ, μ, m and n are constants. **A1ft:** Correct follow through integration of both sides from their $\int \frac{\lambda}{P} + \frac{\mu}{(5-P)}\,\mathrm{d}P = \int K\,\mathrm{d}t$ with or without $+c$ **dM1*:** Use of $t = 0$ and $P = 1$ in an integrated equation containing c **dM1*:** Using ANY of the subtraction (or addition) laws for logarithms CORRECTLY. **dM1*:** Apply logarithms (or take exponentials) to eliminate ln's CORRECTLY from their equation. **dM1*:** A full ACCEPTABLE method of rearranging to make P the subject. (**See below for examples!**) **A1:** $P = \frac{5}{(1 + 4\mathrm{e}^{-\frac{1}{3}t})}$ $\{$where $a = 5, b = 1, c = 4\}$. Also allow any "integer" multiples of this expression. For example: $P = \frac{25}{(5 + 20\mathrm{e}^{-\frac{1}{3}t})}$ **Note: If the first method mark (M1*) is not awarded then the candidate cannot gain any of the six remaining marks for this part of the question.** **Note:** $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \int \frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \ln P - \ln(5-P) = 15t$ is B0M1A1ft. ***dM1* for making P the subject*** **Note there are three type of manipulations here which are considered acceptable to make *P* the subject.** (1) M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \;\Rightarrow\; P(1 + \mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t} \Rightarrow P = \frac{5}{(1 + \mathrm{e}^{-\frac{1}{3}t})}$ (2) M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5-P}{P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} - 1 = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} = \mathrm{e}^{\frac{1}{3}t} + 1 \Rightarrow P = \frac{5}{(1 + \mathrm{e}^{\frac{1}{3}t})}$ (3) M1 for $P(5-P) = 4\mathrm{e}^{\frac{1}{3}t} \Rightarrow P^2 - 5P = -4\mathrm{e}^{\frac{1}{3}t} \Rightarrow \left(P - \frac{5}{2}\right)^2 - \frac{25}{4} = -4\mathrm{e}^{\frac{1}{3}t}$ leading to $P = \ldots$ **Note**: The incorrect manipulation of $\frac{P}{5-P} = \frac{P}{5} - 1$ or equivalent is awarded this dM0*. **Note:** $(P) - (5-P) = \mathrm{e}^{\frac{1}{3}t} \;\Rightarrow\; 2P - 5 = \frac{1}{3}t$ leading to $P = \ldots$ or equivalent is awarded this dM0*	
(c)	**B1:** $1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. **For** $P = \frac{25}{(5 + 20\mathrm{e}^{-\frac{1}{3}t})}$, B1 can be awarded for $5 + 20\mathrm{e}^{-\frac{1}{3}t} > 5$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. B1 can only be obtained if candidates have correct values of a and b in their $P = \frac{a}{(b + c\mathrm{e}^{-\frac{1}{3}t})}$. **Award B0 for:** As $t \to \infty$, $\mathrm{e}^{-\frac{1}{3}t} \to 0$. So $P \to \frac{5}{(1+0)} = 5$, so population cannot exceed 5000, **unless** the candidate also proves that $P = \frac{5}{(1 + 4\mathrm{e}^{-\frac{1}{3}t})}$ oe. is an increasing function. **If unsure here, then send to review!**	
	Alternative method for part (b) **B1M1*A1:** as before for $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \;\; (+c)$ Award 3$^{\text{rd}}$ M1for $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t + c$ Award 4$^{\text{th}}$ M1 for $\frac{P}{5-P} = A\mathrm{e}^{\frac{1}{3}t}$ Award 2$^{\text{nd}}$ M1 for $t = 0, P = 1 \;\Rightarrow\; \frac{1}{5-1} = A\mathrm{e}^{0} \quad \left\{\Rightarrow \;\; A = \frac{1}{4}\right\}$ $\frac{P}{5-P} = \frac{1}{4}\mathrm{e}^{\frac{1}{3}t}$ then award the final M1A1 in the same way.	

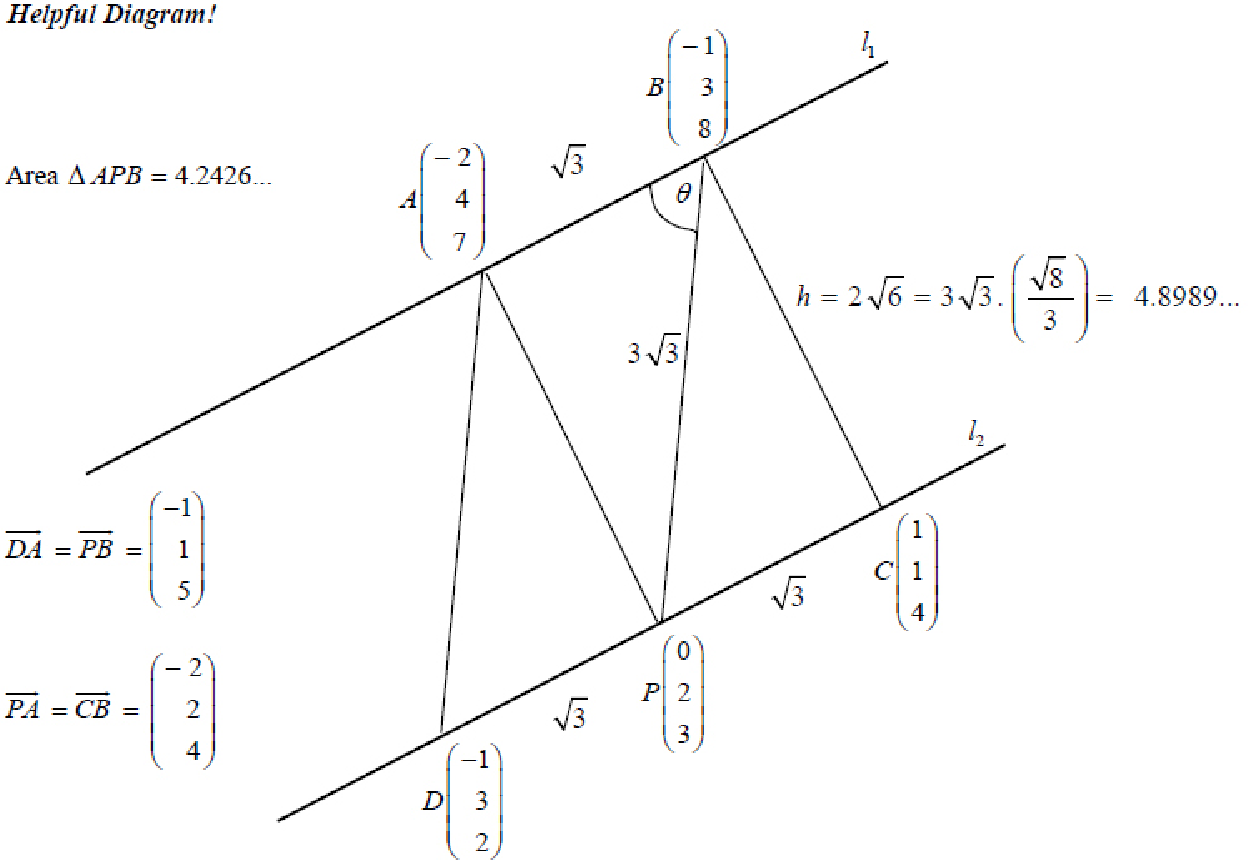
Q4.

Question Number	Scheme	Marks
(a)	$\frac{5}{(x-1)(3x+2)} = \frac{A}{x-1} + \frac{B}{3x+2}$	
	$5 = A(3x+2) + B(x-1)$	
	$x \to 1$ $\quad 5 = 5A \ \Rightarrow \ A = 1$	M1 A1
	$x \to -\frac{2}{3}$ $\quad 5 = -\frac{5}{3}B \ \Rightarrow \ B = -3$	A1 (3)
(b)	$\int \frac{5}{(x-1)(3x+2)} \mathrm{d}x = \int \left(\frac{1}{x-1} - \frac{3}{3x+2} \right) \mathrm{d}x$	
	$= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants	M1 A1ft A1ft
		(3)
(c)	$\int \frac{5}{(x-1)(3x+2)} \mathrm{d}x = \int \left(\frac{1}{y} \right) \mathrm{d}y$	M1
	$\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$	M1 A1
	$y = \frac{K(x-1)}{3x+2}$ depends on first two Ms in (c)	M1 dep
	Using $(2, 8)$ $\quad 8 = \frac{K}{8}$ depends on first two Ms in (c)	M1 dep
	$y = \frac{64(x-1)}{3x+2}$	A1 (6)
		[12]

Q5.

Question Number	Scheme	Marks
(a)	$\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$, $\overrightarrow{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overrightarrow{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{AB} = = \pm((-\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k})); = \mathbf{i} - \mathbf{j} + \mathbf{k}$	M1; A1 [2]
(b)	$\{l_1 : \mathbf{r}\} = \begin{pmatrix}-2\\4\\7\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix}-1\\3\\8\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	B1ft [1]
(c)	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$	M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{PB}\rvert} = \dfrac{\begin{pmatrix}1\\-1\\1\end{pmatrix} \bullet \begin{pmatrix}-1\\1\\5\end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2} . \sqrt{(-1)^2 + (1)^2 + (5)^2}}$ Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{PB}$ or $\overrightarrow{BP})$.	M1
	$\{\cos\theta\} = \dfrac{-1-1+5}{\sqrt{3}.\sqrt{27}} = \dfrac{3}{9} = \underline{\dfrac{1}{3}}$ Correct proof	A1 cso [3]
(d)	$\{l_2 : \mathbf{r} =\} \begin{pmatrix}0\\2\end{pmatrix} + \mu\begin{pmatrix}1\\-1\end{pmatrix}$ $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$, $\mathbf{p} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{p} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$. Correct vector equation.	M1 A1 ft [2]
(e)	$\overrightarrow{OC} = \begin{pmatrix}0\\2\\3\end{pmatrix} + \begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\overrightarrow{OD} = \begin{pmatrix}0\\2\\3\end{pmatrix} - \begin{pmatrix}1\\-1\\1\end{pmatrix}$ Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$	M1
	$\{C(1, 1, 4),\ D(-1, 3, 2)\}$ At least one set of coordinates are correct. Both sets of coordinates are correct.	A1 ft A1 ft [3]
(f) **Way 1**	$\dfrac{h}{\sqrt{(-1)^2 + (1)^2 + (5)^2}} = \sin\theta$ $\dfrac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$	M1
	$h = \sqrt{27}\sin(70.5...)\ \left\{= \sqrt{27}\dfrac{\sqrt{8}}{3} = 2\sqrt{6} = \text{awrt } 4.9\right\}$ $\sqrt{27}\sin(70.5...)$ or $\sqrt{27} . \dfrac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent	A1 oe
	Area $ABCD = \dfrac{1}{2} 2\sqrt{6}\left(\sqrt{3} + 2\sqrt{3}\right)$ $\dfrac{1}{2}$(their h)(their AB + their CD)	dM1
	$\left\{= \dfrac{1}{2} 2\sqrt{6}\left(3\sqrt{3}\right) = 3\sqrt{18}\right\} = \underline{9\sqrt{2}}$ $9\sqrt{2}$	A1 cao [4] 15

(f) ***Helpful Diagram!***



	$\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$, so $BC \perp AB$	Candidates do not need to prove this result for part (f)	
(f) **Way 2**	$h = \lvert\overrightarrow{CB}\rvert = \sqrt{(-2)^2 + (2)^2 + (4)^2} = \sqrt{24} = 2\sqrt{6} = 4.8989...$	Attempts $\lvert\overrightarrow{PA}\rvert$ or $\lvert\overrightarrow{CB}\rvert$	M1
		$\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$	A1 oe
	Area $ABCD = \frac{1}{2}\sqrt{24}\left(\sqrt{3} + 2\sqrt{3}\right)$ or $\frac{1}{2}\sqrt{24}\sqrt{3} + \sqrt{24}\sqrt{3}$	$\frac{1}{2}h(\text{their } AB + \text{their } CD)$	dM1 oe
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 cso
			[4]
Way3 (f)	**Finds the area of either triangle *APB* or *APD* or *BCP* and triples the result.**		
	Area $\Delta APB = \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\theta$	Attempts $\frac{1}{2}$ (their AB)(their PB) $\sin\theta$	M1
	$= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$	$\frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$ or $3\sqrt{2}$ or awrt 4.24 or equivalent	A1
	Area $ABCD = 3\left(3\sqrt{2}\right)$	3 × Area of ΔAPB	dM1
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 cso
			[4]

	Question	Notes
. (a)	**M1**	Finding the difference (either way) between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	A1	$\mathbf{i}-\mathbf{j}+\mathbf{k}$ or $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $(1,-1,1)$ or benefit of the doubt $\begin{matrix}1\\-1\\1\end{matrix}$
(b)	**B1ft**	$\{\mathbf{r}\}=\begin{pmatrix}-2\\4\\7\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\{\mathbf{r}\}=\begin{pmatrix}-1\\3\\8\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$, with $\overrightarrow{AB}$ or $\overrightarrow{BA}$ correctly followed through from (a).
	Note	$\mathbf{r}=$ is not needed.
(c)	**M1**	An attempt to find either the vector $\overrightarrow{PB}$ or $\overrightarrow{BP}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	M1	Applies dot product formula between their $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB}\text{ or }\overrightarrow{BP}\right)$.
	A1	Obtains $\{\cos\theta\}=\frac{1}{3}$ ***by correct solution only.***
	Note	If candidate starts by applying $\frac{\overrightarrow{AB}\bullet\overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert.\lvert\overrightarrow{PB}\rvert}$ correctly (without reference to $\cos\theta=\ldots$) they can gain both 2nd M1 and A1 mark.
	Note	Award the final A1 mark if candidate achieves $\{\cos\theta\}=\frac{1}{3}$ by either taking the dot product between (i) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ or (ii) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly.
	Note	Award final A0, cso for those candidates who take the dot product between (iii) $\begin{pmatrix}1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\end{pmatrix}$ or (iv) $\begin{pmatrix}-1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\end{pmatrix}$ They will usually find $\{\cos\theta\}=-\frac{1}{3}$ or may fudge $\{\cos\theta\}=\frac{1}{3}$. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso

(c)	**<u>Alternative Method 1: The Cosine Rule</u>**		
	$\overrightarrow{PB}=\overrightarrow{OB}-\overrightarrow{OP}=\begin{pmatrix}-1\\3\\8\end{pmatrix}-\begin{pmatrix}0\\2\\3\end{pmatrix}=\begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP}=\begin{pmatrix}1\\-1\\-5\end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Note $\lvert\overrightarrow{PB}\rvert=\sqrt{27}$, $\lvert\overrightarrow{AB}\rvert=\sqrt{3}$ and $\lvert\overrightarrow{PA}\rvert=\sqrt{24}$		
	$\left(\sqrt{24}\right)^2=\left(\sqrt{27}\right)^2+\left(\sqrt{3}\right)^2-2\left(\sqrt{27}\right)\left(\sqrt{3}\right)\cos\theta$	Applies the cosine rule the correct way round	M1 oe
	$\cos\theta=\frac{27+3-24}{18}=\frac{1}{3}$	Correct proof	A1 cso
			[3]

(c)	**Alternative Method 2: Right-Angled Trigonometry**		
	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Either $\left(\sqrt{24}\right)^2 + \left(\sqrt{3}\right)^2 = \left(\sqrt{27}\right)^2$ or $\overrightarrow{AB} \bullet \overrightarrow{PA} = \begin{pmatrix}1\\-1\\1\end{pmatrix} \bullet \begin{pmatrix}-2\\2\\4\end{pmatrix} = -2-2+4=0$	Confirms ΔPAB is right-angled	M1
	So, $\left\{\cos\theta = \frac{AB}{PB} \Rightarrow\right\} \cos\theta = \frac{\sqrt{3}}{\sqrt{27}} = \frac{1}{3}$	Correct proof	A1 cso
			[3]

(d)	**M1**	Writing down a line in the form $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$ with either $\mathbf{a} = \begin{pmatrix}0\\2\\3\end{pmatrix}$ or $\mathbf{d}$ = their $\overrightarrow{AB}$ $\mathbf{d}$ = their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$ found in part (a).
	A1ft	Writing $\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d}$ = their $\overrightarrow{AB}$ or a multiple of their $\overrightarrow{AB}$ found in part (a).
	Note	$\mathbf{r} =$ is not needed.
	Note	Using the same scalar parameter as in part (b) is fine for A1.
(e)	**M1**	Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$.
	A1ft	At least one set of coordinates are correct. Ignore labelling of C, D
	A1ft	Both sets of coordinates are correct. Ignore labelling of C, D
	Note	You can follow through either or both accuracy marks in this part using their $\overrightarrow{AB}$ from part (a).
(f)	**M1**	Way 1: $\frac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$ Way 2: Attempts $\lvert\overrightarrow{PA}\rvert$ or $\lvert\overrightarrow{CB}\rvert$ Way 3: Attempts $\frac{1}{2}$(their PB)(their AB)$\sin\theta$
	Note	Finding AD by itself is M0.
	A1	Either • $h = \sqrt{27}\sin(70.5...)$ or $\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$ or equivalent. (See Way 1 and Way 2) or • the area of either triangle APB or APD or BDP $= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\left(70.5...\right)$ o.e. (See Way 3).
	dM1	**which is dependent on the 1st M1 mark.** A full method to find the area of trapezium $ABCD$. (See Way 1, Way 2 and Way 3).
	A1	$9\sqrt{2}$ from a correct solution only.
	Note	A decimal answer of 12.7279... (without a correct **exact** answer) is A0.

Q6.

Question Number	Scheme	Marks
	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$	M1
	$\theta = \frac{\pi}{3}$ awrt 1.05	A1 **(2)**
	(b) $\frac{dx}{d\theta} = \sec^2\theta,\ \frac{dy}{d\theta} = \cos\theta$	
	$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta}\quad \left(= \cos^3\theta\right)$	M1 A1
	At P, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$ Can be implied	A1
	Using $mm' = -1$, $m' = -8$	M1
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	M1
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$ 1.0625	A1 **(6)**
	(c) $\int y^2\,dx = \int y^2 \frac{dx}{d\theta}\,d\theta = \int \sin^2\theta \sec^2\theta\,d\theta$	M1 A1
	$= \int \tan^2\theta\,d\theta$	A1
	$= \int \left(\sec^2\theta - 1\right)d\theta$	M1
	$= \tan\theta - \theta \quad (+C)$	A1
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\,dx = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - (0 - 0)\right]$	M1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2 \qquad \left(p = 1, q = -\frac{1}{3}\right)$	A1 **(7)**
		[15]

Q7.

Question Number	Scheme		Marks
	Working parametrically:		
	$x=1-\frac{1}{2}t,\quad y=2^t-1$ or $y=\mathrm{e}^{t\ln 2}-1$		
(a)	$\{x=0\Rightarrow\}\ 0=1-\frac{1}{2}t\Rightarrow t=2$	Applies $x=0$ to obtain a value for t.	M1
	When $t=2$, $y=2^2-1=3$	Correct value for y.	A1
			[2]
(b)	$\{y=0\Rightarrow\}\ 0=2^t-1\Rightarrow t=0$	Applies $y=0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t=0$, $x=1-\frac{1}{2}(0)=1$	$x=1$	A1
			[2]
(c)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{1}{2}$ and either $\frac{\mathrm{d}y}{\mathrm{d}t}=2^t\ln 2$ or $\frac{\mathrm{d}y}{\mathrm{d}t}=\mathrm{e}^{t\ln 2}\ln 2$		B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2^t\ln 2}{-\frac{1}{2}}$	Attempts their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$.	M1
	At A, $t=\text{"2"}$, so $m(\mathbf{T})=-8\ln 2\Rightarrow m(\mathbf{N})=\frac{1}{8\ln 2}$	Applies $t=\text{"2"}$ and $m(\mathbf{N})=\frac{-1}{m(\mathbf{T})}$	M1
	$y-3=\frac{1}{8\ln 2}(x-0)$ or $y=3+\frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso
			[5]
(d)	$\text{Area}(R)=\int\left(2^t-1\right).\left(-\frac{1}{2}\right)\mathrm{d}t$	Complete substitution for both y and $\mathrm{d}x$	M1
	$x=-1\rightarrow t=4$ and $x=1\rightarrow t=0$		B1
	$=\left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2}-t\right)$	Either $2^t\rightarrow\frac{2^t}{\ln 2}$ or $\left(2^t-1\right)\rightarrow\frac{(2^t)}{\pm\alpha(\ln 2)}-t$ or $\left(2^t-1\right)\rightarrow\pm\alpha(\ln 2)(2^t)-t$	M1*
		$\left(2^t-1\right)\rightarrow\frac{2^t}{\ln 2}-t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2}-t\right]_4^0\right\}=-\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right)-\left(\frac{16}{\ln 2}-4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$=\frac{15}{2\ln 2}-2$	$\frac{15}{2\ln 2}-2$ or equivalent.	A1
			[6] 15
(a)	**M1:** Applies $x=0$ and obtains a value of t. **A1:** For $y=2^2-1=3$ or $y=4-1=3$ ***Alternative Solution 1:*** **M1:** For substituting $t=2$ into either x or y. **A1:** $x=1-\frac{1}{2}(2)=0$ **and** $y=2^2-1=3$ ***Alternative Solution 2:*** **M1:** Applies $y=3$ and obtains a value of t. **A1:** For $x=1-\frac{1}{2}(2)=0$ or $x=1-1=0$. ***Alternative Solution 3:*** **M1:** Applies $y=3$ or $x=0$ and obtains a value of t. **A1:** Shows that $t=2$ for both $y=3$ and $x=0$.		
(b)	**M1:** Applies $y=0$ and obtains a value of t. Working must be seen in part (b). **A1:** For finding $x=1$. **Note:** Award M1A1 for $x=1$.		
(c)	**B1:** Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. This mark can be implied by later working. **M1:** Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ or their $\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{1}{\text{their}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$. **Note:** their $\frac{\mathrm{d}y}{\mathrm{d}t}$ must be a function of t. **M1:** Uses their value of t found in part (a) and applies $m(\mathbf{N})=\frac{-1}{m(\mathbf{T})}$. **M1:** $y-3=(\text{their normal gradient})x$ **or** $y=(\text{their normal gradient})x+3$ or equivalent. **A1:** $y-3=\frac{1}{8\ln 2}(x-0)$ or $y=3+\frac{1}{8\ln 2}x$ or $y-3=\frac{1}{\ln 256}(x-0)$ or $(8\ln 2)y-24\ln 2=x$ or $\frac{y-3}{(x-0)}=\frac{1}{8\ln 2}$. You can apply isw here. **Working in decimals** is ok for the three method marks. B1, A1 require exact values.		
(d)	**M1:** Complete substitution for both y and $\mathrm{d}x$. So candidate should write down $\int\left(2^t-1\right).\left(\text{their }\frac{\mathrm{d}x}{\mathrm{d}t}\right)$ **B1:** Changes limits from $x\rightarrow t$. $x=-1\rightarrow t=4$ and $x=1\rightarrow t=0$. Note $t=4$ and $t=0$ seen is B1. **M1*:** Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$ ... or integrates $\left(2^t-1\right)$ to give either $\frac{(2^t)}{\pm\alpha(\ln 2)}-t$ or $\pm\alpha(\ln 2)(2^t)-t$. **A1:** Correct integration of $\left(2^t-1\right)$ with respect to t to give $\frac{2^t}{\ln 2}-t$. **dM1*:** **Depends upon the previous method mark.** Substitutes their limits in t and subtracts either way round. **A1:** Exact answer of $\frac{15}{2\ln 2}-2$ or $\frac{15}{\ln 4}-2$ or $\frac{15-4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2}-2$ or $\frac{15}{2}\log_2 \mathrm{e}-2$ or equivalent.		
	<u>Alternative: Converting to a Cartesian equation:</u>		
	$t=2-2x\Rightarrow\quad y=2^{2-2x}-1$		
(a)	$\{x=0\Rightarrow\}\ y=2^2-1$	Applies $x=0$ in their Cartesian equation...	M1
	$y=3$	... to arrive at a correct answer of 3.	A1
			[2]
(b)	$\{y=0\Rightarrow\}\ 0=2^{2-2x}-1\Rightarrow 0=2-2x\Rightarrow x=\ldots$	Applies $y=0$ to obtain a value for x. (Must be seen in part (b)).	M1
	$x=1$	$x=1$	A1
			[2]
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\left(2^{2-2x}\right)\ln 2$	$\pm\lambda 2^{2-2x},\ \lambda\neq 1$	M1
		$-2\left(2^{2-2x}\right)\ln 2$ or equivalent	A1
	At A, $x=0$, so $m(\mathbf{T})=-8\ln 2\Rightarrow m(\mathbf{N})=\frac{1}{8\ln 2}$	Applies $x=0$ and $m(\mathbf{N})=\frac{-1}{m(\mathbf{T})}$	M1
	$y-3=\frac{1}{8\ln 2}(x-0)$ or $y=3+\frac{1}{8\ln 2}x$ or equivalent.	***As in the original scheme.***	M1 A1 oe
			[5]
(d)	$\text{Area}(R)=\int\left(2^{2-2x}-1\right)\mathrm{d}x$	Form the integral of their Cartesian equation of C.	M1
	$=\int_{-1}^{1}\left(2^{2-2x}-1\right)\mathrm{d}x$	For $2^{2-2x}-1$ with limits of $x=-1$ and $x=1$. Ie. $\int_{-1}^{1}\left(2^{2-2x}-1\right)$	B1
	$=\left(\frac{2^{2-2x}}{-2\ln 2}-x\right)$	Either $2^{2-2x}\rightarrow\frac{2^{2-2x}}{-2\ln 2}$ or $\left(2^{2-2x}-1\right)\rightarrow\frac{2^{2-2x}}{\pm\alpha(\ln 2)}-x$ or $\left(2^{2-2x}-1\right)\rightarrow\pm\alpha(\ln 2)(2^{2-2x})-x$	M1*
		$\left(2^{2-2x}-1\right)\rightarrow\frac{2^{2-2x}}{-2\ln 2}-x$	A1
	$\left\{\left[\frac{2^{2-2x}}{-2\ln 2}-x\right]_{-1}^{1}\right\}=\left(\left(\frac{1}{-2\ln 2}-1\right)-\left(\frac{16}{-2\ln 2}+1\right)\right)$	**Depends on the previous method mark.** Substitutes limits of -1 and their x_B and subtracts either way round.	dM1*
	$=\frac{15}{2\ln 2}-2$	$\frac{15}{2\ln 2}-2$ or equivalent.	A1
			[6] 15
(d)	**<u>Alternative method: In Cartesian and applying $u=2-2x$</u>** $\text{Area}(R)=\int\left(2^u-1\right)\{\mathrm{d}x\}$, where $u=2-2x$ $=\int_4^0\left(2^u-1\right)\left(-\tfrac{1}{2}\right)\{\mathrm{d}u\}$	**M0:** Unless a candidate ***writes*** $\int\left(2^{2-2x}-1\right)\{\mathrm{d}x\}$ Then apply the “working parametrically” mark scheme.	
(d)	***Alternative method: For substitution*** $\mathbf{u=2^t}$		
	$\text{Area}(R)=\int\left(2^t-1\right).\left(-\frac{1}{2}\right)\mathrm{d}t$	Complete substitution for both y and $\mathrm{d}x$	M1
	where $u=2^t\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}t}=2^t\ln 2\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}t}=u\ln 2$		
	$x=-1\rightarrow t=4\rightarrow u=16$ and $x=1\rightarrow t=0\rightarrow u=1$	Both correct limits in t or both correct limits in u.	B1
	So $\text{area}(R)=-\frac{1}{2}\int\frac{u-1}{u\ln 2}\mathrm{d}u$ $=-\frac{1}{2}\int\frac{1}{\ln 2}-\frac{1}{u\ln 2}\mathrm{d}u$	If not awarded above, you can award M1 for this integral	
	$=\left\{-\frac{1}{2}\right\}\left(\frac{u}{\ln 2}-\frac{\ln u}{\ln 2}\right)$	Either $2^t\rightarrow\frac{u}{\ln 2}$ or $\left(2^t-1\right)\rightarrow\frac{u}{\pm\alpha(\ln 2)}-\frac{\ln u}{\ln 2}$ or $\left(2^t-1\right)\rightarrow\pm\alpha(\ln 2)(u)-\frac{\ln u}{\ln 2}$	M1*
		$\left(2^t-1\right)\rightarrow\frac{u}{\ln 2}-\frac{\ln u}{\ln 2}$	A1
	$\left\{-\frac{1}{2}\left[\frac{u}{\ln 2}-\frac{\ln u}{\ln 2}\right]_{16}^{1}\right\}=-\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right)-\left(\frac{16}{\ln 2}-\frac{\ln 16}{\ln 2}\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits ***in u*** and subtracts either way round.	dM1*
	$=\frac{15}{2\ln 2}-\frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2}-2$	$\frac{15}{2\ln 2}-\frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2}-2$ or equivalent.	A1
			[6]

Q8.

Question Number	Scheme	Marks
Q (a)	$\int \sin^2\theta \, \mathrm{d}\theta = \frac{1}{2}\int (1-\cos 2\theta)\, \mathrm{d}\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1 (2)
(b)	$x = \tan\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = \sec^2\theta$	
	$\pi\int y^2 \, \mathrm{d}x = \pi\int y^2 \frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta \, \mathrm{d}\theta$	M1 A1
	$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta}\mathrm{d}\theta$	M1
	$= 16\pi\int \sin^2\theta \, \mathrm{d}\theta$ $\qquad k = 16\pi$	A1
	$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1 (5)
	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta \, \mathrm{d}\theta\right)$	
(c)	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1
	$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ Use of correct limits	M1
	$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3}$ $\qquad p = \frac{4}{3},\ q = -2$	A1 (3)
		[10]

Q9.

Question Number	Scheme		Marks
(a)	$\frac{d\theta}{dt} = \lambda(120-\theta)$, $\theta \leqslant 100$		
	$\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$ or $\int \frac{1}{\lambda(120-\theta)}\,d\theta = \int dt$		B1
	$-\ln(120-\theta); = \lambda t + c$ or $-\frac{1}{\lambda}\ln(120-\theta); = t + c$	See notes	M1 A1; M1 A1
	$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$	See notes	M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$		
	then either... $-\lambda t = \ln(120-\theta) - \ln 100$ $-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$ $e^{-\lambda t} = \frac{120-\theta}{100}$ *or...* $\lambda t = \ln 100 - \ln(120-\theta)$ $\lambda t = \ln\left(\frac{100}{120-\theta}\right)$ $e^{\lambda t} = \frac{100}{120-\theta}$		dddM1
	$100e^{-\lambda t} = 120-\theta$ *or...* $(120-\theta)e^{\lambda t} = 100$ $\Rightarrow 120-\theta = 100e^{-\lambda t}$ leading to $\theta = 120 - 100e^{-\lambda t}$		A1 *
			[8]
(b)	$\{\lambda = 0.01,\ \theta = 100 \Rightarrow\}\quad 100 = 120 - 100e^{-0.01t}$		M1
	$\Rightarrow 100e^{-0.01t} = 120 - 100 \Rightarrow -0.01t = \ln\left(\frac{120-100}{100}\right)$ $t = \frac{1}{-0.01}\ln\left(\frac{120-100}{100}\right)$	Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = ...$ and $t = A\ln B$, where $B > 0$	dM1
	$\left\{t = \frac{1}{-0.01}\ln\left(\frac{1}{5}\right) = 100\ln 5\right\}$		
	$t = 160.94379... = 161$ (s) (nearest second)	awrt 161	A1
			[3] 11

Notes for Question		
(a)	**B1:** Separates variables as shown. $\mathrm{d}\theta$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. *Either* **M1:** $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$ **A1:** $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln(120-\theta)$ **M1:** $\int \lambda\,\mathrm{d}t \to \lambda t$ *or* $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$, A is a constant. $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120-\theta)$ or $-\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$, $\int 1\,\mathrm{d}t \to t$ **A1:** $\int \lambda\,\mathrm{d}t \to \lambda t + c$ or $\int 1\,\mathrm{d}t \to t + c$ The $+c$ can appear on either side of the equation. **IMPORTANT:** $+c$ can be on either side of their equation for the 2nd A1 mark. **M1:** Substitutes $t = 0$ AND $\theta = 20$ in an integrated or changed equation containing c (or A or $\ln A$). **Note** that this mark can be implied by the correct value of c. $\{$ Note that $-\ln 100 = -4.60517...\}$. **dddM1:** Uses their value of c which must be a ln term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on all three previous method marks being awarded. **A1*:** This is a given answer. All previous marks must have been scored and there must not be any errors in the candidate's working. Do not accept huge leaps in working at the end. So a minimum of either: (1): $e^{-\lambda t} = \frac{120-\theta}{100} \Rightarrow 100e^{-\lambda t} = 120 - \theta \Rightarrow \theta = 120 - 100e^{-\lambda t}$ or (2): $e^{\lambda t} = \frac{100}{120-\theta} \Rightarrow (120-\theta)e^{\lambda t} = 100 \Rightarrow 120-\theta = 100e^{-\lambda t} \Rightarrow \theta = 120 - 100e^{-\lambda t}$ is required for A1. **Note:** $\int \frac{1}{(120\lambda-\lambda\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$ is ok for the first M1A1 in part (a).	
(b)	**M1:** Substitutes $\lambda = 0.01$ and $\theta = 100$ into the printed equation or one of their earlier equations connecting θ and t. This mark can be implied by subsequent working. **dM1:** Candidate uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to $t = ...$ **Note:** that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (b). **A1:** awrt 161 or "awrt" 2 minutes 41 seconds. (Ignore incorrect units).	
Aliter **(a)** **Way 2**	$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$	B1
	$-\ln(120-\theta) = \lambda t + c$ See notes	M1 A1; M1 A1
	$-\ln(120-\theta) = \lambda t + c$ $\ln(120-\theta) = -\lambda t + c$ $120 - \theta = Ae^{-\lambda t}$ $\theta = 120 - Ae^{-\lambda t}$	
	$\{t = 0, \theta = 20 \Rightarrow\}\ 20 = 120 - Ae^{0}$	M1
	$A = 120 - 20 = 100$ So, $\theta = 120 - 100e^{-\lambda t}$	dddM1 A1 * [8]

Notes for Question Continued			
(a)	**B1M1A1M1A1:** Mark as in the original scheme. **M1:** Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. **Note** that this mark can be implied by the correct value of c or A. **dddM1:** Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration. **Note:** This mark is dependent on all three previous method marks being awarded. **Note:** $\ln(120-\theta) = -\lambda t + c$ leading to $120 - \theta = \mathrm{e}^{-\lambda t} + \mathrm{e}^{c}$ or $120 - \theta = \mathrm{e}^{-\lambda t} + A$, would be dddM0. **A1*:** Same as the original scheme. **Note:** The jump from $\ln(120-\theta) = -\lambda t + c$ to $120 - \theta = A\mathrm{e}^{-\lambda t}$ ***with no incorrect working*** is condoned in part (a).		
Aliter (a) Way 3	$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t \quad \left\{\Rightarrow \int \frac{-1}{\theta-120}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t\right\}$		B1
	$-\ln\lvert\theta-120\rvert = \lambda t + c$	*Modulus required for 1st A1.*	M1 **A1** M1 A1
	$\{t = 0,\ \theta = 20 \Rightarrow\}\ -\ln\lvert 20-120\rvert = \lambda(0) + c$	*Modulus not required here!*	M1
	$\Rightarrow c = -\ln 100 \Rightarrow -\ln\lvert\theta-120\rvert = \lambda t - \ln 100$		

then either...	*or...*
$-\lambda t = \ln\lvert\theta-120\rvert - \ln 100$	$\lambda t = \ln 100 - \ln\lvert\theta-120\rvert$
$-\lambda t = \ln\left\lvert\frac{\theta-120}{100}\right\rvert$	$\lambda t = \ln\left\lvert\frac{100}{\theta-120}\right\rvert$
As $\theta \leqslant 100$	
$-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$	$\lambda t = \ln\left(\frac{100}{120-\theta}\right)$
$\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100}$	$\mathrm{e}^{\lambda t} = \frac{100}{120-\theta}$
$100\mathrm{e}^{-\lambda t} = 120-\theta$	$(120-\theta)\mathrm{e}^{\lambda t} = 100$ $\Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t}$
leading to $\theta = 120 - 100\mathrm{e}^{-\lambda t}$	

Understanding of modulus is required here! **dddM1**

A1 *

[8]

B1: Mark as in the original scheme.
M1: Mark as in the original scheme ignoring the modulus.
A1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \rightarrow -\ln\lvert\theta-120\rvert$. ***(The modulus is required here).***
M1A1: Mark as in the original scheme.
M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Mark as in the original scheme ignoring the modulus.
dddM1: Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert\theta-120\rvert$ to $\ln(120-\theta)$ in their working. **Note:** This mark is dependent on all three previous method marks being awarded.
A1: Mark as in the original scheme.

Notes for Question Continued		
Aliter (a) Way 4	*Use of an integrating factor*	
	$\frac{d\theta}{dt} = \lambda(120 - \theta) \Rightarrow \frac{d\theta}{dt} + \lambda\theta = 120\lambda$	
	$\text{IF} = e^{\lambda t}$	B1
	$\frac{d}{dt}\left(e^{\lambda t}\theta\right) = 120\lambda e^{\lambda t}$,	M1A1
	$e^{\lambda t}\theta = 120\lambda e^{\lambda t} + k$	M1A1
	$\theta = 120 + Ke^{-\lambda t}$	M1
	$\{t = 0 , \theta = 20 \Rightarrow\} -100 = K$	
	$\theta = 120 - 100e^{-\lambda t}$	M1A1

Q10.

Question Number	Scheme		Marks
	$\int y\,dy = \int \frac{3}{\cos^2 x}\,dx$ $= \int 3\sec^2 x\,dx$	Can be implied. Ignore integral signs	B1
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1
	$y = 2, x = \frac{\pi}{4}$ $\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$		M1
	Leading to $C = -1$		
	$\frac{1}{2}y^2 = 3\tan x - 1$	or equivalent	A1 (5) [5]

Q11.

Question Number	Scheme	Marks
	$\frac{dV}{dt}=80\pi$, $V=4\pi h(h+4)=4\pi h^2+16\pi h$, $\frac{dV}{dh}=8\pi h+16\pi$ $\quad$ $\pm\alpha h\pm\beta,\ \alpha\neq 0,\ \beta\neq 0$ $8\pi h+16\pi$	M1 A1
	$\left\{\frac{dV}{dh}\times\frac{dh}{dt}=\frac{dV}{dt}\Rightarrow\right\}$ $(8\pi h+16\pi)\frac{dh}{dt}=80\pi$ $\quad$ $\left(\text{Candidate's }\frac{dV}{dh}\right)\times\frac{dh}{dt}=80\pi$ $\left\{\frac{dh}{dt}=\frac{dV}{dt}\div\frac{dV}{dh}\Rightarrow\right\}$ $\frac{dh}{dt}=80\pi\times\frac{1}{8\pi h+16\pi}$ $\quad$ or $80\pi\div\text{Candidate's }\frac{dV}{dh}$	M1 oe
	When $h=6$, $\left\{\frac{dh}{dt}=\right\}\frac{1}{8\pi(6)+16\pi}\times 80\pi$ $\left\{=\frac{80\pi}{64\pi}\right\}$ $\quad$ **dependent on the previous M1 see notes** $\frac{dh}{dt}=\underline{1.25}\ (\text{cms}^{-1})$ $\quad$ 1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	dM1 A1 oe **[5]** **5**
	Alternative Method for the first M1A1 Product rule: $\left\{\begin{array}{ll} u=4\pi h & v=h+4 \\ \frac{du}{dh}=4\pi & \frac{dv}{dh}=1\end{array}\right\}$ $\frac{dV}{dh}=4\pi(h+4)+4\pi h$ $\quad$ $\pm\alpha h\pm\beta,\ \alpha\neq 0,\ \beta\neq 0$ $4\pi(h+4)+4\pi h$	M1 A1

Question Notes

	M1	An expression of the form $\pm\alpha h\pm\beta,\ \alpha\neq 0,\ \beta\neq 0$. **Can be simplified or un-simplified.**
	A1	Correct simplified or un-simplified differentiation of V. eg. $8\pi h+16\pi$ or $4\pi(h+4)+4\pi h$ or $8\pi(h+2)$ or equivalent.
	Note	Some candidates will use the product rule to differentiate V with respect to h. **(See Alt Method 1).**
	Note	$\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.
	M1	$\left(\text{Candidate's }\frac{dV}{dh}\right)\times\frac{dh}{dt}=80\pi$ or $80\pi\div\text{Candidate's }\frac{dV}{dh}$
	Note	Also allow 2nd M1 for $\left(\text{Candidate's }\frac{dV}{dh}\right)\times\frac{dh}{dt}=\mathbf{80}$ or $\mathbf{80}\div\text{Candidate's }\frac{dV}{dh}$
	Note	Give 2nd M0 for $\left(\text{Candidate's }\frac{dV}{dh}\right)\times\frac{dh}{dt}=\mathbf{80\pi t}$ **or 80k** or **80πt or 80k** $\div$ Candidate's $\frac{dV}{dh}$
	dM1	**which is dependent on the previous M1 mark.** Substitutes $h=6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and 80π (or 80)
	A1	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
	Note	$\frac{80\pi}{64\pi}$ as a final answer is A0.
	Note	Substituting $h=6$ into a correct $\frac{dV}{dh}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)

Q12.

Question Number	Scheme	Marks
(a)	From question, $V=\frac{4}{3}\pi r^3$, $S=4\pi r^2$, $\frac{\mathrm{d}V}{\mathrm{d}t}=3$ $\left\{V=\frac{4}{3}\pi r^3 \Rightarrow\right\}\ \frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ $\qquad$ $\frac{\mathrm{d}V}{\mathrm{d}r}=4\cancel{\pi}r^2$ (Can be implied)	B1 oe
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\Rightarrow\right\}\ \left(4\pi r^2\right)\frac{\mathrm{d}r}{\mathrm{d}t}=3$ $\qquad$ $\left(\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}r}\right)\times\frac{\mathrm{d}r}{\mathrm{d}t}=3$ $\left\{\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}r}\Rightarrow\right\}\ \frac{\mathrm{d}r}{\mathrm{d}t}=(3)\frac{1}{4\pi r^2};\ \left\{=\frac{3}{4\pi r^2}\right\}$ $\qquad$ or $3\div\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}r}$;	M1 oe
	When $r=4\text{cm}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{3}{4\pi(4)^2}\ \left\{=\frac{3}{64\pi}\right\}$ $\qquad$ **dependent on previous M1.** **see notes**	dM1
	Hence, $\frac{\mathrm{d}r}{\mathrm{d}t}=0.01492077591...(\text{cm}^2\ \text{s}^{-1})$ $\qquad$ anything that rounds to 0.0149	A1 **[4]**
(b)	$\left\{\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{\mathrm{d}S}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=\right\}\Rightarrow\frac{\mathrm{d}S}{\mathrm{d}t}=\ 8\pi r\times\frac{3}{4\pi r^2}$ $\left\{\text{or }\frac{6}{r}\ \text{ or }8\pi r\times 0.0149...\right\}$ $\qquad$ $8\pi r\times\text{Candidate's }\frac{\mathrm{d}r}{\mathrm{d}t}$ When $r=4\text{cm}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=8\pi(4)\times\frac{3}{4\pi(4)^2}$ or $\frac{6}{4}$ or $8\pi(4)\times 0.0149...$	M1; oe
	Hence, $\frac{\mathrm{d}S}{\mathrm{d}t}=\ 1.5\ (\text{cm}^2\ \text{s}^{-1})$ $\qquad$ anything that rounds to 1.5	A1 cso **[2]** **6**

	Question Notes	
(a)	**B1**	$\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ Can be implied by later working.
	M1	$\left(\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}r}\right)\times\frac{\mathrm{d}r}{\mathrm{d}t}=3$ or $3\div\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}r}$
	dM1	**(dependent on the previous method mark)** Substitutes $r=4$ into an expression which is a result of a quotient of "3" and their $\frac{\mathrm{d}V}{\mathrm{d}r}$.
	A1	anything that rounds to 0.0149 (units are not required)
(b)	**M1**	$8\pi r\times\text{Candidate's }\frac{\mathrm{d}r}{\mathrm{d}t}$
	A1	anything that rounds to 1.5 (units are not required). **Correct solution only.**
	Note	Using $\frac{\mathrm{d}r}{\mathrm{d}t}=0.0149$ gives $\frac{\mathrm{d}S}{\mathrm{d}t}=\ 1.4979...$ which is fine for A1.

Q13.

Question Number	Scheme	Marks
	(a) $V=\pi r^2 h \quad \left(\text{or base }\left(\pi r^2\right)\times \text{ height}\right)$	B1 (1)
	As $h=r$, $V=\pi r^3$ *	
	(b) $\frac{dV}{dr}=3\pi r^2$	B1 (1)
	(c) $V=\int \frac{2t}{2+t^2}dt=\ln\left(2+t^2\right)+C$ Require C for the A	M1 A1
	$t=0, V=3 \Rightarrow 3=\ln 2+C$	M1
	$V=\ln\left(2+t^2\right)-\ln 2+3$	A1 (4)
	(d) $V=\ln 3-\ln 2+3 \quad (=3.405\,46\,...)$	M1 A1
	$r=\sqrt[3]{\frac{1}{\pi}(\ln 3-\ln 2+3)}\approx 1.03$ awrt 1.03	M1 A1 (4)
	(e) $\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}$	M1
	$=\frac{2t}{3\pi r^2\left(2+t^2\right)}$	A1 (2)
	(f) $\frac{dr}{dt}=\frac{2}{9\pi r^2}\approx 0.0670$ awrt 0.067	M1 A1 (2)
		(14)

Q14.

Question Number	Scheme	Marks
	(a) $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$	M1 A1
	$V = 9\pi h \quad \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = 9\pi \frac{\mathrm{d}h}{\mathrm{d}t}$	B1
	$9\pi \frac{\mathrm{d}h}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$	M1
	Leading to $75\frac{\mathrm{d}h}{\mathrm{d}t} = 4 - 5h$ * cso	A1 **(5)**
	(b) $\int \frac{75}{4-5h}\,\mathrm{d}h = \int 1\,\mathrm{d}t$ separating variables	M1
	$-15\ln(4-5h) = t \quad (+C)$	M1 A1
	$-15\ln(4-5h) = t + C$	
	When $t = 0$, $h = 0.2$	
	$-15\ln 3 = C$	M1
	$t = 15\ln 3 - 15\ln(4-5h)$	
	When $h = 0.5$	
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	M1 A1
	Alternative for last 3 marks	
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$	
	$= -15\ln 1.5 + 15\ln 3$	M1 M1
	$= 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	A1 **(6)**

Q15.

Question Number	Scheme	Marks
(a)	$\int x\sin 3x\,\mathrm{d}x = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,\{\mathrm{d}x\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\,\{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x\,\mathrm{d}x = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\,\{\mathrm{d}x\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)\ \{+c\}$	A1 isw
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x\ \{+c\}\right\}$ *Ignore subsequent working*	**[3]**
		6
(a)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \to u' = 1$ and $v' = \sin 3x \to v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$). This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant. If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,\{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$. **A1:** $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without + c. Can be un-simplified.	
(b)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \to u' = 2x$ or x and $v' = \cos 3x \to v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$). This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$ or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$. If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\,\{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$. **A1:** $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without + c, can be un-simplified. You can ignore subsequent working here. **Special Case:** If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}(\text{their follow through part(a) answer})$.	

Q16.

Question Number	Scheme		Marks
	Volume $= \underline{\pi \int_0^2 \left(\sqrt{\left(\frac{2x}{3x^2+4}\right)}\right)^2 \, \mathrm{d}x}$	Use of $V = \underline{\pi \int y^2} \, \mathrm{d}x$.	$\underline{\text{B1}}$
	$= (\pi)\left[\frac{1}{3}\ln\left(3x^2+4\right)\right]_0^2$	$\pm k\ln\left(3x^2+4\right)$	M1
		$\frac{1}{3}\ln\left(3x^2+4\right)$	A1
	$= (\pi)\left[\left(\frac{1}{3}\ln 16\right)-\left(\frac{1}{3}\ln 4\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{1}{3}\pi\ln 4}$	$\underline{\frac{1}{3}\pi\ln 4}$ or $\underline{\frac{2}{3}\pi\ln 2}$	A1 oe isw
			[5] 5

NOTE: π is required for the B1 mark and the final A1 mark. It is not required for the 3 intermediate marks.

B1: For applying $\pi\int y^2$. Ignore limits and $\mathrm{d}x$. This can be implied by later working,

but the pi and $\int \frac{2x}{3x^2+4}$ must appear on one line somewhere in the candidate's working.

B1 can also be implied by a correct final answer. **Note:** $\pi\left(\int y\right)^2$ would be B0.

Working in x

M1: For $\pm k\ln\left(3x^2+4\right)$ or $\pm k\ln\left(x^2+\frac{4}{3}\right)$ where k is a constant and k can be 1.

Note: M0 for $\pm k\,x\ln\left(3x^2+4\right)$.

Note: M1 can also be given for $\pm k\ln\left(p\left(3x^2+4\right)\right)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln\left(3x^2+4\right)$ or $\frac{1}{3}\ln\left(\frac{1}{3}\left(3x^2+4\right)\right)$ or $\frac{1}{3}\ln\left(x^2+\frac{4}{3}\right)$ or $\frac{1}{3}\ln\left(p(3x^2+4)\right)$.

You may allow M1 A1 for $\frac{1}{3}\left(\frac{x}{x}\right)\ln\left(3x^2+4\right)$ or $\frac{1}{3}\left(\frac{2x}{6x}\right)\ln\left(3x^2+4\right)$

dM1: Substitutes limits of 2 and 0 and subtracts the correct way round. Working in decimals is fine for dM1.

A1: For either $\frac{1}{3}\pi\ln 4$, $\frac{1}{3}\ln 4^{\pi}$, $\frac{2}{3}\pi\ln 2$, $\pi\ln 4^{\frac{1}{3}}$, $\pi\ln 2^{\frac{2}{3}}$, $\frac{1}{3}\pi\ln\left(\frac{16}{4}\right)$, $2\pi\ln\left(\frac{16^{\frac{1}{6}}}{4^{\frac{1}{6}}}\right)$, etc.

Note: $\frac{1}{3}\pi(\ln 16 - \ln 4)$ would be A0.

Working in u: where $u = 3x^2+4$,

M1: For $\pm k\ln u$ where k is a constant and k can be 1.

Note: M1 can also be given for $\pm k\ln(pu)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln u$ or $\frac{1}{3}\ln 3u$ or $\frac{1}{3}\ln pu$.

dM1: Substitutes limits of 16 and 4 in u or limits of 2 and 0 in x and subtracts the correct way round.

A1: As above!

Q17.

Question Number	Scheme		Marks
(a)	$\int \tan^2 x\ \mathrm{d}x$		
	$\left[NB: \underline{\sec^2 A = 1+\tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1}\right]$	The correct <u>underlined identity</u>.	M1 oe
	$= \int \sec^2 x - 1\ \mathrm{d}x$		
	$= \underline{\tan x\ -\ x}\ (+\ c)$	Correct integration with/without + c	A1
			(2)
(b)	$\int \frac{1}{x^3}\ln x\ \mathrm{d}x$		
	$\left\{\begin{array}{lll} u = \ln x & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = x^{-3} & \Rightarrow & v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array}\right\}$		
	$= -\frac{1}{2x^2}\ln x - \int -\frac{1}{2x^2}.\frac{1}{x}\ \mathrm{d}x$	Use of 'integration by parts' formula in the correct direction. Correct direction means that $u = \ln x$.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\int \frac{1}{x^3}\ \mathrm{d}x$	An attempt to multiply through $\frac{k}{x^n}, n \in \square\ , n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)}(+\ c)$	... "integrate"(process the result);	M1
		<u>correct solution</u> with/without + c	A1 oe
			(4)

Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x}\ \mathrm{d}x$		
	$\left\{u = 1+e^x \Rightarrow \underline{\frac{\mathrm{d}u}{\mathrm{d}x} = e^x},\ \underline{\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{e^x}},\ \underline{\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{u-1}}\right\}$	Differentiating to find any one of the <u>three underlined</u>	<u>B1</u>
	$= \int \frac{e^{2x}.e^x}{1+e^x}\ \mathrm{d}x = \int \frac{(u-1)^2.e^x}{u}.\frac{1}{e^x}\ \mathrm{d}u$ or $= \int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\ \mathrm{d}u$	Attempt to substitute for $e^{2x} = \mathrm{f}(u)$, their $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{e^x}$ and $u = 1+e^x$ or $e^{3x} = \mathrm{f}(u)$, their $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{u-1}$ and $u = 1+e^x$.	M1*
	$= \int \frac{(u-1)^2}{u}\ \mathrm{d}u$	$\underline{\int \frac{(u-1)^2}{u}\ \mathrm{d}u}$	A1
	$= \int \frac{u^2-2u+1}{u}\ \mathrm{d}u$ $= \int u - 2 + \frac{1}{u}\ \mathrm{d}u$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u\ \ (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$	Substitutes $u = 1+e^x$ back into their integrated expression with at least two terms.	dM1*
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ **AG**	$\underline{\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a $+c$ and " $-\frac{3}{2}$ " combined.	A1 cso
			(7)
			[13]

Q18.

Question Number	Scheme	Marks
	$\int x\sin 2x\,dx = -\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx$ $= \quad \ldots \quad + \frac{\sin 2x}{4}$ $[\ \ldots\ ]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	M1 A1 A1 M1 M1 A1 **[6]**

Q19.

Question Number	Scheme	Marks
(a)	$x = 3 \Rightarrow y = 0.1847$ awrt $x = 5 \Rightarrow y = 0.1667$ awrt or $\frac{1}{6}$	B1 B1 (2)
(b)	$I \approx \frac{1}{\underline{\underline{2}}}\left[0.2 + 0.1667 + 2(0.1847 + 0.1745)\right]$ ≈ 0.543 0.542 or 0.543	<u>B1</u> M1 A1ft A1 (4)
(c)	$\frac{dx}{du} = 2(u-4)$ $\int \frac{1}{4+\sqrt{(x-1)}}\,dx = \int \frac{1}{u} \times 2(u-4)\,du$ $= \int \left(2 - \frac{8}{u}\right) du$ $= 2u - 8\ln u$ $x = 2 \Rightarrow u = 5,\ x = 5 \Rightarrow u = 6$ $[2u - 8\ln u]_5^6 = (12 - 8\ln 6) - (10 - 8\ln 5)$ $= 2 + 8\ln\left(\frac{5}{6}\right)$	B1 M1 A1 M1 A1 B1 M1 A1 (8) **[14]**

Q20.

Question Number	Scheme		Marks	
	$\frac{du}{dx} = -\sin x$		B1	
	$\int \sin x\, e^{\cos x+1} dx = -\int e^u du$		M1 A1	
	$= -e^u$	ft sign error	A1ft	
	$= -e^{\cos x+1}$			
	$\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - \left(-e^2\right)$	or equivalent with u	M1	
	$= e(e-1)$ ✱	cso	A1	**(6)**
				[6]

Q21.

Question Number	Scheme		Marks	
	(a) $f(\theta) = 4\cos^2\theta - 3\sin^2\theta$			
	$= 4\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)$		M1 M1	
	$= \frac{1}{2}+\frac{7}{2}\cos 2\theta$ ✱	cso	A1	**(3)**
	(b) $\int \theta\cos 2\theta\, d\theta = \frac{1}{2}\theta\sin 2\theta - \frac{1}{2}\int \sin 2\theta\, d\theta$		M1 A1	
	$= \frac{1}{2}\theta\sin 2\theta + \frac{1}{4}\cos 2\theta$		A1	
	$\int \theta f(\theta) d\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta\sin 2\theta + \frac{7}{8}\cos 2\theta$		M1 A1	
	$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16}+0-\frac{7}{8}\right] - \left[0+0+\frac{7}{8}\right]$		M1	
	$= \frac{\pi^2}{16} - \frac{7}{4}$		A1	**(7)**
				[10]

Q22.

Question Number	Scheme	Marks
	(a) $\frac{\mathrm{d}x}{\mathrm{d}u}=-2\sin u$	B1
	$\int\frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x=\int\frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}}\times-2\sin u\,\mathrm{d}u$	M1
	$=\int\frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\,\mathrm{d}u$ Use of $1-\cos^2 u=\sin^2 u$	M1
	$=-\frac{1}{4}\int\frac{1}{\cos^2 u}\,\mathrm{d}u$ $\pm k\int\frac{1}{\cos^2 u}\,\mathrm{d}u$	M1
	$=-\frac{1}{4}\tan u\ \ (+C)$ $\pm k\tan u$	M1
	$x=\sqrt{2}\ \Rightarrow\sqrt{2}=2\cos u\ \Rightarrow\ u=\frac{\pi}{4}$	
	$x=1\ \Rightarrow 1=2\cos u\ \Rightarrow\ u=\frac{\pi}{3}$	M1
	$\left[-\frac{1}{4}\tan u\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}=-\frac{1}{4}\left(\tan\frac{\pi}{4}-\tan\frac{\pi}{3}\right)$	
	$=-\frac{1}{4}\left(1-\sqrt{3}\right)\ \ \left(=\frac{\sqrt{3}-1}{4}\right)$	A1 (7)
	(b) $V=\pi\int_{1}^{\sqrt{2}}\left(\frac{4}{x\left(4-x^2\right)^{\frac{1}{4}}}\right)^2\mathrm{d}x$	M1
	$=16\pi\int_{1}^{\sqrt{2}}\frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x$ $16\pi\times$ integral in (a)	M1
	$=16\pi\left(\frac{\sqrt{3}-1}{4}\right)$ $16\pi\times$ their answer to part (a)	A1ft (3)
		[10]

Q23.

Question Number	Scheme	Marks
Q (a)	$\int \sqrt{(5-x)}\,\mathrm{d}x = \int (5-x)^{\frac{1}{2}}\,\mathrm{d}x = \dfrac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} \quad (+C)$	M1 A1 (2)
	$\left(= -\dfrac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$	
(b)	(i) $\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x = -\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \dfrac{2}{3}\int (5-x)^{\frac{3}{2}}\,\mathrm{d}x$	M1 A1ft
	$= \quad \ldots \quad + \dfrac{2}{3} \times \dfrac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} \quad (+C)$	M1
	$= -\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}} \quad (+C)$	A1 (4)
	(ii) $\left[-\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5 = (0-0) - \left(0 - \dfrac{4}{15} \times 4^{\frac{5}{2}}\right)$	
	$= \dfrac{128}{15}\left(= 8\dfrac{8}{15} \approx 8.53\right)$ awrt 8.53	M1 A1 (2)
		[8]

Q24.

Question Number	Scheme		
(a)	$\int \frac{1}{x^3}\ln x\,\mathrm{d}x$, $\left\{\begin{matrix} u=\ln x & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x}=x^{-3} & \Rightarrow & v=\frac{x^{-2}}{-2}=\frac{-1}{2x^2} \end{matrix}\right\}$		
		In the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu\frac{1}{x^2}.\frac{1}{x}$	M1
	$=\underline{\frac{-1}{2x^2}\ln x}\ \underline{\underline{-\int\frac{-1}{2x^2}.\frac{1}{x}}}\,\mathrm{d}x$	$\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified.	$\underline{\text{A1}}$
		$\underline{\underline{-\int\frac{-1}{2x^2}.\frac{1}{x}}}$ simplified or un-simplified.	$\underline{\underline{\text{A1}}}$
	$\left\{=\ \frac{-1}{2x^2}\ln x+\frac{1}{2}\int\frac{1}{x^3}\ \mathrm{d}x\right\}$		
	$=-\frac{1}{2x^2}\ln x+\frac{1}{2}\left(-\frac{1}{2x^2}\right)\ \{+c\}$	$\pm\int\mu\frac{1}{x^2}.\frac{1}{x}\ \rightarrow \pm\beta x^{-2}.$	dM1
		Correct answer, with/without $+\ c$	A1
			[5]
(b)	$\left\{\left[-\frac{1}{2x^2}\ln x-\frac{1}{4x^2}\right]_1^2\right\}=\left(-\frac{1}{2(2)^2}\ln 2-\frac{1}{4(2)^2}\right)-\left(-\frac{1}{2(1)^2}\ln 1-\frac{1}{4(1)^2}\right)$	Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round.	M1
	$=\frac{3}{16}-\frac{1}{8}\ln 2$ or $\frac{3}{16}-\ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3-2\ln 2)$, etc, or awrt 0.1	or equivalent.	A1
			[2] 7
(a)	**M1**: Integration by parts is applied in the form $\frac{\pm\lambda}{x^2}\ln x\pm\int\mu\frac{1}{x^2}.\frac{1}{x}$ or equivalent. $\underline{\textbf{A1}}$: $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified. $\underline{\underline{\textbf{A1}}}$: $\underline{\underline{-\int\frac{-1}{2x^2}.\frac{1}{x}}}$ or equivalent. You can ignore the $\mathrm{d}x$. **dM1**: Depends on the previous M1. $\pm\int\mu\frac{1}{x^2}.\frac{1}{x}\ \rightarrow\pm\beta x^{-2}.$ **A1**: $-\frac{1}{2x^2}\ln x+\frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$ or $=-\frac{1}{2x^2}\ln x-\frac{1}{4x^2}\{+c\}$ or $\frac{x^{-2}}{-2}\ln x-\frac{x^{-2}}{4}\{+c\}$ or $\frac{-1-2\ln x}{4x^2}\{+c\}$ or equivalent. You can ignore subsequent working after a correct stated answer.		
(b)	**M1**: Some evidence of applying limits of 2 and 1 to their part (a) answer and subtracts the correct way round. **A1**: *Two term exact answer* of either $\frac{3}{16}-\frac{1}{8}\ln 2$ or $\frac{3}{16}-\ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3-2\ln 2)$ or $\frac{\ln\left(\frac{1}{4}\right)+3}{16}$ or $0.1875-0.125\ln 2$. Also allow awrt 0.1. Also note the fraction terms must be combined. **Note**: Award the final A0 in part (b) for a candidate who achieves awrt 0.1 in part (b), when their answer to part (a) is incorrect.		
(b) ctd	**Note**: Decimal answer is 0.100856... in part (b).		
	<u>Alternative Solution</u>		
	$\int\frac{1}{x^3}\ln x\,\mathrm{d}x$, $\left\{\begin{matrix} u=x^{-3} & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x}=-3x^{-4} \\ \frac{\mathrm{d}v}{\mathrm{d}x}=\ln x & \Rightarrow & v=x\ln x-x \end{matrix}\right\}$		
	$\int\frac{1}{x^3}\ln x\,\mathrm{d}x=\frac{1}{x^3}(x\ln x-x)-\int(x\ln x-x)\frac{-3}{x^4}\mathrm{d}x$		
		$k\int\frac{1}{x^3}\ln x\,\mathrm{d}x=\frac{1}{x^3}(x\ln x-x)\pm\int\frac{\lambda}{x^3}\mathrm{d}x$ where $k\neq 1$	M1
	$-2\int\frac{1}{x^3}\ln x\,\mathrm{d}x=\frac{1}{x^3}(x\ln x-x)-\int\frac{3}{x^3}\mathrm{d}x$	Any one of $\frac{1}{x^3}(x\ln x-x)$ or $-\int\frac{3}{x^3}\mathrm{d}x$	A1
		$\frac{1}{x^3}(x\ln x-x)-\int\frac{3}{x^3}\mathrm{d}x$ and $k=-2$	A1
	$-2\int\frac{1}{x^3}\ln x\,\mathrm{d}x=\frac{1}{x^3}(x\ln x-x)+\frac{3}{2x^2}\ \{+c\}$	$\pm\int\mu\frac{1}{x^3}\ \rightarrow\pm\beta x^{-2}.$	dM1
	$\int\frac{1}{x^3}\ln x\,\mathrm{d}x=-\frac{1}{2x^3}(x\ln x-x)-\frac{3}{4x^2}\ \{+c\}$	$-\frac{1}{2x^3}(x\ln x-x)-\frac{3}{4x^2}$ or equivalent with/without $+\ c$.	A1
	$=-\frac{1}{2x^2}\ln x-\frac{1}{4x^2}\{+c\}$		

Q25.

Question Number	Scheme	Marks
(a)	$\{y=0\Rightarrow\}\ 1-2\cos x=0$ $1-2\cos x=0$, seen or implied.	M1
	At least one correct value of x. (See notes).	A1
	$\Rightarrow x=\frac{\pi}{3},\frac{5\pi}{3}$ Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$	A1 **cso**
		[3]
(b)	$V=\pi\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}}(1-2\cos x)^2\,\mathrm{d}x$ For $\pi\int(1-2\cos x)^2$. Ignore limits and $\mathrm{d}x$	B1
	$\left\{\int(1-2\cos x)^2\,\mathrm{d}x\right\}=\int(1-4\cos x+4\cos^2 x)\mathrm{d}x$ $=\int 1-4\cos x+4\left(\frac{1+\cos 2x}{2}\right)\mathrm{d}x$ $\cos 2x=2\cos^2 x-1$ See notes. $=\int(3-4\cos x+2\cos 2x)\,\mathrm{d}x$	M1
	Attempts $\int y^2$ to give any two of $\pm A\to\pm Ax$, $\pm B\cos x\to\pm B\sin x$ or $\pm\lambda\cos 2x\to\pm\mu\sin 2x$. $=3x-4\sin x+\frac{2\sin 2x}{2}$	M1
	Correct integration.	A1
	$V=\{\pi\}\left(\left(3(\frac{5\pi}{3})-4\sin(\frac{5\pi}{3})+\frac{2\sin(\frac{10\pi}{3})}{2}\right)-\left(3(\frac{\pi}{3})-4\sin(\frac{\pi}{3})+\frac{2\sin(\frac{2\pi}{3})}{2}\right)\right)$ Applying limits the correct way round. Ignore π.	ddM1
	$=\pi\left(\left(5\pi+2\sqrt{3}-\frac{\sqrt{3}}{2}\right)-\left(\pi-2\sqrt{3}+\frac{\sqrt{3}}{2}\right)\right)$ $=\pi((18.3060...)-(0.5435...))=17.7625\pi=55.80$	
	$=\pi(4\pi+3\sqrt{3})$ or $4\pi^2+3\pi\sqrt{3}$ Two term exact answer.	A1
		[6] **9**
(a)	**M1:** $1-2\cos x=0$. This can be implied by either $\cos x=\frac{1}{2}$ or any one of the correct values for x in radians or in degrees. **1st A1:** Any one of either $\frac{\pi}{3}$ or $\frac{5\pi}{3}$ or 60 or 300 or awrt 1.05 or 5.23 or awrt 5.24 . **2nd A1:** Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.	
(b)	**B1:** (M1 on epen) For $\pi\int(1-2\cos x)^2$. Ignore limits and $\mathrm{d}x$. **1st M1:** Any correct form of $\cos 2x=2\cos^2 x-1$ used or written down in the same variable. This can be implied by $\cos^2 x=\frac{1+\cos 2x}{2}$ or $4\cos^2 x\to 2+2\cos 2x$ or $\cos 2A=2\cos^2 A-1$. **2nd M1:** Attempts $\int y^2$ to give any two of $\pm A\to\pm Ax$, $\pm B\cos x\to\pm B\sin x$ or $\pm\lambda\cos 2x\to\pm\mu\sin 2x$. Do not worry about the signs when integrating $\cos x$ or $\cos 2x$ for this mark. Note: $\int(1-2\cos x)^2=\int 1+4\cos^2 x$ is ok for an attempt at $\int y^2$. **1st A1:** Correct integration. Eg. $3x-4\sin x+\frac{2\sin 2x}{2}$ or $x-4\sin x+\frac{2\sin 2x}{2}+2x$ oe. **3rd ddM1:** Depends on both of the two previous method marks. (Ignore π). Some evidence of substituting their $x=\frac{5\pi}{3}$ and their $x=\frac{\pi}{3}$ and subtracting the correct way round. You will need to use your calculator to check for correct substitution of their limits into their integrand if a candidate does not explicitly give **some evidence.** **Note:** For correct integral and limits decimals gives: $\pi((18.3060...)-(0.5435...))=17.7625\pi=55.80$ **2nd A1:** ***Two term*** exact answer of either $\pi(4\pi+3\sqrt{3})$ or $4\pi^2+3\pi\sqrt{3}$ or equivalent. **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** Decimal answer of 58.802... without correct exact answer is A0. **Note:** Applying $\int(1-2\cos x)\,\mathrm{d}x$ will usually be given no marks in this part.	

Q26.

Question Number	Scheme		Marks
(a)	$\int x^2e^x\,dx$, 1st Application: $\left\{\begin{matrix} u=x^2 & \Rightarrow & \frac{du}{dx}=2x \\ \frac{dv}{dx}=e^x & \Rightarrow & v=e^x \end{matrix}\right\}$, 2nd Application: $\left\{\begin{matrix} u=x & \Rightarrow & \frac{du}{dx}=1 \\ \frac{dv}{dx}=e^x & \Rightarrow & v=e^x \end{matrix}\right\}$		
	$= x^2e^x - \int 2xe^x dx$	$x^2e^x - \int \lambda xe^x\{dx\},\ \lambda > 0$	M1
		$x^2e^x - \int 2xe^x\{dx\}$	A1 oe
	$= x^2e^x - 2\left(xe^x - \int e^x dx\right)$	**Either** $\pm Ax^2e^x \pm Bxe^x \pm C\int e^x\{dx\}$ **or** for $\pm K\int xe^x\{dx\} \to \pm K\left(xe^x - \int e^x\{dx\}\right)$	M1
	$= x^2e^x - 2(xe^x - e^x)\ \{+c\}$	$\pm Ax^2e^x \pm Bxe^x \pm Ce^x$	M1
		Correct answer, with/without $+c$	A1
			[5]
(b)	$\left\{\left[x^2e^x - 2(xe^x - e^x)\right]_0^1\right\}$ $= \left(1^2e^1 - 2(1e^1 - e^1)\right) - \left(0^2e^0 - 2(0e^0 - e^0)\right)$ $= e - 2$	Applies limits of 1 and 0 to an expression of the form $\pm Ax^2e^x \pm Bxe^x \pm Ce^x$, $A \neq 0$, $B \neq 0$ and $C \neq 0$ and subtracts the correct way round.	M1
		$e - 2$ cso	A1 oe
			[2] 7

Notes for Question

(a)	**M1:** Integration by parts is applied in the form $x^2e^x - \int \lambda xe^x\{dx\}$, where $\lambda > 0$. (must be in this form). **A1:** $x^2e^x - \int 2xe^x\{dx\}$ or equivalent. **M1:** **Either** achieving a result in the form $\pm Ax^2e^x \pm Bxe^x \pm C\int e^x\{dx\}$ (can be implied) (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **or** for $\pm K\int xe^x\{dx\} \to \pm K\left(xe^x - \int e^x\{dx\}\right)$ **M1:** $\pm Ax^2e^x \pm Bxe^x \pm Ce^x$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **A1:** $x^2e^x - 2(xe^x - e^x)$ or $x^2e^x - 2xe^x + 2e^x$ or $(x^2 - 2x + 2)e^x$ or equivalent with/without $+c$.
(b)	**M1:** **Complete method** of applying limits of 1 and 0 to their part (a) answer in the form $\pm Ax^2e^x \pm Bxe^x \pm Ce^x$, (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round. Evidence of a proper consideration of the limit of 0 (as detailed above) is needed for M1. So, just subtracting zero is M0. **A1:** $e - 2$ or $e^1 - 2$ or $-2 + e$. Do not allow $e - 2e^0$ unless simplified to give $e - 2$. **Note:** that 0.718... without seeing $e - 2$ or equivalent is A0. **WARNING:** Please note that this A1 mark is for correct solution only. So incorrect $[......]_0^1$ leading to $e - 2$ is A0. **Note:** If their part (a) is correct candidates can get M1A1 in part (b) for $e - 2$ from no working. **Note:** 0.718... from no working is M0A0

Q27.

Question Number	Scheme		Marks
	$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x$, $u = 2+\sqrt{(2x+1)}$		
	$\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = u-2$	Either $\frac{\mathrm{d}u}{\mathrm{d}x} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = \pm\lambda(u-2)$	M1
		Either $\frac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = (u-2)$	A1
	$\left\{\int \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\right\} = \int \frac{1}{u}(u-2)\,\mathrm{d}u$	Correct substitution (Ignore integral sign and $\mathrm{d}u$).	A1
	$= \int\left(1-\frac{2}{u}\right)\mathrm{d}u$	An attempt to divide each term by u.	dM1
		$\pm Au \pm B\ln u$	ddM1
	$= u - 2\ln u$	$u - 2\ln u$	A1 ft
	$\left\{\text{So } [u-2\ln u]_3^5\right\} = (5-2\ln 5)-(3-2\ln 3)$	Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.	M1
	$= 2+2\ln\left(\frac{3}{5}\right)$	$2+2\ln\left(\frac{3}{5}\right)$	A1 **cao cso**
			[8] 8

Notes for Question	
	M1: Also allow $\mathrm{d}u = \pm\lambda\frac{1}{(u-2)}\mathrm{d}x$ or $(u-2)\mathrm{d}u = \pm\lambda\,\mathrm{d}x$ **Note:** The expressions must contain $\mathrm{d}u$ and $\mathrm{d}x$. They can be simplified or un-simplified. **A1:** Also allow $\mathrm{d}u = \frac{1}{(u-2)}\mathrm{d}x$ or $(u-2)\mathrm{d}u = \pm\lambda\,\mathrm{d}x$ **Note:** The expressions must contain $\mathrm{d}u$ and $\mathrm{d}x$. They can be simplified or un-simplified. **A1:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u$. (Ignore integral sign and $\mathrm{d}u$). **dM1:** An attempt to divide each term by u. **Note** that this mark is dependent on the previous M1 mark being awarded. **Note** that this mark can be implied by later working. **ddM1:** $\pm Au \pm B\ln u$, $A \neq 0$, $B \neq 0$ **Note** that this mark is dependent on the two previous M1 marks being awarded. **A1ft:** $u - 2\ln u$ or $\pm Au \pm B\ln u$ being correctly followed through, $A \neq 0$, $B \neq 0$ **M1:** Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round. **A1:** cso and cao. $2+2\ln\left(\frac{3}{5}\right)$ or $2+2\ln(0.6)$, $\left(= A+2\ln B, \text{ so } A = 2, B = \frac{3}{5}\right)$ **Note:** $2-2\ln\left(\frac{3}{5}\right)$ is A0.

Notes for Question Continued	
ctd	**Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = u-2\ln u$ with no working is 2nd M1, 3rd M1, 3rd A1. but **Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = (u-2)\ln u$ with no working is 2nd M0, 3rd M0, 3rd A0.

Q28.

Question Number	Scheme		Marks
(a)	6.248046798... $=$ 6.248 (3dp)	6.248 or awrt 6.248	B1
			[1]
(b)	Area $\approx \frac{1}{2}\times 2\,;\times\underline{\left[3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223\right]}$		B1; <u>M1</u>
	$= 49.369 = 49.37$ (2 dp)	49.37 or awrt 49.37	A1
			[3]
(c)	$\left\{\int(4te^{-\frac{1}{3}t}+3)\,\mathrm{d}t\right\} = -12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}\,\{\mathrm{d}t\}$	$\pm Ate^{-\frac{1}{3}t} \pm B\int e^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A\neq 0$, $B\neq 0$	M1
		See notes.	A1
	$+\,3t$	$3\rightarrow 3t$	B1
	$= -12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}\ \{+\,3t\}$	$-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$	A1
	$\left[-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t} + 3t\right]_0^8 =$		
	$= \left(-12(8)e^{-\frac{1}{3}(8)} - 36e^{-\frac{1}{3}(8)} + 3(8)\right) - \left(-12(0)e^{-\frac{1}{3}(0)} - 36e^{-\frac{1}{3}(0)} + 3(0)\right)$	Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.	dM1
	$= \left(-96e^{-\frac{8}{3}} - 36e^{-\frac{8}{3}} + 24\right) - \left(0 - 36 + 0\right)$		
	$= 60 - 132e^{-\frac{8}{3}}$	$60 - 132e^{-\frac{8}{3}}$	A1
			[6]
(d)	Difference $= \left\lvert 60 - 132e^{-\frac{8}{3}} - 49.37\right\rvert = 1.458184439... = 1.46$ (2 dp)	1.46 or awrt 1.46	B1
			[1] **11**

	Notes for Question
(a)	**B1:** 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2}\times 2$ or 1 **M1:** <u>For structure of trapezium rule</u>$\left[\ \underline{\ldots\ldots\ldots\ldots}\ \right]$. Allow one miscopy of their values. **A1:** 49.37 or anything that rounds to 49.37 **Note:** It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37) <u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>**Note**</u>: actual area is 50.828… <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for $1 + 3 + 2\left(7.107 + 7.218 + \text{their } 6.248\right) + 5.223$ (nb: answer of 50.369).

	Notes for Question Continued
(b) ctd	*Alternative method for part (b): Adding individual trapezia* Area $\approx 2\times\left[\frac{3+7.107}{2}+\frac{7.107+7.218}{2}+\frac{7.218+6.248}{2}+\frac{6.248+5.223}{2}\right]= 49.369$ **B1:** 2 and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 49.37
(c)	**M1:** For $4te^{-\frac{1}{3}t} \to \pm Ate^{-\frac{1}{3}t} \pm B\int e^{-\frac{1}{3}t}\{dt\}$, $A \neq 0, B \neq 0$ **A1:** For $te^{-\frac{1}{3}t} \to \left(-3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ (some candidates lose the 4 and this is fine for the first A1 mark). or $4te^{-\frac{1}{3}t} \to 4\left(-3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ or $-12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}$ or $12\left(-te^{-\frac{1}{3}t} - \int -e^{-\frac{1}{3}t}\right)$ These results can be implied. They can be simplified or un-simplified. **B1:** $3 \to 3t$ or $3 \to 3x$ (bod) . **Note:** Award B0 for 3 integrating to $12t$ (implied), which is a common error when taking out a factor of 4. **Be careful** some candidates will factorise out 4 and have $4\left(....+\frac{3}{4}\right) \to 4\left(....+\frac{3}{4}t\right)$ which would then be fine for B1. **Note:** Allow B1 for $\int_0^8 3\,dt = 24$ **A1:** For correct integration of $4te^{-\frac{1}{3}t}$ to give $-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$ or $4\left(-3te^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t}\right)$ or equivalent. This can be simplified or un-simplified. **dM1:** Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round. **Note:** Evidence of a proper consideration of the limit of 0 (as detailed in the scheme) is needed for dM1. So, just subtracting zero is M0. **A1:** An exact answer of $60 - 132e^{-\frac{8}{3}}$. A decimal answer of 50.82818444... without a correct answer is A0. **Note:** A decimal answer of 50.82818444... without a correct exact answer is A0. **Note:** If a candidate gains M1A1B1A1 and then writes down 50.8 or awrt 50.8 with no method for substituting limits of 8 and 0, then award the final M1A0. **IMPORTANT:** that is fine for candidates to work in terms of *x* rather than *t* in part (c). **Note:** The "3*t*" is needed for B1 and the final A1 mark.
(d)	**B1:** 1.46 or awrt 1.46 or -1.46 or awrt -1.46. Candidates may give correct decimal answers of 1.458184439... or 1.459184439... **Note:** You can award this mark whether or not the candidate has answered part (c) correctly.

Q29.

Question Number	Scheme	Marks
	$x = 27\sec^3 t,\quad y = 3\tan t,\quad 0 \leqslant t \leqslant \frac{\pi}{3}$	
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 81\sec^2 t\sec t\tan t\,,\quad \frac{\mathrm{d}y}{\mathrm{d}t} = 3\sec^2 t$ At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct.	B1
	Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct.	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2 t}{81\sec^3 t\tan t}\left\{= \frac{1}{27\sec t\tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t}\right\}$ Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$	M1;
	At $t = \frac{\pi}{6},\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72}\left\{= \frac{3}{54} = \frac{1}{18}\right\}$ $\frac{4}{72}$	A1 cao cso
		[4]
(b)	$\left\{1 + \tan^2 t = \sec^2 t\right\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\frac{x}{27}\right)}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$	M1
	$\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow\ 9 + y^2 = x^{\frac{2}{3}}\ \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *	A1 * cso
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$ $a = 27$ and $b = 216$	B1
		[3]
(c)	$V = \pi\int_{27}^{125}\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2 \mathrm{d}x$ or $\pi\int_{27}^{125}\left(x^{\frac{2}{3}} - 9\right)\mathrm{d}x$ For $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\}\left[\frac{3}{5}x^{\frac{5}{3}} - 9x\right]_{27}^{125}$ Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\frac{3}{5}x^{\frac{5}{3}}$ oe	M1
	$\frac{3}{5}x^{\frac{5}{3}} - 9x$ oe	A1
	$= \{\pi\}\left(\left(\frac{3}{5}(125)^{\frac{5}{3}} - 9(125)\right) - \left(\frac{3}{5}(27)^{\frac{5}{3}} - 9(27)\right)\right)$ Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.	dM1
	$= \{\pi\}\left((1875 - 1125) - (145.8 - 243)\right)$	
	$= \frac{4236\pi}{5}$ or 847.2π $\frac{4236\pi}{5}$ or 847.2π	A1
		[5] **12**

Notes for Question

(a)	**B1:** At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. **Note:** that this mark can be implied from their working. **B1:** Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct. **Note:** that this mark can be implied from their working. **M1:** Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$, where both $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$ are trigonometric functions of t. **A1:** $\frac{4}{72}$ or any equivalent correct rational answer not involving surds. Allow $0.0\dot{5}$ with the recurring symbol.

Notes for Question Continued

Note: Please check that their $\frac{\mathrm{d}x}{\mathrm{d}t}$ is differentiated correctly.

Eg. Note that $x = 27\sec^3 t = 27(\cos t)^{-3} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = -81(\cos t)^{-2}(-\sin t)$ is correct.

(b)

M1: Either:

- Applying a correct trigonometric identity (usually $1 + \tan^2 t = \sec^2 t$) to give a Cartesian equation in x and y only.
- Starting from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.
- Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity.

A1*: For a correct proof of $y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$.

Note this result is printed on the Question Paper, so no incorrect working is allowed.

B1: Both $a = 27$ and $b = 216$. **Note** that $27 \leqslant x \leqslant 216$ is also fine for B1.

(c)

B1: For a correct statement of $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$. Ignore limits and $\mathrm{d}x$. Can be implied.

M1: Either integrates to give $\pm Ax^{\frac{5}{3}} \pm Bx$, $A \neq 0$, $B \neq 0$ or integrates $x^{\frac{2}{3}}$ correctly to give $\frac{3}{5}x^{\frac{5}{3}}$ oe

A1: $\frac{3}{5}x^{\frac{5}{3}} - 9x$ or. $\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} - 9x$ oe.

dM1: Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.
Note: that this mark is dependent upon the previous method mark being awarded.

A1: A correct exact answer of $\frac{4236\pi}{5}$ or 847.2π.

Note: The π in the volume formula is only required for the B1 mark and the final A1 mark.
Note: A decimal answer of 2661.557... without a correct exact answer is A0.
Note: If a candidate gains the first B1M1A1 and then writes down 2661 or awrt 2662 with no method for substituting limits of 125 and 27, then award the final M1A0.

(a)

Way 2

Alternative response using the Cartesian equation in part (a)

	$\left\{y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \Rightarrow\right\} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm Kx^{-\frac{1}{3}}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}$	M1
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ oe	A1
	At $t = \frac{\pi}{6}$, $x = 27\sec^3\left(\frac{\pi}{6}\right) = 24\sqrt{3}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(\left(24\sqrt{3}\right)^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}\left(24\sqrt{3}\right)^{-\frac{1}{3}}\right)$	Uses $t = \frac{\pi}{6}$ to find x and substitutes their x into an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.	dM1
	So, $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{3\sqrt{3}}\right) = \frac{1}{18}$	$\frac{1}{18}$	A1 cao cso

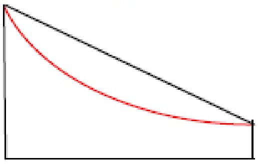
Note: Way 2 is marked as M1 A1 dM1 A1
Note: For way 2 the second M1 mark is dependent on the first M1 being gained.

	Notes for Question Continued		
(b) **Way 2**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE RHS**		
	$\{\text{RHS} =\}\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}=\sqrt{\left(27\sec^3 t\right)^{\frac{2}{3}}-9}=\sqrt{9\sec^2 t-9}=\sqrt{9\tan^2 t}$	For applying $1+\tan^2 t=\sec^2 t$ oe to achieve $\sqrt{9\tan^2 t}$	M1
	$=3\tan t=y\ \{=\text{LHS}\}$ **cso**	Correct proof from $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$ to y.	A1*
	M1: Starts from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.		
(b) **Way 3**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE LHS**		
	$\{\text{LHS} =\}\ y=3\tan t=\sqrt{\left(9\tan^2 t\right)}=\sqrt{9\sec^2 t-9}$	For applying $1+\tan^2 t=\sec^2 t$ oe to achieve $\sqrt{9\sec^2 t-9}$	M1
	$=\sqrt{9\left(\frac{x}{27}\right)^{\frac{2}{3}}-9}=\sqrt{9\left(\frac{x^{\frac{2}{3}}}{9}\right)-9}=\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$ **cso**	Correct proof from y to $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$.	A1*
	M1: Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t-9}$ by using a correct trigonometric identity.		
(c) **Way 2**	***Alternative response for part (c) using parametric integration***		
	$V=\pi\int 9\tan^2 t\left(81\sec^2 t\sec t\tan t\right)\,dt$	$\pi\int 3\tan t\left(81\sec^2 t\sec t\tan t\right)\,dt$ Ignore limits and dx. Can be implied.	B1
	$=\{\pi\}\int 729\sec^2 t\tan^2 t\sec t\tan t\ dt$		
	$=\{\pi\}\int 729\sec^2 t\left(\sec^2 t-1\right)\sec t\tan t\ dt$		
	$=\{\pi\}\int 729\left(\sec^4 t-\sec^2 t\right)\sec t\tan t\ dt$		
	$=\{\pi\}\int 729\left(\sec^4 t-\sec^2 t\right)\sec t\tan t\ dt$		
	$=\{\pi\}\left[729\left(\frac{1}{5}\sec^5 t-\frac{1}{3}\sec^3 t\right)\right]$	$\pm A\sec^5 t\pm B\sec^3 t$	M1
		$729\left(\frac{1}{5}\sec^5 t-\frac{1}{3}\sec^3 t\right)$	A1
	$V=\{\pi\}\left[729\left(\frac{1}{5}\left(\frac{5}{3}\right)^5-\frac{1}{3}\left(\frac{5}{3}\right)^3\right)-729\left(\frac{1}{5}1^5-\frac{1}{3}1^3\right)\right]$	Substitutes $\sec t=\frac{5}{3}$ and $\sec t=1$ into an integrated function and subtracts the correct way round.	dM1
	$=729\pi\left[\left(\frac{250}{243}\right)-\left(-\frac{2}{15}\right)\right]$		
	$=\frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1
			[5]

Q30.

Question Number	Scheme	Marks
	$\begin{array}{c\|c\|c\|c\|c} x & 1 & 2 & 3 & 4 \\ \hline y & 1.42857 & 0.90326 & \boldsymbol{0.682116...} & 0.55556 \end{array}$ $\quad y = \dfrac{10}{2x + 5\sqrt{x}}$	
(a)	$\{\text{At } x = 3,\}\ y = 0.68212$ (5 dp) 0.68212	B1 cao **[1]**
(b)	$\dfrac{1}{2}\times 1 \times \underline{\left[1.42857 + 0.55556 + 2\left(0.90326 + \text{their } 0.68212\right)\right]}$ Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$ <u>For structure of</u> $\underline{[..............]}$	B1 aef M1
	$\left\{= \frac{1}{2}(5.15489)\right\} = 2.577445 = 2.5774$ (4 dp) anything that rounds to 2.5774	A1 **[3]**
(c)	• Overestimate and a reason such as • {top of} <u>trapezia lie above the curve</u> • a diagram which gives reference to the extra area • concave or convex • $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ (can be implied) • bends inwards • curves downwards	B1 **[1]**
(d)	$\left\{u = \sqrt{x} \Rightarrow\right\} \dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u} = 2u$	B1
	$\displaystyle\int \frac{10}{2u^2 + 5u}.\, 2u\ \mathrm{d}u$ Either $\left\{\displaystyle\int\right\}\dfrac{\pm ku}{\alpha u^2 \pm \beta u}\ \{\mathrm{d}u\}$ or $\left\{\displaystyle\int\right\}\dfrac{\pm k}{u\left(\alpha u^2 \pm \beta u\right)}\ \{\mathrm{d}u\}$	M1
	$\left\{= \displaystyle\int \frac{20}{2u + 5}\ \mathrm{d}u\right\} = \dfrac{20}{2}\ln(2u + 5)$ $\pm\lambda\ln(2u + 5)$ or $\pm\lambda\ln\left(u + \frac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**	M1
	$\dfrac{20}{2u + 5} \rightarrow \dfrac{20}{2}\ln(2u + 5)$ or $10\ln\left(u + \dfrac{5}{2}\right)$	A1 cso
	$\left\{\left[\dfrac{20}{2}\ln(2u + 5)\right]_1^2\right\} = 10\ln\left(2(2) + 5\right) - 10\ln\left(2(1) + 5\right)$ Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1
	$10\ln 9 - 10\ln 7$ or $10\ln\left(\dfrac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$	A1 oe cso **[6]** **11**

	Question Notes	
(a)	**B1**	0.68212 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\dfrac{1}{2}\times 1$ or $\dfrac{1}{2}$ or equivalent.
	M1	<u>For structure of trapezium rule</u>[.............]
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	anything that rounds to 2.5774
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428…)

(b) contd	**Note**	Award B1M1A1 for $\frac{1}{2}(1.42857+0.55556)+(0.90326+\text{their }0.68212)=2.577445$ **<u>Bracketing mistake</u>: Unless the final answer implies that the calculation has been done correctly** award B1M0A0 for $\frac{1}{2}\times 1\ +1.42857+2(0.90326+\text{their }0.68212)+0.55556$ (nb: answer of 5.65489). award B1M0A0 for $\frac{1}{2}\times 1\ (1.42857+0.55556)+2(0.90326+\text{their }0.68212)$ (nb: answer of 4.162825). **<u>Alternative method: Adding individual trapezia</u>** $\text{Area}\approx 1\times\left[\frac{1.42857+0.90326}{2}+\frac{0.90326+\text{"}0.68212\text{"}}{2}+\frac{\text{"}0.68212\text{"}+0.55556}{2}\right]=2.577445$
	B1	**B1:** 1 and a divisor of 2 on all terms inside brackets.
	M1	**M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1	**A1:** anything that rounds to 2.5774
(c)	**B1**	Overestimate **and** either trapezia lie above curve **or** a diagram that gives reference to the extra area **eg.** This diagram is sufficient. It must show the top of a trapezium lying above the curve.  **or** concave **or** convex **or** $\frac{d^2y}{dx^2}>0$ (can be implied) **or** bends inwards **or** curves downwards.
	Note	Reason of "gradient is negative" by itself is B0
(d)	**B1**	$\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or $du=\frac{1}{2\sqrt{x}}dx$ or $2\sqrt{x}\,du=dx$ or $dx=2u\,du$ or $\frac{dx}{du}=2u$ o.e.
	M1	Applying the substitution and achieving $\left\{\int\right\}\frac{\pm ku}{\alpha u^2\pm\beta u}\{du\}$ or $\left\{\int\right\}\frac{\pm k}{u(\alpha u^2\pm\beta u)}\{du\}$, $k,\alpha,\beta\neq 0$. Integral sign and du not required for this mark.
	M1	Cancelling u and integrates to achieve $\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\frac{5}{2}\right)$, $\lambda\neq 0$ **with no other terms.**
	A1	cso. Integrates $\frac{20}{2u+5}$ to give $\frac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\frac{5}{2}\right)$, un-simplified or simplified.
	Note	BE CAREFUL! Candidates must be integrating $\frac{20}{2u+5}$ or equivalent. So $\int\frac{10}{2u+5}du=10\ln(2u+5)$ WOULD BE A0 and final A0.
	M1	Applies limits of 2 and 1 in u or 4 and 1 in x in their (i.e. any) changed function and subtracts the correct way round.
	A1	Exact answers of either $10\ln 9-10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3-10\ln 7$ or $20\ln\left(\frac{3}{\sqrt{7}}\right)$ or $\ln\left(\frac{9^{10}}{7^{10}}\right)$ or equivalent. **Correct solution only.**
	Note	You can ignore subsequent working which follows from a correct answer.
	Note	A decimal answer of 2.513144283... (without a correct **exact** answer) is A0.

Q31.

Question Number	Scheme		Marks
(i)	$\int xe^{4x}\,\mathrm{d}x = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{\mathrm{d}x\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{\mathrm{d}x\},\quad \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{\mathrm{d}x\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\ \{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\,\mathrm{d}x = \frac{8(2x-1)^{-2}}{(2)(-2)}\ \{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\ \{+c\}\}$	*{Ignore subsequent working}.*	[2]
(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{x}\operatorname{cosec}2y\operatorname{cosec}y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec}2y\operatorname{cosec}y}\,\mathrm{d}y = \int e^{x}\,\mathrm{d}x$ or $\int \sin 2y\sin y\,\mathrm{d}y = \int e^{x}\,\mathrm{d}x$		B1 oe
	$\int 2\sin y\cos y\sin y\,\mathrm{d}y = \int e^{x}\,\mathrm{d}x$	Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = e^{x}\ \{+c\}$	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$e^{x} \to e^{x}$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = e^{0} + c$ or $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = e^{x} - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = e^{x} - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec}2y\operatorname{cosec}y}\,\mathrm{d}y = \int e^{x}\,\mathrm{d}x$ or $\int \sin 2y\sin y\,\mathrm{d}y = \int e^{x}\,\mathrm{d}x$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,\mathrm{d}y = \int e^{x}\,\mathrm{d}x$	$\sin 2y\sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = e^{x}\ \{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$e^{x} \to e^{x}$ *as part of solving their DE.*	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = e^{0} + c$ or $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^{x} - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^{x} - \frac{11}{12}$	A1
			[7]
			12

	Question	Notes
(i)	**M1**	Integration by parts is applied in the form $\pm\alpha xe^{4x} - \int \beta e^{4x}\{dx\}$, where $\alpha \neq 0$, $\beta > 0$. (must be in this form).
	A1	$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\{dx\}$ or equivalent.
	A1	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$ with/without $+ c$. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{dv}{dx} = e^{4x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	**M1**	$\pm\lambda(2x-1)^{-2}$, $\lambda \neq 0$. **Note that** λ can be 1.
	A1	$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or $-2(2x-1)^{-2}$ or $\frac{-2}{(2x-1)^2}$ with/without $+ c$. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.
(iii)	**B1**	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Allow B1 for** $\int \frac{1}{\text{cosec}\,2y\ \text{cosec}\,y} = \int e^x$ or $\int \sin 2y \sin y = \int e^x$
	M1	$\frac{1}{\text{cosec}2y} \to 2\sin y\cos y$ **or** $\sin 2y \to 2\sin y \cos y$ **or** $\sin 2y \sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$ seen anywhere in the candidate's working to (iii).
	M1	Integrates to give $\pm\mu\sin^3 y$, $\mu \neq 0$ **or** $\pm\alpha\sin 3y \pm \beta\sin y$, $\alpha \neq 0$, $\beta \neq 0$
	[illegible]	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$ (with no extra terms) or integrates to give $\frac{1}{2}\left(\frac{1}{3}\sin 3y\right.$ [illegible] $\left.\sin y\right)$
	B1	Evidence that e^x has been integrated to give e^x ***as part of solving their DE.***
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$ or $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$ **or any equivalent correct answer.**
	Note	You can ignore subsequent working which follows from a correct answer.

Alternative Method 2 (Using integration by parts twice)

$\int \sin 2y \sin y\, dy = \int e^x\, dx$		B1 oe
	Applies integration by parts twice to give $\pm\alpha\cos y\sin 2y \pm \beta\sin y\cos 2y$	**M2**
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = e^x\ \{+ c\}$	$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y$ (simplified or un-simplified)	A1
	$e^x \to e^x$ ***as part of solving their DE.***	B1
	as in the main scheme	M1
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
		[7]

Q32.

Question Number	Scheme	Marks
(a)	Area $\approx \frac{1}{2}\times 0.5\,;\times\left[\underline{2+2(4.077+7.389+10.043)+0}\right]$	B1; <u>M1</u>
	$=\frac{1}{4}\times 45.018 \;=\; 11.2545 \;= 11.25\,(2\text{ dp})$ 11.25	A1 **cao**
		[3]
(b)	Any one of • Increase the number of strips • Use more trapezia • Make h smaller • Increase the **number of** x and/or y values used • Shorter /smaller intervals for x • More values of y. • More intervals of x • Increase n	B1
		[1]
(c)	$\left\{\int (2-x)e^{2x}\,dx\right\}$, $\left\{\begin{array}{l} u=2-x \;\Rightarrow\; \frac{du}{dx}=-1 \\ \frac{dv}{dx}=e^{2x} \;\Rightarrow\; v=\frac{1}{2}e^{2x} \end{array}\right\}$	
	$=\frac{1}{2}(2-x)e^{2x}-\int -\frac{1}{2}e^{2x}\{dx\}$ Either $(2-x)e^{2x}\to \pm\lambda(2-x)e^{2x}\pm\int \mu e^{2x}\{dx\}$ or $\pm xe^{2x}\to\pm\lambda xe^{2x}\pm\int \mu e^{2x}\{dx\}$	M1
	$(2-x)e^{2x}\to \frac{1}{2}(2-x)e^{2x}-\int -\frac{1}{2}e^{2x}\{dx\}$	A1
	$=\frac{1}{2}(2-x)e^{2x}+\frac{1}{4}e^{2x}$ $\frac{1}{2}(2-x)e^{2x}+\frac{1}{4}e^{2x}$	A1 oe
	Area $=\left\{\left[\frac{1}{2}(2-x)e^{2x}+\frac{1}{4}e^{2x}\right]_0^2\right\}$	
	$=\left(0+\frac{1}{4}e^4\right)-\left(\frac{1}{2}(2)e^0+\frac{1}{4}e^0\right)$ Applies limits of 2 and 0 *to all terms* and subtracts the correct way round.	dM1
	$=\frac{1}{4}e^4-\frac{5}{4}$ $\frac{1}{4}e^4-\frac{5}{4}$ or $\frac{e^4-5}{4}$ **cao**	A1 oe
		[5] **9**

		Question Notes
(a)	**B1**	Outside brackets $\frac{1}{2}\times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$.
	M1	<u>For structure of trapezium rule</u>$\left[\;\ldots\ldots\ldots\ldots\;\right]$. Condone missing 0.
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	11.25 **cao**
	Note	Working must be seen to demonstrate the use of the trapezium rule. The actual area is 12.39953751…
	Note	Award B1M1A1 for $\frac{0.5}{2}(2+0)+\frac{1}{2}(4.077+7.389+10.043)=11.25$

(a) contd		**Bracketing mistake: Unless the final answer implies that the calculation has been done correctly.** Award B1M0A0 for $\frac{1}{2}\times 0.5+2+2(4.077+7.389+10.043)+0$ (nb: answer of 45.268).
		Alternative method for part (a): Adding individual trapezia Area $\approx 0.5\times\left[\frac{2+4.077}{2}+\frac{4.077+7.389}{2}+\frac{7.389+10.043}{2}+\frac{10.043+0}{2}\right]=11.2545=11.25$ (2 dp) cao
	B1	0.5 and a divisor of 2 on all terms inside brackets.
	M1	First and last ordinates once and the middle ordinates twice inside brackets ignoring the 2.
	A1	11.25 **cao**
(b)	**B0**	Give B0 for • smaller values of x and/or y. • use more decimal places
(c)	**M1**	Either $(2-x)e^{2x}\to\pm\lambda(2-x)e^{2x}\pm\int\mu e^{2x}\{dx\}$ or $\pm xe^{2x}\to\pm\lambda xe^{2x}\pm\int\mu e^{2x}\{dx\}$
	A1	$(2-x)e^{2x}\to\frac{1}{2}(2-x)e^{2x}-\int-\frac{1}{2}e^{2x}\{dx\}$ either un-simplified or simplified.
	A1	Correct expression, i.e. $\frac{1}{2}(2-x)e^{2x}+\frac{1}{4}e^{2x}$ or $\frac{5}{4}e^{2x}-xe^{2x}$ (or equivalent)
	dM1	**which is dependent on the 1st M1 mark being awarded.** **Complete method** of applying limits of 2 and 0 **to all terms** and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So, just subtracting zero is M0.
	A1	$\frac{1}{4}e^4-\frac{5}{4}$ or $\frac{e^4-5}{4}$. Do not allow $\frac{1}{4}e^4-\frac{5}{4}e^0$ unless simplified to give $\frac{1}{4}e^4-\frac{5}{4}$
	Note	12.39953751... **without seeing** $\frac{1}{4}e^4-\frac{5}{4}$ is A0.
	Note	12.39953751... from **NO** working is M0A0A0M0A0.

Q33.

Question Number	Scheme	Marks
	(a) $\int x^2 \ln x \,\mathrm{d}x = \frac{1}{3}x^3 \ln x - \int \left(\frac{1}{3}x^3 \times \frac{1}{x}\right)\mathrm{d}x$	M1 A1
	$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \quad (+C)$	M1 A1 (4)
	(b) $\int \left(\sec 2x \tan 2x + \sec^2 x\right)\mathrm{d}x = \frac{1}{2}\sec 2x + \tan x \qquad (+C)$	M1 A1 + B1 (3)
	(c) $u = 2 + \cos\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta} = -\sin\theta$	M1
	$\theta = 0 \Rightarrow u = 3;\ \theta = \frac{\pi}{2} \Rightarrow u = 2$	B1 B1
	$\int \frac{\sin 2\theta}{2 + \cos\theta}\mathrm{d}\theta = \int \frac{2\sin\theta\cos\theta}{2+\cos\theta}\mathrm{d}\theta = -\int \frac{2(u-2)}{u}\mathrm{d}u$	M1
	$= \int \left(4u^{-1} - 2\right)\mathrm{d}u = 4\ln u - 2u$	M1 A1
	$\left[4\ln u - 2u\right]_3^2 = 4\ln 2 - 4\ln 3 + 2$ Order of limits not essential for M	M1
	$= 4\ln\frac{2}{3} + 2$ Accept exact equivalents	A1 (8)
		(15)

Q34.

Question Number	Scheme	Marks
(a)	$\left\{x = u^2 \Rightarrow\right\} \frac{\mathrm{d}x}{\mathrm{d}u} = 2u$ or $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$	B1
	$\left\{\int \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x\right\} = \int \frac{1}{u^2(2u-1)}\,2u\,\mathrm{d}u$	M1
	$= \int \frac{2}{u(2u-1)}\,\mathrm{d}u$	A1 * cso
		[3]
(b)	$\frac{2}{u(2u-1)} \equiv \frac{A}{u} + \frac{B}{(2u-1)} \Rightarrow 2 \equiv A(2u-1) + Bu$	
	$u = 0 \Rightarrow 2 = -A \Rightarrow A = -2$ $u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4$ See notes	M1 A1
	So $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = \int \frac{-2}{u} + \frac{4}{(2u-1)}\,\mathrm{d}u$ Integrates $\frac{M}{u} + \frac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ to obtain any one of $\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$	M1
	$= -2\ln u + 2\ln(2u-1)$ At least one term correctly followed through $-2\ln u + 2\ln(2u-1)$.	A1 ft A1 cao
	So, $\left[-2\ln u + 2\ln(2u-1)\right]_1^3$	
	$= (-2\ln 3 + 2\ln(2(3)-1)) - (-2\ln 1 + 2\ln(2(1)-1))$ Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round.	M1
	$= -2\ln 3 + 2\ln 5 - (0)$	
	$= 2\ln\left(\frac{5}{3}\right)$ $2\ln\left(\frac{5}{3}\right)$	A1 cso cao
		[7] 10

Notes for Question

(a)	**B1:** $\frac{\mathrm{d}x}{\mathrm{d}u} = 2u$ or $\mathrm{d}x = 2u\,\mathrm{d}u$ or $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$ or $\mathrm{d}u = \frac{\mathrm{d}x}{2\sqrt{x}}$	
	M1: A full substitution producing an integral in u only (including the $\mathrm{d}u$) (Integral sign not necessary). The candidate needs to deal with the "x", the "$(2\sqrt{x}-1)$" and the "$\mathrm{d}x$" and converts from an integral term in x to an integral in u. (Remember the integral sign is not necessary for M1).	
	A1*: leading to the result printed on the question paper (including the $\mathrm{d}u$). (Integral sign is needed).	
(b)	**M1:** Writing $\frac{2}{u(2u-1)} \equiv \frac{A}{u} + \frac{B}{(2u-1)}$ or writing $\frac{1}{u(2u-1)} \equiv \frac{P}{u} + \frac{Q}{(2u-1)}$ and a complete method for finding the value of at least one of their A or their B (or their P or their Q).	
	A1: Both their $A = -2$ **and** their $B = 4$. (Or their $P = -1$ **and** their $Q = 2$ with the multiplying factor of 2 in front of the integral sign).	
	M1: Integrates $\frac{M}{u} + \frac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ (i.e. ***a two term partial fraction***) to obtain any one of $\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$ or $\pm\mu\ln\left(u - \frac{1}{2}\right)$	
	A1ft: At least one term correctly followed through from their A or from their B (or their P and their Q).	
	A1: $-2\ln u + 2\ln(2u-1)$	

Notes for Question Continued

(b) ctd	**M1:** Applies limits of 3 and 1 in u or 9 and 1 in x in their (i.e. any) changed function and subtracts the	

correct way round.

Note: If a candidate just writes $(-2\ln 3 + 2\ln(2(3)-1))$ oe , this is ok for M1.

A1: $2\ln\left(\frac{5}{3}\right)$ **correct answer only.** (Note: $a = 5, b = 3$).

Important note: Award M0A0M1A1A0 for a candidate who writes

$$\int \frac{2}{u(2u-1)}\,\mathrm{d}u = \int \frac{2}{u} + \frac{2}{(2u-1)}\,\mathrm{d}u \;=\; 2\ln u + \ln(2u-1)$$

AS EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ **AS PARTIAL FRACTIONS IS GIVEN.**

Important note: Award M0A0M0A0A0 for a candidate who writes down either

$$\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + 2\ln(2u-1) \quad \textbf{or} \quad \int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$$

WITHOUT ANY EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ as partial fractions.

Important note: Award M1A1M1A1A1 for a candidate who writes down

$$\int \frac{2}{u(2u-1)}\,\mathrm{d}u = -2\ln u + 2\ln(2u-1)$$

WITHOUT ANY EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ as partial fractions.

Note: In part (b) if they lose the "2" and find $\int \frac{1}{u(2u-1)}\,\mathrm{d}u$ we can allow a maximum of

M1A0 M1A1ftA0 M1A0.

Q35.

Question Number	Scheme		Marks
(a)	$\frac{dN}{dt} = \frac{(kt-1)(5000-N)}{t}$, $\quad t > 0,\ 0 < N < 5000$		
	$\int \frac{1}{5000-N}\,dN = \int \frac{(kt-1)}{t}\,dt \quad \left\{\text{or } = \int\left(k - \frac{1}{t}\right)dt\right\}$	See notes	B1
	$-\ln(5000-N) = kt - \ln t; +c$	See notes	M1 A1; A1
	then eg either... $-kt + c = \ln(5000-N) - \ln t$ $-kt + c = \ln\left(\frac{5000-N}{t}\right)$ $e^{-kt+c} = \frac{5000-N}{t}$		
	or... $kt + c = \ln t - \ln(5000-N)$ $kt + c = \ln\left(\frac{t}{5000-N}\right)$ $e^{kt+c} = \frac{t}{5000-N}$		
	or... $\ln(5000-N) = -kt + \ln t + c$ $5000 - N = e^{-kt + \ln t + c}$ $5000 - N = t\,e^{-kt+c}$		
	leading to $N = 5000 - Ate^{-kt}$ with no incorrect working/statements. See notes		A1 * cso **[5]**
(b)	$\{t = 1, N = 1200 \Rightarrow\}\quad 1200 = 5000 - Ae^{-k}$ $\{t = 2, N = 1800 \Rightarrow\}\quad 1800 = 5000 - 2Ae^{-2k}$	At least one correct statement written down using the boundary conditions	B1
	So $\quad Ae^{-k} = 3800$ and $\quad 2Ae^{-2k} = 3200$ or $Ae^{-2k} = 1600$ Eg. $\quad \frac{e^{-k}}{2e^{-2k}} = \frac{3800}{3200}$ or $\frac{2e^{-2k}}{e^{-k}} = \frac{3200}{3800}$ $\frac{1}{2} \cdot \frac{3800}{3200} \quad \cdot \frac{3200}{3800}$	An attempt to eliminate A by producing an equation in only k.	M1
	$k = \ln\left(\frac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\frac{19}{8}\right)\right\}$	At least one of $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or exact equivalent	A1
	$\left\{A = 3800(e^{k}) = 3800\left(\frac{19}{8}\right) \Rightarrow\right\}\ A = 9025$	Both $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or exact equivalent	A1 **[4]**
	Alternative Method for the M1 mark in (b) $e^{-k} = \frac{3800}{A}$ $2A\left(\frac{3800}{A}\right)^2 = 3200$	An attempt to eliminate k by producing an equation in only A	M1
(c)	$\left\{t = 5,\ N = 5000 - 9025(5)e^{-5\ln\left(\frac{19}{8}\right)}\right\}$ $N = 4402.828401... = 4400$ (fish) (nearest 100)	anything that rounds to 4400	B1 **[1]** **10**

	Question Notes	
(a)		
	B1	Separates variables as shown. dN and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	M1	**Either** $\pm\lambda\ln(5000-N)$ or $\pm\lambda\ln(N-5000)$ **or** $kt-\ln t$ where $\lambda \neq 0$ is a constant.
	A1	For $-\ln(5000-N) = kt - \ln t$ or $\ln(5000-N) = -kt+\ln t$ or $-\frac{1}{k}\ln(5000-N) = t - \frac{1}{k}\ln t$ oe
	A1	**which is dependent on the 1st M1 mark being awarded.** For applying a constant of integration, eg. $+c$ or $+\ln e^c$ or $+\ln c$ or A to their integrated equation
	Note	$+c$ can be on either side of their equation for the 2nd A1 mark.
	A1	Uses a constant of integration eg. "c" or " $\ln e^c$ " "$\ln c$" or and applies a fully correct method to prove the result $N = 5000 - Ate^{-kt}$ with no incorrect working seen. (Correct solution only.)
	NOTE	**IMPORTANT** There needs to be an intermediate stage of justifying the A **and** the e^{-kt} in Ate^{-kt} by for example - **either** $5000 - N = e^{\ln t - kt + c}$ - **or** $5000 - N = te^{-kt+c}$ - **or** $5000 - N = te^{-kt}e^{c}$ or equivalent needs to be stated before achieving $N = 5000 - Ate^{-kt}$
(b)	**B1**	At least one of **either** $1200 = 5000 - Ae^{-k}$ (or equivalent) **or** $1800 = 5000 - 2Ae^{-2k}$ (or equivalent)
	M1	- **Either** an attempt to eliminate A by producing an equation in only k. - or an attempt to eliminate k by producing an equation in only A
	A1	At least one of $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or equivalent
	A1	Both $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or equivalent
	Note	Alternative correct values for k are $k = \ln\left(\frac{19}{8}\right)$ or $k = -\ln\left(\frac{8}{19}\right)$ or $k = \ln 7600 - \ln 3200$ or $k = -\ln\left(\frac{3800}{9025}\right)$ or equivalent.
	Note	$k = 0.8649...$ without a correct exact equivalent is A0.
(c)	**B1**	anything that rounds to 4400

Q36.

Question Number	Scheme	Marks
(a)	$\frac{25}{x^2(2x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)}$ $B = 25,\ C = 100$ At least one of "B" or "C" correct. Breaks up their partial fraction correctly into *three terms* **and both** "B" $= 25$ and "C" $= 100$. **See notes.**	B1 B1 cso
	$25 \equiv Ax(2x+1) + B(2x+1) + Cx^2$ $x = 0, \quad 25 = B$ $x = -\frac{1}{2}, \quad 25 = \frac{1}{4}C \Rightarrow C = 100$ x^2 terms: $\ 0 = 2A + C$ $0 = 2A + 100 \ \Rightarrow \ A = -50$ $x^2 : 0 = 2A + C, \quad x : 0 = A + 2B,$ constant: $\ 25 = B$ Writes down **a correct identity** and attempts to **find the value** of either one of "A", "B" or "C".	M1
	leading to $A = -50$ $\left\{\frac{25}{x^2(2x+1)} \equiv -\frac{50}{x} + \frac{25}{x^2} + \frac{100}{(2x+1)}\right\}$ Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.	A1 [4]
(b)	$V = \pi \int_1^4 \left(\frac{5}{x\sqrt{(2x+1)}}\right)^2 \mathrm{d}x$ For $\pi \int \left(\frac{5}{x\sqrt{(2x+1)}}\right)^2$	B1
	For their partial fraction	
	$\left\{\int \frac{25}{x^2(2x+1)} \mathrm{d}x = \int -\frac{50}{x} + \frac{25}{x^2} + \frac{100}{(2x+1)} \mathrm{d}x\right\}$ $= -50\ln x + \frac{25x^{-1}}{(-1)} + \frac{100}{2}\ln(2x+1)\ \{+c\}$ Either $\pm\frac{A}{x} \to \pm a\ln x$ or $\pm a\ln kx$ or $\pm\frac{B}{x^2} \to \pm bx^{-1}$ or $\frac{C}{(2x+1)} \to \pm c\ln(2x+1)$ *At least* two terms correctly integrated *All three* terms correctly integrated.	M1 A1ft A1ft
	$\left\{\int_1^4 \frac{25}{x^2(2x+1)} \mathrm{d}x\right\} = \left[-50\ln x - \frac{25}{x} + 50\ln(2x+1)\right]_1^4$ $= \left(-50\ln 4 - \frac{25}{4} + 50\ln 9\right) - (0 - 25 + 50\ln 3)$ Applies limits of 4 and 1 and subtracts the correct way round. $= 50\ln 9 - 50\ln 4 - 50\ln 3 - \frac{25}{4} + 25$ $= 50\ln\left(\frac{3}{4}\right) + \frac{75}{4}$ So, $V = \frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$	dM1 A1 oe [6] 10

		Question Notes
(a)		**BE CAREFUL!** Candidates will assign *their own* "A, B and C" for this question.
	B1	At least one of "B" or "C" are correct.
	B1	Breaks up their partial fraction correctly into three terms **and** both $"B" = 25$ and $"C" = 100$.
	Note	If a candidate does not give partial fraction decomposition then: - the 2nd B1 mark can follow from a correct identity.
	M1	Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C". This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.
	A1	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.
	Note	If a candidate does not give partial fraction decomposition then the final A1 mark can be awarded for a correct "A" if a candidate writes out their partial fractions at the end.
	Note	The correct partial fraction from no working scores B1B1M1A1.
	Note	A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.
	Note	Award SC B1B0M0A0 for $\frac{25}{x^2(2x+1)} \equiv \frac{B}{x^2} + \frac{C}{(2x+1)}$ leading to $"B" = 25$ **or** $"C" = 100$
(b)	**B1**	For a correct statement of $\pi\int\left(\frac{5}{x\sqrt{(2x+1)}}\right)^2$ or $\pi\int\frac{25}{x^2(2x+1)}$. Ignore limits and d$x$. Can be implied.
	Note	The π can only be recovered later from a correct expression.
		For their partial fraction, (not $\sqrt{\text{their partial fraction}}$), where A, B, C are "their" part (a) constants
	M1	Either $\pm\frac{A}{x} \to \pm a\ln x$ or $\pm\frac{B}{x^2} \to \pm bx^{-1}$ or $\frac{C}{(2x+1)} \to \pm c\ln(2x+1)$.
	Note	$\sqrt{\frac{B}{x^2}} \to \frac{\sqrt{B}}{x}$ which integrates to $\sqrt{B}\ln x$ is **not** worthy of M1.
	A1ft	At least two terms from any of $\pm\frac{A}{x}$ or $\pm\frac{B}{x^2}$ or $\frac{C}{(2x+1)}$ correctly integrated. Can be un-simplified.
	A1ft	All 3 terms from $\pm\frac{A}{x}$, $\pm\frac{B}{x^2}$ and $\frac{C}{(2x+1)}$ correctly integrated. Can be un-simplified.
	Note	The 1st A1 and 2nd A1 marks in part (b) are both follow through accuracy marks.
	dM1	**Dependent on the previous M mark.** Applies limits of 4 and 1 and subtracts the correct way round.
	A1	Final correct **exact** answer in the form $a + b\ln c$. i.e. either $\frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or $50\pi\ln\left(\frac{3}{4}\right) + \frac{75}{4}\pi$ or $50\pi\ln\left(\frac{9}{12}\right) + \frac{75}{4}\pi$ or $\frac{75}{4}\pi - 50\pi\ln\left(\frac{4}{3}\right)$ or $\frac{75}{4}\pi + 25\pi\ln\left(\frac{9}{16}\right)$ etc. Also allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$ or equivalent.
	Note	A candidate who achieves full marks in (a), but then mixes up the correct constants when writing their partial fraction can only achieve a maximum of B1M1A1A0M1A0 in part (b).
	Note	The π in the volume formula is only required for the B1 mark and the final A1 mark.

(b)	**Alternative method of integration** $V = \pi \int_1^4 \left(\frac{5}{x\sqrt{(2x+1)}}\right)^2 \mathrm{d}x$	**B1**	For $\pi \int \left(\frac{5}{x\sqrt{(2x+1)}}\right)^2$ Ignore limits and $\mathrm{d}x$. Can be implied.
	$\int \frac{25}{x^2(2x+1)}\,\mathrm{d}x \;;\; u = \frac{1}{x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{x^2}$ $= \int \frac{-25}{\left(\frac{2}{u}+1\right)}\,\mathrm{d}u \;=\; \int \frac{-25}{\left(\frac{2+u}{u}\right)}\,\mathrm{d}u \;=\; \int \frac{-25u}{(2+u)}\,\mathrm{d}u \;=\; -25\int \frac{2+u-2}{(2+u)}\,\mathrm{d}u$		
	$= -25\int 1 - \frac{2}{(2+u)}\,\mathrm{d}u = -25\,(u - 2\ln(2+u))$	**M1**	Achieves $\pm\alpha \pm \frac{\beta}{(k+u)}$ **and integrates to give** either $\pm\alpha u$ or $\pm\beta\ln(k+u)$
		A1	**Dependent on the M mark.** Either $-25u$ or $50\ln(2+u)$
		A1	$-25\,(u - 2\ln(2+u))$
	$\left\{\int_1^4 \frac{25}{x^2(2x+1)}\,\mathrm{d}x = \left[-25u + 50\ln(2+u)\right]_1^{\frac{1}{4}}\right\}$ $= \left(-\frac{25}{4} + 50\ln\left(\frac{9}{4}\right)\right) - (-25 + 50\ln 3)$ $= 50\ln\left(\frac{9}{4}\right) - 50\ln 3 - \frac{25}{4} + 25$ $= 50\ln\left(\frac{3}{4}\right) + \frac{75}{4}$	**dM1**	Applies limits of $\frac{1}{4}$ and 1 in u or 4 and 1 in x in their integrated function and subtracts the correct way round.
	So, $V = \frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$	**A1**	$\frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$

Q37.

Question Number	Scheme	Marks
	(a) $\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}x = \dfrac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1 **(2)**
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
	(b) $\displaystyle\int \frac{1}{\sqrt{(4y+3)}}\,\mathrm{d}y = \int \frac{1}{x^2}\,\mathrm{d}x$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}y = \int x^{-2}\,\mathrm{d}x$	
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \dfrac{1}{2}(4\times 1.5+3)^{\frac{1}{2}} = -\dfrac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \dfrac{2}{x}$	M1
	$y = \dfrac{1}{4}\left(2 - \dfrac{2}{x}\right)^2 - \dfrac{3}{4}$ or equivalent	A1 **(6)**
		[8]

Q38.

Question Number	Scheme	Marks
	(a) $\int \frac{9x+6}{x}\,\mathrm{d}x = \int\left(9+\frac{6}{x}\right)\mathrm{d}x$	M1
	$= 9x+6\ln x \ \ (+C)$	A1 (2)
	(b) $\int \frac{1}{y^{\frac{1}{3}}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$ Integral signs not necessary	B1
	$\int y^{-\frac{1}{3}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$	
	$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x+6\ln x \ \ (+C)$ $\pm ky^{\frac{2}{3}} =$ their (a)	M1
	$\frac{3}{2}y^{\frac{2}{3}} = 9x+6\ln x \ \ (+C)$ ft their (a)	A1ft
	$y=8,\ x=1$	
	$\frac{3}{2}8^{\frac{2}{3}} = 9+6\ln 1+C$	M1
	$C=-3$	A1
	$y^{\frac{2}{3}} = \frac{2}{3}(9x+6\ln x-3)$	
	$y^2 = (6x+4\ln x-2)^3 \quad \left(=8(3x+2\ln x-1)^3\right)$	A1 (6)
		[8]

Q39.

Question Number	Scheme	Marks
(a)	$\left\{\frac{d\theta}{dt} = \frac{(3-\theta)}{125}\right\} \Rightarrow \int \frac{1}{3-\theta}\,d\theta = \int \frac{1}{125}\,dt$ or $\int \frac{125}{3-\theta}\,d\theta = \int dt$	B1
	$-\ln(\theta-3) = \frac{1}{125}t\ \{+c\}$ or $-\ln(3-\theta) = \frac{1}{125}t\ \{+c\}$ See notes.	M1 A1
	$\ln(\theta-3) = -\frac{1}{125}t + c$ $\theta - 3 = e^{-\frac{1}{125}t+c}$ or $e^{-\frac{1}{125}t}e^{c}$ $\theta = Ae^{-0.008t} + 3$ * ***Correct*** completion to $\theta = Ae^{-0.008t}+3$.	A1
		[4]
(b)	$\{t=0\ ,\ \theta=16 \Rightarrow\}$ $16 = Ae^{-0.008(0)} + 3$; $\Rightarrow \underline{A=13}$ ***See notes.***	M1; A1
	$10 = 13e^{-0.008t} + 3$ Substitutes $\theta=10$ into an equation **of the form** $\theta = Ae^{-0.008t}+3$, or equivalent. ***See notes.***	M1
	$e^{-0.008t} = \frac{7}{13} \Rightarrow -0.008t = \ln\left(\frac{7}{13}\right)$ Correct algebra to $-0.008t = \ln k$, where k is a positive value. ***See notes.***	M1
	$\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute) awrt 77	A1
		[5] **9**
(a)	**B1: (M1 on epen)** Separates variables as shown. $d\theta$ and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1: Both** $\pm\lambda\ln(3-\theta)$ or $\pm\lambda\ln(\theta-3)$ **and** $\pm\mu t$ where λ and μ are constants. **A1:** For $-\ln(\theta-3) = \frac{1}{125}t$ or $-\ln(3-\theta) = \frac{1}{125}t$ or $-125\ln(\theta-3) = t$ or $-125\ln(3-\theta) = t$ **Note:** $+c$ is not needed for this mark. **A1:** Correct completion to $\theta = Ae^{-0.008t}+3$. **Note:** $+c$ is needed for this mark. **Note:** $\ln(\theta-3) = -\frac{1}{125}t + c$ leading to $\theta - 3 = e^{-\frac{1}{125}t} + e^{c}$ or $\theta - 3 = e^{-\frac{1}{125}t} + A$, would be final A0. **Note:** From $-\ln(\theta-3) = \frac{1}{125}t + c$, then $\ln(\theta-3) = -\frac{1}{125}t + c$ $\Rightarrow \theta - 3 = e^{-\frac{1}{125}t+c}$ **or** $\theta - 3 = e^{-\frac{1}{125}t}e^{c} \Rightarrow \theta = Ae^{-0.008t}+3$ is required for A1. **Note:** From $-\ln(3-\theta) = \frac{1}{125}t + c$, then $\ln(3-\theta) = -\frac{1}{125}t + c$ $\Rightarrow 3-\theta = e^{-\frac{1}{125}t+c}$ **or** $3-\theta = e^{-\frac{1}{125}t}e^{c} \Rightarrow \theta = Ae^{-0.008t}+3$ is sufficient for A1. **Note:** The jump from $3-\theta = Ae^{-\frac{1}{125}t}$ to $\theta = Ae^{-0.008t}+3$ is fine. **Note:** $\ln(\theta-3) = -\frac{1}{125}t + c \Rightarrow \theta - 3 = Ae^{-\frac{1}{125}t}$, where candidate writes $A = e^{c}$ is also acceptable.	
(b)	**M1:** (B1 on epen) Substitutes $\theta = 16, t = 0$, into either their equation containing an unknown constant or the printed equation. **Note:** You can imply this method mark. **A1:** (M1 on epen) $A = 13$. **Note:** $\theta = 13e^{-0.008t}+3$ without any working implies the first two marks, M1A1. **M1:** Substitutes $\theta = 10$ into an equation **of the form** $\theta = Ae^{-0.008t}+3$, or equivalent. where A is a positive or negative numerical value and A can be equal to 1 or -1. **M1:** Uses correct algebra to rearrange **their equation** into the form $-0.008t = \ln k$, where k is ***a positive numerical value***. **A1:** awrt 77 or awrt 1 hour 17 minutes. ***<u>Alternative Method 1 for part (b)</u>*** $\int \frac{1}{3-\theta}\,d\theta = \int \frac{1}{125}\,dt \Rightarrow -\ln(\theta-3) = \frac{1}{125}t + c$ $\{t=0\ ,\ \theta=16 \Rightarrow\}$ $-\ln(16-3) = \frac{1}{125}(0) + c$ $\Rightarrow c = -\ln 13$ **M1:** Substitutes $t = 0, \theta = 16$, into $-\ln(\theta-3) = \frac{1}{125}t + c$ **A1:** $c = -\ln 13$ $-\ln(\theta-3) = \frac{1}{125}t - \ln 13$ or $\ln(\theta-3) = -\frac{1}{125}t + \ln 13$ $-\ln(10-3) = \frac{1}{125}t - \ln 13$ **M1:** Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(\theta-3) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values. $\ln 13 - \ln 7 = \frac{1}{125}t$ **M1:** Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***. $t = 77.3799... = 77$ (nearest minute) **A1:** awrt 77. ***<u>Alternative Method 2 for part (b)</u>*** $\int \frac{1}{3-\theta}\,d\theta = \int \frac{1}{125}\,dt \Rightarrow -\ln\lvert 3-\theta\rvert = \frac{1}{125}t + c$ $\{t=0\ ,\ \theta=16 \Rightarrow\}$ $-\ln\lvert 3-16\rvert = \frac{1}{125}(0) + c$ $\Rightarrow c = -\ln 13$ **M1:** Substitutes $t = 0, \theta = 16$, into $-\ln(3-\theta) = \frac{1}{125}t + c$ **A1:** $c = -\ln 13$ $-\ln\lvert 3-\theta\rvert = \frac{1}{125}t - \ln 13$ or $\ln\lvert 3-\theta\rvert = -\frac{1}{125}t + \ln 13$ $-\ln(3-10) = \frac{1}{125}t - \ln 13$ **M1:** Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(3-\theta) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values. $\ln 13 - \ln 7 = \frac{1}{125}t$ **M1:** Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***. $t = 77.3799... = 77$ (nearest minute) **A1:** awrt 77.	
(b)	***<u>Alternative Method 3 for part (b)</u>*** $\int_{16}^{10} \frac{1}{3-\theta}\,d\theta = \int_{0}^{t} \frac{1}{125}\,dt$ $= \left[-\ln\lvert 3-\theta\rvert\right]_{16}^{10} = \left[\frac{1}{125}t\right]_{0}^{t}$ **M1A1:** $\ln 13$ **M1:** Substitutes limit of $\theta = 10$ correctly. $-\ln 7 - -\ln 13 = \frac{1}{125}t$ **M1:** Uses correct algebra to rearrange **their own equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***. $t = 77.3799... = 77$ (nearest minute) **A1:** awrt 77. ***<u>Alternative Method 4 for part (b)</u>*** **M1*:** Writes down a pair of equations in A and t, for $\theta = 16$ and $\theta = 10$ with either A unknown or A being a positive or negative value. $\{\theta = 16 \Rightarrow\}$ $16 = Ae^{-0.008t} + 3$ $\{\theta = 10 \Rightarrow\}$ $10 = Ae^{-0.008t} + 3$ **A1:** Two equations with an unknown A. $-0.008t = \ln\left(\frac{13}{A}\right)$ or $-0.008t = \ln\left(\frac{7}{A}\right)$ **M1:** Uses ***correct algebra*** to solve both of **their equations** leading to answers of the form $-0.008t = \ln k$, where k is ***a positive numerical value***. $t_{(1)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008}$ and $t_{(2)} = \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$ $t = t_{(1)} - t_{(2)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008} - \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$ **M1:** Finds difference between the two times. (either way round). $\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute) **A1:** awrt 77. Correct solution only.	

Q40.

Question Number	Scheme	Marks
(a)	1.0981	**B1 cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times 1\,;\times\underline{\left[0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333\right]}$	B1; <u>M1</u>
	$=\frac{1}{2}\times 5.6863 \;=\; 2.84315 \;=\; 2.843\;\;(3\text{ dp})$ 2.843 or awrt 2.843	A1 **[3]**
(c)	$\left\{u=1+\sqrt{x}\right\} \Rightarrow \underline{\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}}}$ or $\underline{\frac{\mathrm{d}x}{\mathrm{d}u}=2(u-1)}$	<u>B1</u>
	$\left\{\int\frac{x}{1+\sqrt{x}}\,\mathrm{d}x=\right\}\int\frac{(u-1)^2}{u}.\,2(u-1)\;\mathrm{d}u$ $\int\frac{(u-1)^2}{u}\ldots\ldots$	M1
	$\int\frac{(u-1)^2}{u}.\,2(u-1)$	A1
	$=2\int\frac{(u-1)^3}{u}\,\mathrm{d}u\;=\{2\}\int\frac{(u^3-3u^2+3u-1)}{u}\,\mathrm{d}u$ Expands to give a "four term" cubic in u. Eg: $\pm Au^3\pm Bu^2\pm Cu\pm D$	M1
	$=\{2\}\int\left(u^2-3u+3-\frac{1}{u}\right)\mathrm{d}u$ An attempt to divide at least three terms in ***their cubic*** by u. See notes.	M1
	$=\{2\}\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$ $\int\frac{(u-1)^3}{u}\to\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$	A1
	$\text{Area}(R)=\left[\frac{2u^3}{3}-3u^2+6u-2\ln u\right]_2^3$	
	$=\left(\frac{2(3)^3}{3}-3(3)^2+6(3)-2\ln 3\right)-\left(\frac{2(2)^3}{3}-3(2)^2+6(2)-2\ln 2\right)$ Applies limits of 3 and 2 in u or 4 and 1 in x and subtracts either way round.	M1
	$=\frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$, etc Correct exact answer or equivalent.	A1 **[8]** **12**

(a) **B1:** 1.0981 correct answer only. Look for this on the table or in the candidate's working.

(b) **B1:** Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$

M1: <u>For structure of trapezium rule</u>$\left[\;\underline{\ldots\ldots\ldots\ldots}\;\right]$

A1: anything that rounds to 2.843

<u>Note:</u> Working must be seen to demonstrate the use of the trapezium rule. **<u>Note</u>**: actual area is 2.85573645…

<u>Note:</u> Award B1M1 A1 for $\frac{1}{2}(0.5+1.3333)+(0.8284+\text{their }1.0981)=2.84315$

<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**

Award B1M0A0 for $\frac{1}{2}\times 1\;+0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333$ (nb: answer of 6.1863).

Award B1M0A0 for $\frac{1}{2}\times 1\;(0.5+1.3333)+2\left(0.8284+\text{their }1.0981\right)$ (nb: answer of 4.76965).

(b) ctd *<u>Alternative method for part (b): Adding individual trapezia</u>*

$$\text{Area}\approx 1\times\left[\frac{0.5+0.8284}{2}+\frac{0.8284+1.0981}{2}+\frac{1.0981+1.3333}{2}\right]=\;2.84315$$

B1: 1 and a divisor of 2 on all terms inside brackets.
M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
A1: anything that rounds to 2.843

(c)

B1: $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}}$ or $\mathrm{d}u=\frac{1}{2\sqrt{x}}\,\mathrm{d}x$ or $2\sqrt{x}\,\mathrm{d}u=\mathrm{d}x$ or $\mathrm{d}x=2(u-1)\mathrm{d}u$ or $\frac{\mathrm{d}x}{\mathrm{d}u}=2(u-1)$ oe.

1st M1: $\frac{x}{1+\sqrt{x}}$ becoming $\frac{(u-1)^2}{u}$ (Ignore integral sign).

1st A1 (B1 on epen): $\frac{x}{1+\sqrt{x}}\,\mathrm{d}x$ becoming $\frac{(u-1)^2}{u}.\,2(u-1)\{\mathrm{d}u\}$ or $\frac{(u-1)^2}{u}.\,\frac{2}{(u-1)^{-1}}\{\mathrm{d}u\}$.

You can ignore the integral sign and the $\mathrm{d}u$.

2nd M1: Expands to give a "four term" cubic in u, $\pm Au^3\pm Bu^2\pm Cu\pm D$
where $A\neq 0$, $B\neq 0$, $C\neq 0$ and $D\neq 0$ The cubic does not need to be simplified for this mark.

3rd M1: An attempt to divide at least three terms in *their cubic* by u.
Ie. $\frac{(u^3-3u^2+3u-1)}{u}\to u^2-3u+3-\frac{1}{u}$

2nd A1: $\int\frac{(u-1)^3}{u}\,\mathrm{d}u\to\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$

4th M1: Some evidence of limits of 3 and 2 in u and subtracting either way round.

3rd A1: Exact answer of $\frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$ or $2\left(\frac{11}{6}+\ln 2-\ln 3\right)$
or $\frac{22}{6}+2\ln\left(\frac{2}{3}\right)$, etc. **Note**: that fractions must be combined to give either $\frac{11}{3}$ or $\frac{22}{6}$ or $3\frac{2}{3}$

<u>Alternative method for 2nd M1 and 3rd M1 mark</u>

$\{2\}\int\frac{(u-1)^2}{u}.\,(u-1)\;\mathrm{d}u\;=\;\{2\}\int\frac{(u^2-2u+1)}{u}.\,(u-1)\;\mathrm{d}u$

$=\{2\}\int\left(u-2+\frac{1}{u}\right).\,(u-1)\;\mathrm{d}u\;=\;\{2\}\int\left(u^2-\ldots\right)\;\mathrm{d}u$

An attempt to expand $(u-1)^2$, then divide the result by u and then go on to multiply by $(u-1)$. **2nd M1**

$=\{2\}\int\left(u^2-2u+1-u+2-\frac{1}{u}\right)\;\mathrm{d}u$

to give three out of four of $\pm Au^2$, $\pm Bu$, $\pm C$ or $\pm\frac{D}{u}$ **3rd M1**

$=\{2\}\int\left(u^2-3u+3-\frac{1}{u}\right)\;\mathrm{d}u$

(c) ctd **<u>Final two marks in part (c):</u>** $u=1+\sqrt{x}$

$$\text{Area}(R)=\left[\frac{2\left(1+\sqrt{x}\right)^3}{3}-3\left(1+\sqrt{x}\right)^2+6\left(1+\sqrt{x}\right)-2\ln\left(1+\sqrt{x}\right)\right]_1^4$$

$$=\left(\frac{2\left(1+\sqrt{4}\right)^3}{3}-3\left(1+\sqrt{4}\right)^2+6\left(1+\sqrt{4}\right)-2\ln\left(1+\sqrt{4}\right)\right)$$
$$-\left(\frac{2\left(1+\sqrt{1}\right)^3}{3}-3\left(1+\sqrt{1}\right)^2+6\left(1+\sqrt{1}\right)-2\ln\left(1+\sqrt{1}\right)\right)$$

M1: Applies limits of 4 and 1 in x and subtracts either way round.

$$=\left(18-27+18-2\ln 3\right)-\left(\frac{16}{3}-12+12-2\ln 2\right)$$

$$=\frac{11}{3}+2\ln 2-2\ln 3\quad\text{or}\quad\frac{11}{3}+2\ln\left(\frac{2}{3}\right)\quad\text{or}\quad\frac{11}{3}-\ln\left(\frac{9}{4}\right),\ \text{etc}$$

A1: Correct exact answer or equivalent.

<u>Alternative method for the final 5 marks in part (b)</u>

$$\int\frac{(u-1)^3}{u}\,\mathrm{d}u,\qquad\left\{\begin{array}{lll}"u"=u^{-1} & \Rightarrow & \frac{\mathrm{d}"u"}{\mathrm{d}x}=-u^{-2}\\ \frac{\mathrm{d}v}{\mathrm{d}x}=(u-1)^3 & \Rightarrow & v=\frac{(u-1)^4}{4}\end{array}\right\}$$

$$=\frac{(u-1)^4}{4u}\;--\frac{1}{4}\int\frac{(u-1)^4}{u^2}\,\mathrm{d}u$$

$$=\frac{(u-1)^4}{4u}\;+\;\frac{1}{4}\int\frac{u^4-4u^3+6u^2-4u+1}{u^2}\,\mathrm{d}u$$

M1: Applies integration by parts and expands to give a five term quartic.

$$=\frac{(u-1)^4}{4u}\;+\;\frac{1}{4}\int u^2-4u+6-\frac{4}{u}+\frac{1}{u^2}\,\mathrm{d}u$$

M1: Dividing at least 4 terms.

$$=\frac{(u-1)^4}{4u}\;+\;\frac{1}{4}\left(\frac{u^3}{3}-2u^2+6u-4\ln u-\frac{1}{u}\right)$$

A1: Correct Integration.

$$\int_2^3\frac{(u-1)^3}{u}\,\mathrm{d}u=\left[\frac{(u-1)^4}{4u}+\frac{u^3}{12}-\frac{u^2}{2}+\frac{3u}{2}-\ln u-\frac{1}{4u}\right]_2^3$$

$$=\left(\frac{16}{12}+\frac{27}{12}-\frac{9}{2}+\frac{9}{2}-\ln 3-\frac{1}{12}\right)-\left(\frac{1}{8}+\frac{8}{12}-\frac{4}{2}+\frac{6}{2}-\ln 2-\frac{1}{8}\right)$$

M1

$$=\left(7-\ln 3\right)-\left(\frac{5}{3}-\ln 2\right)$$

$$=\frac{11}{6}+\ln\frac{2}{3}$$

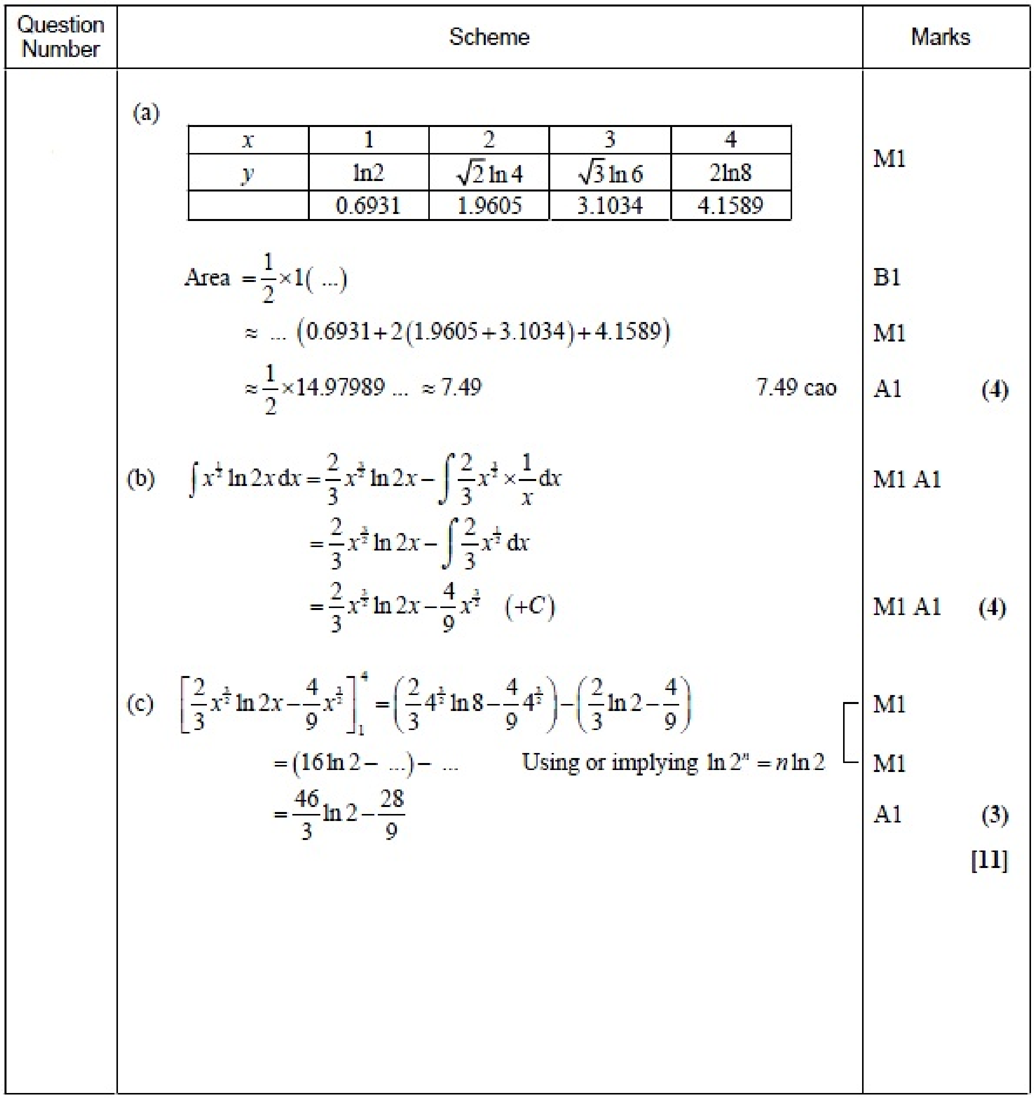
$$\text{Area}(R)=2\int_2^3\frac{(u-1)^3}{u}\,\mathrm{d}u\;=\;2\left(\frac{11}{6}+\ln\frac{2}{3}\right)$$

A1

Q41.

Question Number	Scheme		Marks
(a)	0.73508		**B1 cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{8}\,;\times\underline{\left[0+2\left(\text{their } 0.73508+1.17157+1.02280\right)+0\right]}$		B1 $\underline{\text{M1}}$
	$=\frac{\pi}{16}\times 5.8589... \;=\; 1.150392325... = 1.1504 \;\; (4\text{ dp})$	awrt 1.1504	A1 [3]
(c)	$\underline{\{u=1+\cos x\} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=-\sin x}$		$\underline{\text{B1}}$
	$\left\{\int \frac{2\sin 2x}{(1+\cos x)}\,\mathrm{d}x = \right\} \int \frac{2(2\sin x\cos x)}{(1+\cos x)}\,\mathrm{d}x$	$\sin 2x = 2\sin x\cos x$	B1
	$= \int \frac{4(u-1)}{u}.(-1)\,\mathrm{d}u \quad \left\{= \; 4\int \frac{(1-u)}{u}\,\mathrm{d}u\right\}$		M1
	$= 4\int\left(\frac{1}{u}-1\right)\mathrm{d}u \;=\; 4\left(\ln u - u\right)+c$		dM1
	$= 4\ln(1+\cos x) - 4(1+\cos x) + c \quad = 4\ln(1+\cos x) - 4\cos x + k$	**AG**	A1 **cso** **[5]**
(d)	$= \left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - \left[4\ln(1+\cos 0) - 4\cos 0\right]$	Applying limits $x=\frac{\pi}{2}$ and $x=0$ either way round.	M1
	$= [4\ln 1 - 0] - [4\ln 2 - 4]$		
	$= 4 - 4\ln 2 \; \{= 1.227411278...\}$	$\pm 4(1-\ln 2)$ or $\pm(4-4\ln 2)$ or awrt ± 1.2, ***however*** found.	A1
	Error $= \lvert(4-4\ln 2) - 1.1504...\rvert$ $= 0.0770112776... \;=\; 0.077 \;(2\text{sf})$	awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 cso [3]
			12
(a)	**B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.		
(b)	**B1**: Outside brackets $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196		
	M1: <u>For structure of trapezium rule</u>$\left[\;\underline{............}\;\right]$; (0 can be implied).		
	A1: anything that rounds to 1.1504		
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**		
	Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8} + 2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 6.0552).		
	Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}\;(0+0)+2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 5.8589).		
	<u>*Alternative method for part (b): Adding individual trapezia*</u>		
	Area $\approx \frac{\pi}{8}\times\left[\frac{0+0.73508}{2}+\frac{0.73508+1.17157}{2}+\frac{1.17157+1.02280}{2}+\frac{1.02280+0}{2}\right] = 1.150392325...$		
	B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets.		
	M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.		
	A1: anything that rounds to 1.1504		
(c)	**B1:** $\frac{\mathrm{d}u}{\mathrm{d}x}=-\sin x$ or $\mathrm{d}u=-\sin x\,\mathrm{d}x$ or $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{-\sin x}$ oe.		
	B1: For seeing, applying or implying $\sin 2x = 2\sin x\cos x$.		
	M1: After applying substitution candidate achieves $\pm k\int\frac{(u-1)}{u}(\mathrm{d}u)$ or $\pm k\int\frac{(1-u)}{u}(\mathrm{d}u)$.		
	Allow M1 for "invisible" brackets here, eg: $\pm\int\frac{(\lambda u-1)}{u}(\mathrm{d}u)$ or $\pm\int\frac{(-\lambda+u)}{u}(\mathrm{d}u)$, where λ is a positive constant.		
	dM1: An attempt to divide through each term by u and $\pm k\int\left(\frac{1}{u}-1\right)\mathrm{d}u \rightarrow \pm k\left(\ln u - u\right)$ with/without $+c$. Note that this mark is dependent on the previous M1 mark being awarded.		
	<u>Alternative method:</u> Candidate can also gain this mark for applying integration by parts followed by a correct method for integrating $\ln u$. (See below).		
	A1: Correctly combines their $+c$ and $"-4"$ together to give $\underline{4\ln(1+\cos x) - 4\cos x + k}$		
	As a minimum candidate must write either $4\ln(1+\cos x) - 4(1+\cos x) + c \rightarrow 4\ln(1+\cos x) - 4\cos x + k$ or $4\ln(1+\cos x) - 4(1+\cos x) + k \rightarrow 4\ln(1+\cos x) - 4\cos x + k$		
	Note: that this mark is also for a correct solution only.		
	Note: those candidates who attempt to find the value of k will usually achieve A0.		
(d)	**M1:** Substitutes limits of $x=\frac{\pi}{2}$ and $x=0$ into $\{4\ln(1+\cos x) - 4\cos x\}$ or their answer from part (c) and subtracts the either way round. **Note that**: $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - [0]$ is M0.		
	A1: $4(1-\ln 2)$ or $4-4\ln 2$ or awrt 1.2, however found. This mark can be implied by the final answer of either awrt ± 0.077 or awrt ± 6.3		
	A1: For either awrt ± 0.077 or awrt ± 6.3 (for percentage error). Note this mark is for a **correct solution only**. Therefore if there if a candidate substitutes limits the incorrect way round and final achieves (usually fudges) the final correct answer then this mark can be withheld. Note that awrt 6.7 (for percentage error) is A0.		
	<u>*Alternative method for dM1 in part (c)*</u>		
	$\int\frac{(1-u)}{u}\,\mathrm{d}u = \left((1-u)\ln u - \int -\ln u\,du\right) = \left((1-u)\ln u + u\ln u - \int\frac{u}{u}\,\mathrm{d}u\right) = \left((1-u)\ln u + u\ln u - u\right)$		
	or $\int\frac{(u-1)}{u}\,\mathrm{d}u = \left((u-1)\ln u - \int\ln u\,\mathrm{d}u\right) = \left((u-1)\ln u - \left(u\ln u - \int\frac{u}{u}\,\mathrm{d}u\right)\right) = \left((u-1)\ln u - u\ln u + u\right)$		
	So **dM1** is for $\int\frac{(1-u)}{u}\,\mathrm{d}u$ going to $\left((1-u)\ln u + u\ln u - u\right)$ or $\left((u-1)\ln u - u\ln u + u\right)$ oe.		
	<u>*Alternative method for part (d)*</u>		
	M1A1 for $\left\{4\int_2^1\left(\frac{1}{u}-1\right)\mathrm{d}u = \right\} 4\left[\ln u - u\right]_2^1 = 4\left[(\ln 1 - 1) - (\ln 2 - 2)\right] = 4(1-\ln 2)$		
	<u>*Alternative method for part (d): Using an extra constant λ from their integration.*</u>		
	$\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2} + \lambda\right] - \left[4\ln(1+\cos 0) - 4\cos 0 + \lambda\right]$		
	λ is usually -4, but can be a value of k that the candidate has found in part (d).		
	Note: The extra constant λ should cancel out and so the candidate can gain all three marks using this method, even the final A1 cso.		

Q42.

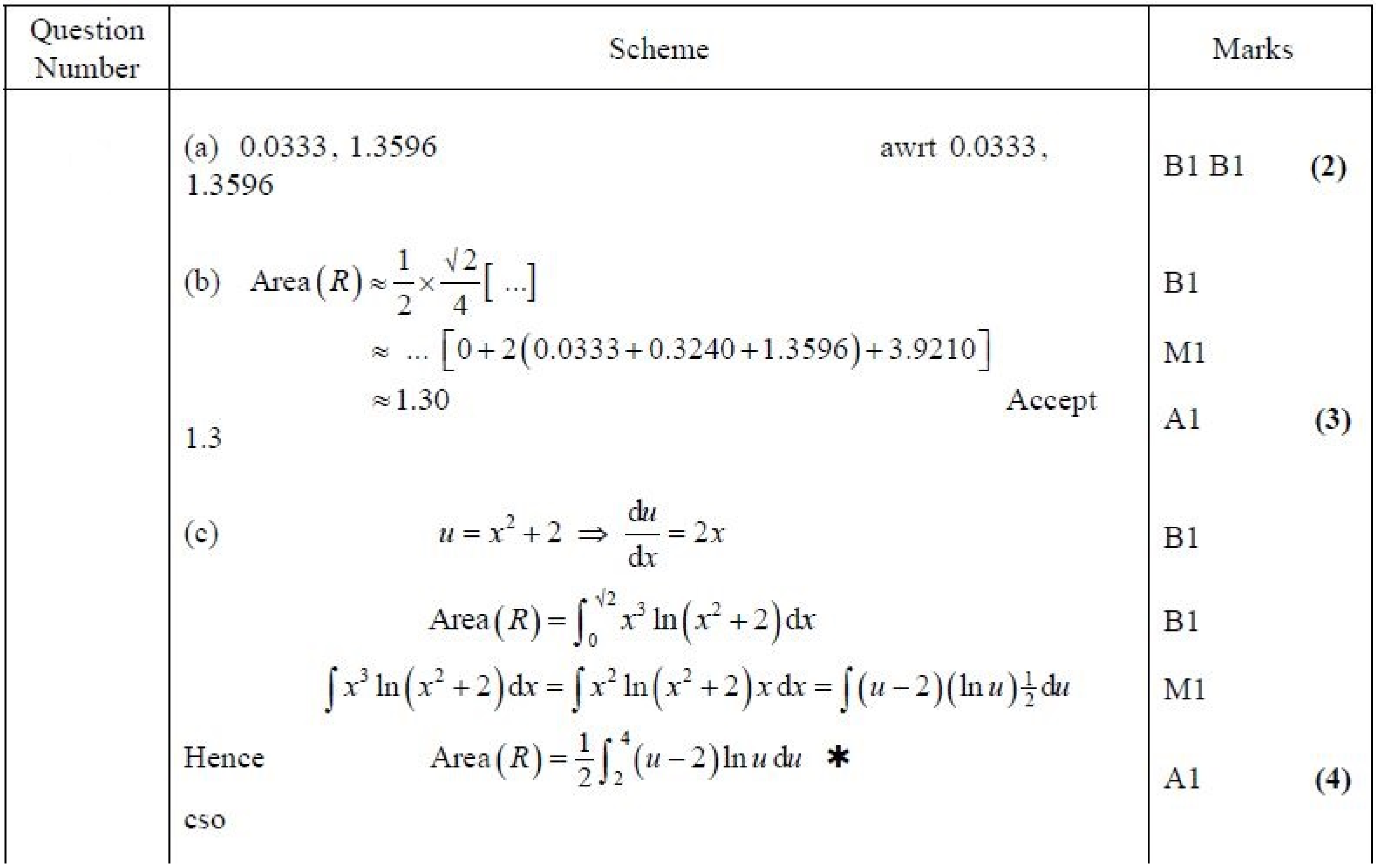


Question Number	Scheme	Marks	
	(a)		

x	1	2	3	4
y	ln2	$\sqrt{2}\ln 4$	$\sqrt{3}\ln 6$	2ln8
	0.6931	1.9605	3.1034	4.1589

	(table)	M1	
	Area $=\frac{1}{2}\times 1(\ \dots)$	B1	
	$\approx\ \dots\ \left(0.6931+2(1.9605+3.1034)+4.1589\right)$	M1	
	$\approx\frac{1}{2}\times 14.97989\ \dots\ \approx 7.49$ 7.49 cao	A1	(4)
	(b) $\int x^{\frac{1}{2}}\ln 2x\,\mathrm{d}x=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\int\frac{2}{3}x^{\frac{3}{2}}\times\frac{1}{x}\mathrm{d}x$	M1 A1	
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\int\frac{2}{3}x^{\frac{1}{2}}\,\mathrm{d}x$		
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\frac{4}{9}x^{\frac{3}{2}}\quad(+C)$	M1 A1	(4)
	(c) $\left[\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\frac{4}{9}x^{\frac{3}{2}}\right]_1^4=\left(\frac{2}{3}4^{\frac{3}{2}}\ln 8-\frac{4}{9}4^{\frac{3}{2}}\right)-\left(\frac{2}{3}\ln 2-\frac{4}{9}\right)$	M1	
	$=(16\ln 2-\ \dots)-\ \dots$ Using or implying $\ln 2^n=n\ln 2$	M1	
	$=\frac{46}{3}\ln 2-\frac{28}{9}$	A1	(3)
			[11]

Q43.



Question Number	Scheme	Marks
	(a) 0.0333, 1.3596 awrt 0.0333, 1.3596	B1 B1 **(2)**
	(b) Area $(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4}[\ \dots]$	B1
	$\approx\ \dots\left[0+2(0.0333+0.3240+1.3596)+3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)**
	(c) $u = x^2+2 \ \Rightarrow\ \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$	B1
	Area $(R) = \int_0^{\sqrt{2}} x^3 \ln\left(x^2+2\right)\mathrm{d}x$	B1
	$\int x^3 \ln\left(x^2+2\right)\mathrm{d}x = \int x^2 \ln\left(x^2+2\right) x\,\mathrm{d}x = \int (u-2)(\ln u)\tfrac{1}{2}\,\mathrm{d}u$	M1
	Hence Area $(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,\mathrm{d}u$ ✱ cso	A1 **(4)**

Q44.

Question Number	Scheme	Marks
	(a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec} x \, \mathrm{d}x = \left[-\ln\left(\operatorname{cosec} x + \cot x\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1 A1
	$= -\ln\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) + \ln\left(2+\sqrt{3}\right) \approx 0.768$ awrt 0.768	A1 (3)
	(b) $y\left(\frac{\pi}{6}\right) = 2,\ y\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$ awrt 1.15	B1
	$y\left(\frac{\pi}{4}\right) = \sqrt{2}$ awrt 1.41	B1
	$A \approx \frac{\pi}{24}\left(2 + 2\sqrt{2} + \frac{2}{\sqrt{3}}\right)$	M1 A1
	≈ 0.783 cao	A1 (5)
	(c) Error is $0.783 - 0.768 = 0.015$ Accept awrt 0.015	B1 (1)
	$\left(or \text{ Error is } \frac{0.783 - 0.768}{0.768} \times 100 \approx 2\% \right)$	
	(d) $V = (\pi)\int_{\dots}^{\dots} \operatorname{cosec}^2 x \, \mathrm{d}x$	M1
	$= (\pi)\left[-\cot x\right]_{\dots}^{\dots}$	A1
	$\pi\left[-\cot x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \pi\left(-\frac{1}{\sqrt{3}} + \sqrt{3}\right)$	M1
	$= \frac{2}{3}\pi\sqrt{3}$	A1 (4)
		(13)
	Alternative to (a)	
	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec} x \, \mathrm{d}x = \left[-\ln\left(\tan\frac{x}{2}\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1 A1
	$\ln\tan\frac{\pi}{6} - \ln\tan\frac{\pi}{12} \approx 0.768$	A1 (3)

Q45.

Question Number	Scheme	Marks
	(a) 1.386, 2.291 awrt 1.386, 2.291	B1 B1 (2)
	(b) $A \approx \frac{1}{2}\times 0.5(\ \dots\ )$	B1
	$= \dots \left(0+2(0.608+1.386+2.291+3.296+4.385)+5.545\right)$	M1
	$= 0.25\left(0+2(0.608+1.386+2.291+3.296+4.385)+5.545\right)$ ft their (a)	A1ft
	$=0.25\times 29.477\dots \approx 7.37$ cao	A1 (4)
	(c)(i) $\int x\ln x\,\mathrm{d}x = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\times\frac{1}{x}\,\mathrm{d}x$	M1 A1
	$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,\mathrm{d}x$	
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4}\ (+C)$	M1 A1
	(ii) $\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^4 = (8\ln 4 - 4) - \left(-\frac{1}{4}\right)$	M1
	$= 8\ln 4 - \frac{15}{4}$	
	$= 8(2\ln 2) - \frac{15}{4}$ $\ln 4 = 2\ln 2$ seen or implied	M1
	$= \frac{1}{4}(64\ln 2 - 15)$ $a = 64, b = -15$	A1 (7)
		[13]