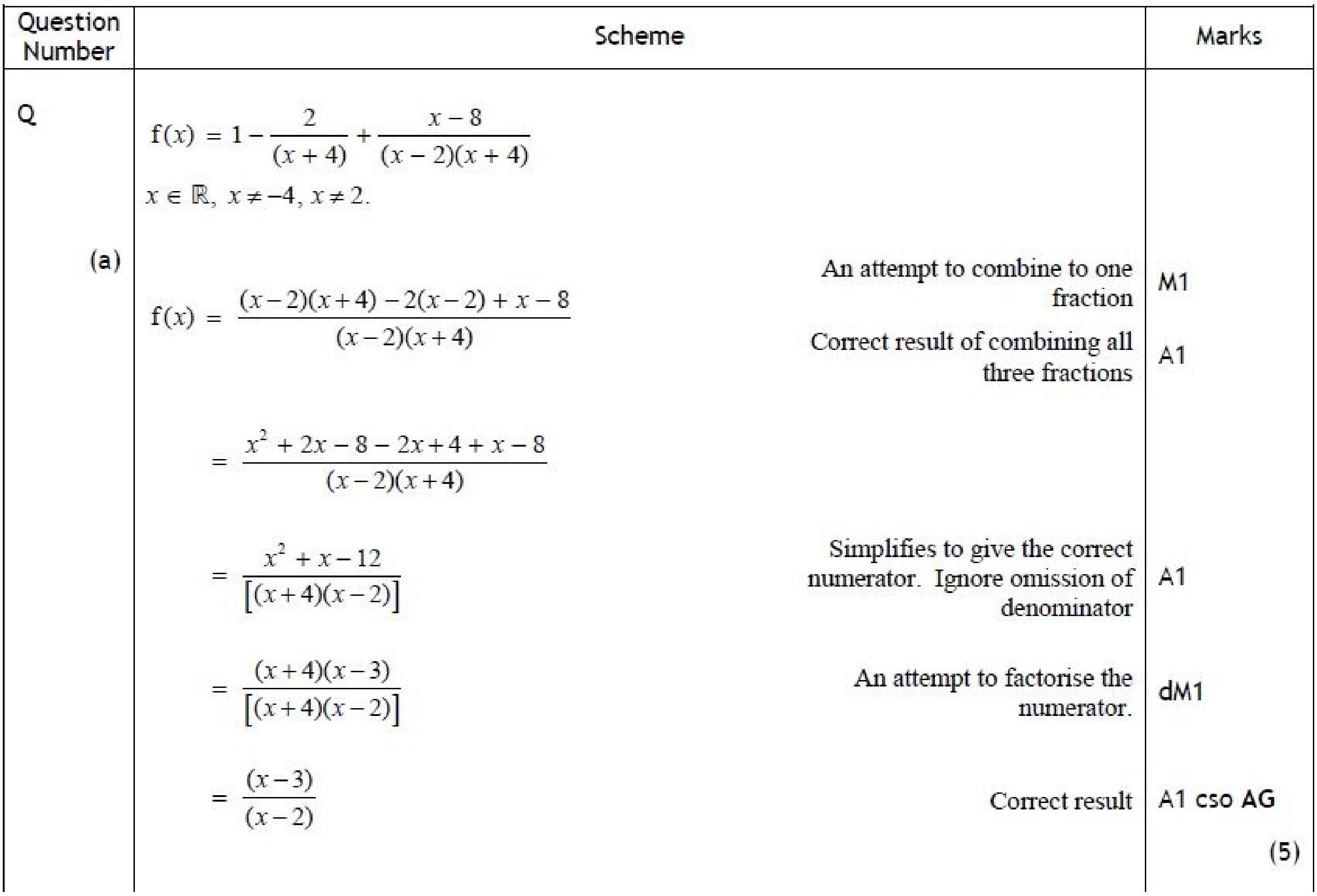
Mark Scheme

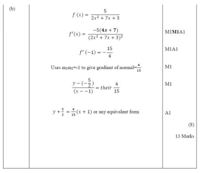
Q1.



Question Number	Scheme		Marks
Q	$\mathrm{f}(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$		
(a)	$\mathrm{f}(x) = \frac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	An attempt to combine to one fraction Correct result of combining all three fractions	M1 A1
	$= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$		
	$= \frac{x^2 + x - 12}{\left[(x+4)(x-2)\right]}$	Simplifies to give the correct numerator. Ignore omission of denominator	A1
	$= \frac{(x+4)(x-3)}{\left[(x+4)(x-2)\right]}$	An attempt to factorise the numerator.	dM1
	$= \frac{(x-3)}{(x-2)}$	Correct result	A1 cso AG
			(5)

Q2.

Question Number	Scheme	Marks
(a)	$x^2 - 9 = (x+3)(x-3)$	B1
	$\frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{(x+3)(x-3)}$	
	$= \frac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)} - \frac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$= \frac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$= \frac{5\cancel{(x-3)}}{(2x+1)\cancel{(x-3)}(x+3)} = \frac{5}{(2x+1)(x+3)}$	A1*
		(5)



(b)	$f(x) = \frac{5}{2x^2+7x+3}$	
	$f'(x) = \frac{-5(4x+7)}{(2x^2+7x+3)^2}$	M1**M1**A1
	$f'(-1) = -\frac{15}{4}$	M1A1
	Uses m_1m_2=-1 to give gradient of normal=$\frac{4}{15}$	M1
	$\frac{y-(-\frac{5}{2})}{(x--1)} = their\ \frac{4}{15}$	M1
	$y + \frac{5}{2} = \frac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		13 Marks

Q3.

Question Number	Scheme	Marks
(a)	$\frac{2x+2}{x^2-2x-3}-\frac{x+1}{x-3}=\frac{2x+2}{(x-3)(x+1)}-\frac{x+1}{x-3}$	
	$=\frac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$	M1 A1
	$=\frac{(x+1)(1-x)}{(x-3)(x+1)}$	M1
	$=\frac{1-x}{x-3}$ Accept $-\frac{x-1}{x-3},\ \frac{x-1}{3-x}$	A1 (4)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1-x}{x-3}\right)=\frac{(x-3)(-1)-(1-x)1}{(x-3)^2}$	M1 A1
	$=\frac{-x+3-1+x}{(x-3)^2}=\frac{2}{(x-3)^2}$ $\ast$ cso	A1 (3)
		[7]
	Alternative to (a)	
	$\frac{2x+2}{x^2-2x-3}=\frac{2(x+1)}{(x-3)(x+1)}=\frac{2}{x-3}$	M1 A1
	$\frac{2}{x-3}-\frac{x+1}{x-3}=\frac{2-(x+1)}{x-3}$	M1
	$=\frac{1-x}{x-3}$	A1 (4)
	Alternatives to (b)	
	① $\mathrm{f}(x)=\frac{1-x}{x-3}=-1-\frac{2}{x-3}=-1-2(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(-2)(x-3)^{-2}$	M1 A1
	$=\frac{2}{(x-3)^2}$ $\ast$ cso	A1 (3)
	② $\mathrm{f}(x)=(1-x)(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(x-3)^{-1}+(1-x)(-1)(x-3)^{-2}$	M1
	$=-\frac{1}{x-3}-\frac{1-x}{(x-3)^2}=\frac{-(x-3)-(1-x)}{(x-3)^2}$	A1
	$=\frac{2}{(x-3)^2}$ $\ast$	A1 (3)

Q4.

Question Number	Scheme	Marks
(a)	$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$	
	$= \frac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$ An attempt to form a single fraction	M1
	$= \frac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$ Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \frac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$ An attempt to factorise a 3 term quadratic numerator	M1
	$= \frac{4x+1}{2x-1}$	A1 **(4)**
(b)	$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$	
	$f(x) = \frac{(4x+1)}{(2x-1)} - 2$	
	$= \frac{(4x+1)-2(2x-1)}{(2x-1)}$ An attempt to form a single fraction	M1
	$= \frac{4x+1-4x+2}{(2x-1)}$	
	$= \frac{3}{(2x-1)}$ Correct result	A1 * **(2)**
(c)	$f(x) = \frac{3}{(2x-1)} = 3(2x-1)^{-1}$	
	$f'(x) = 3(-1)(2x-1)^{-2}(2)$	
	$\pm k(2x-1)^{-2}$	M1
		A1 aef
	$f'(2) = \frac{-6}{9} = -\frac{2}{3}$ Either $\frac{-6}{9}$ or $-\frac{2}{3}$	A1
		(3) **[9]**

Q5.

Question Number	Scheme	Marks
	(a) $\frac{2}{x+2}+\frac{4}{x^2+5}-\frac{18}{(x+2)(x^2+5)}=\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\frac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\frac{2x}{(x^2+5)}$	A1*
		(4)
	(b) $\text{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1A1
	$\text{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
	(c) Maximum occurs when $\text{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	(c) Maximum occurs when $\text{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5}\Rightarrow \text{h}(x)=\frac{\sqrt{5}}{5}$	M1,A1
	Range of h(x) is $0\le h(x)\le\frac{\sqrt{5}}{5}$	A1ft
		(5) **(12 marks)**

(a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are

$\frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$ Eg 2An example of an error (on middle term), 1st term has been adapted

$\frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.
$\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.
Eg $\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4(x+2)}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator.
A factor of $(x+2)$ must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A, B>0$$

A1 Correct unsimplified answer $\mathrm{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}$ $\frac{-2(x^2-5)}{(x^2+5)^2}$, $\frac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$.
Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0 \le \mathrm{h}(x) \le \frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0 \le y \le \frac{\sqrt{5}}{5}$, $0 \le Range \le \frac{\sqrt{5}}{5}$, $\left[0, \frac{\sqrt{5}}{5}\right]$ but not $0 \le x \le \frac{\sqrt{5}}{5}$, $\left(0, \frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $\mathrm{h}(x)=2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being $2x$ and $(x^2+5)^{-1}$
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1}\times A+2x\times \pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1}\times 2+2x\times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept
$\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}=\frac{2(5-x^2)}{(x^2+5)^2}=\frac{-2(x^2-5)}{(x^2+5)^2}=(10-2x^2)(x^2+5)^{-2}$

Q6.

Question Number	Scheme	Marks	
(a)	$\mathrm{g}(x) \geq 1$	B1	(1)
(b)	$\mathrm{fg}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1	
	$= x^2 + 3\mathrm{e}^{x^2}$ ✱	A1	(2)
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}\right)$		
(c)	$\mathrm{fg}(x) \geq 3$	B1	(1)
(d)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\mathrm{e}^{x^2}\right) = 2x + 6x\,\mathrm{e}^{x^2}$	M1 A1	
	$2x + 6x\,\mathrm{e}^{x^2} = x^2\,\mathrm{e}^{x^2} + 2x$		
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1	
	$\mathrm{e}^{x^2} \neq 0$, $6x - x^2 = 0$	A1	
	$x = 0, 6$	A1 A1	(6) **[10]**

Q7.

Question Number	Scheme		Marks
	$x = 2\sin t,\quad y = 1 - \cos 2t\quad \left\{= 2\sin^2 t\right\},\qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t,\quad \frac{dy}{dt} = 2\sin 2t\ \text{ or }\ \frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t}\ \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
	At $t = \frac{\pi}{6},\ \frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)};\ = 1$	Correct value for $\frac{dy}{dx}$ of 1	A1 cao cso
			[4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$		M1
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 cso isw
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1
			[3]
(c)	Range: $0 \leqslant \mathrm{f}(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant \mathrm{f} \leqslant 2$	See notes	B1 B1
			[2] **9**

Notes for Question

(a)

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$.

This mark may be implied by their final answer.

Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied).

A1: For an answer of 1 ***by correct solution only.***

Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.

Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2\cos t,\ \frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$

which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

Notes for Question Continued

(b)

M1: Uses the correct double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1.

A1: Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents **in the form $y = \mathrm{f}(x)$**. For example:

$$y = \frac{2x^2}{4} \quad \text{or} \quad y = 2\left(\frac{x}{2}\right)^2 \quad \text{or} \quad y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right) \quad \text{or} \quad y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$$

and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation.
IMPORTANT: Please check working as this result can be fluked from an incorrect method.
Award A0 if there is a $+c$ added to their answer.

B1: Either $k = 2$ or a candidate writes down $-2 \leqslant x \leqslant 2$. Note: $-2 \leqslant k \leqslant 2$ unless k stated as 2 is B0.

(c)

Note: The values of 0 and/or 2 need to be evaluated in this part

B1: Achieves an inclusive upper or lower limit, using acceptable notation. Eg: $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$

B1: $0 \leqslant \mathrm{f}(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant \mathrm{f} \leqslant 2$

Special Case: SC: B1B0 for either $0 < \mathrm{f}(x) < 2$ or $0 < \mathrm{f} < 2$ or $0 < y < 2$ or $(0, 2)$

Special Case: SC: B1B0 for $0 \leqslant x \leqslant 2$.

IMPORTANT: Note that: Therefore candidates can use either y or f in place of $\mathrm{f}(x)$

Examples:

$0 \leqslant x \leqslant 2$ is SC: B1B0
$x \geqslant 0$ is B0B0
$\mathrm{f}(x) > 0$ is B0B0
$x > 0$ is B0B0
$0 \geqslant \mathrm{f}(x) \geqslant 2$ is B0B0
$0 \leqslant \mathrm{f}(x) < 2$ is B1B0.
$\mathrm{f}(x) \leqslant 2$ is B1B0
$2 \leqslant \mathrm{f}(x) \leqslant 2$ is B0B0
$|\mathrm{f}(x)| \leqslant 2$ is B1B0
$1 \leqslant \mathrm{f}(x) \leqslant 2$ is B1B0
$0 \leqslant \mathrm{f}(x) \leqslant 4$ is B1B0
$0 \leqslant$ Range $\leqslant 2$ is B1B0
$0 <$ Range < 2 is B0B0.
Range $\leqslant 2$ is B1B0
$[0, 2]$ is B1B1

$0 < x < 2$ is B0B0
$x \leqslant 2$ is B0B0
$\mathrm{f}(x) < 2$ is B0B0
$x < 2$ is B0B0
$0 < \mathrm{f}(x) \leqslant 2$ is B1B0
$\mathrm{f}(x) \geqslant 0$ is B1B0
$\mathrm{f}(x) \geqslant 0$ and $\mathrm{f}(x) \leqslant 2$ is B1B1. Must state AND {or} $\cap$
$\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ is B1B0.
$|\mathrm{f}(x)| \geqslant 2$ is B0B0
$1 < \mathrm{f}(x) < 2$ is B0B0
$0 < \mathrm{f}(x) < 4$ is B0B0
Range is in between 0 and 2 is B1B0
Range $\geqslant 0$ is B1B0
Range $\geqslant 0$ and Range $\leqslant 2$ is B1B0.
$(0, 2)$ is SC B1B0

Aliter (a) Way 2	$\frac{\mathrm{d}x}{\mathrm{d}t} = 2\cos t$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\sin 2t$, At $t = \frac{\pi}{6}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$ Hence $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$	So B1, B1. So implied M1, A1.

Notes for Question Continued			
Aliter (a) Way 3	$y=\frac{1}{2}x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=x$	Correct differentiation of their Cartesian equation.	B1ft
		Finds $\frac{\mathrm{d}y}{\mathrm{d}x}=x$, using the correct Cartesian equation only.	B1
	At $t=\frac{\pi}{6}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin\left(\frac{\pi}{6}\right)$	Finds the value of "x" when $t=\frac{\pi}{6}$ and substitutes this into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$=1$	Correct value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ of 1	A1
Aliter (b) Way 2	$y=1-\cos 2t=1-(2\cos^2 t-1)$ $y=2-2\cos^2 t \Rightarrow \cos^2 t=\frac{2-y}{2} \Rightarrow 1-\sin^2 t=\frac{2-y}{2}$		M1
	$1-\left(\frac{x}{2}\right)^2=\frac{2-y}{2}$ $y=2-2\left(1-\left(\frac{x}{2}\right)^2\right)$	(Must be in the form $y=\mathrm{f}(x)$).	A1
Aliter (b) Way 3	$x=2\sin t \Rightarrow t=\sin^{-1}\left(\frac{x}{2}\right)$	Rearranges to make t the subject and substitutes the result into y.	M1
	So, $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	$y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	A1 oe
Aliter (b) Way 4	$y=1-\cos 2t \Rightarrow \cos 2t=1-y \Rightarrow t=\frac{1}{2}\cos^{-1}(1-y)$ So, $x=\pm 2\sin\left(\frac{1}{2}\cos^{-1}(1-y)\right)$	Rearranges to make t the subject and substitutes the result into y.	M1
	So, $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	$y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	A1 oe
Aliter (b) Way 5	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin t=x \Rightarrow y=\frac{1}{2}x^2+c$	$\frac{\mathrm{d}y}{\mathrm{d}x}=x \Rightarrow y=\frac{1}{2}x^2+c$	M1
	Eg: when eg: $t=0$ (nb: $-\frac{\pi}{2}\leqslant t\leqslant\frac{\pi}{2}$), $x=0, y=1-1=0 \Rightarrow c=0 \Rightarrow y=\frac{1}{2}x^2$	Full method of finding $y=\frac{1}{2}x^2$ using a value of t: $-\frac{\pi}{2}\leqslant t\leqslant\frac{\pi}{2}$	A1
	Note: $\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin t=x \Rightarrow y=\frac{1}{2}x^2$, with no attempt to find c is M1A0.		

Q8.

Question Number	Scheme		Marks
	$x = t - 4\sin t, \quad y = 1 - 2\cos t, \quad -\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3}$ $A(k, 1)$ lies on the curve, $k > 0$		
(a)	$\{\text{When } y = 1,\}\ 1 = 1 - 2\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}$ $k \text{ (or } x) = \frac{\pi}{2} - 4\sin\left(\frac{\pi}{2}\right)$ or $x = -\frac{\pi}{2} - 4\sin\left(-\frac{\pi}{2}\right)$	Sets $y = 1$ to find t and uses their t to find x.	M1
	$\left\{\text{When } t = -\frac{\pi}{2}, k > 0,\right\}$ so $k = 4 - \frac{\pi}{2}$ or $\frac{8 - \pi}{2}$	x or $k = 4 - \frac{\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt} = 1 - 4\cos t, \quad \frac{dy}{dt} = 2\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin t}{1 - 4\cos t}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$.	M1;
	At $t = -\frac{\pi}{2}, \ \frac{dy}{dx} = \frac{2\sin\left(-\frac{\pi}{2}\right)}{1 - 4\cos\left(-\frac{\pi}{\ }\right)}$; $= -2$	Correct value for $\frac{dy}{dx}$ of -2	A1
			[4]
(c)	$\frac{2\sin t}{1 - 4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	gives $4\sin t - 4\cos t = -1$	**See notes**	A1
	So $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right); = -1$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right); = -1$	**See notes**	M1; A1
	$t = \sin^{-1}\left(\frac{-1}{4\sqrt{2}}\right) + \frac{\pi}{4}$ or $t = \cos^{-1}\left(\frac{1}{4\sqrt{2}}\right) - \frac{\pi}{4}$	**See notes**	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6] **12**

	Question Notes	
(c)	**NOTE**	**VERY IMPORTANT NOTE FOR PART (c)** Candidates who state $t = 0.6077$ with no intermediate working from $4\sin t - 4\cos t = -1$ will get 2nd M0, 2nd A0, 3rd M0, 3rd A0. They will not express $4\sin t - 4\cos t$ as either $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$. OR use any acceptable alternative method to achieve $t = 0.6077$
	NOTE	Alternative methods for part (c) are given on the next page.

	Question Alternative Methods for Part (c)		
(c)	**Alternative Method 1:**		
	$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$	M1
	eg. $\left(\frac{2\sin t}{1-4\cos t}\right)^2 = \frac{1}{4}$ or $(4\sin t)^2 = (4\cos t - 1)^2$ or $(4\sin t + 1)^2 = (4\cos t)^2$ etc.	Squaring to give a correct equation. This mark can be implied by a "squared" correct equation.	A1
		Note: You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	
		Squares their equation, applies $\sin^2 t + \cos^2 t = 1$ and achieves a **three term quadratic equation** of the form $\pm a\cos^2 t \pm b\cos t \pm c = 0$ or $\pm a\sin^2 t \pm b\sin t \pm c = 0$ or eg. $\pm a\cos^2 t \pm b\cos t = \pm c$ where $a \neq 0, b \neq 0$ and $c \neq 0$.	M1
	• **Either** $32\cos^2 t - 8\cos t - 15 = 0$ • **or** $32\sin^2 t + 8\sin t - 15 = 0$	For a correct **three term quadratic equation.**	A1
	• **Either** $\cos t = \frac{8 \pm \sqrt{1984}}{64} = \frac{1+\sqrt{31}}{8} \Rightarrow t = \cos^{-1}(...)$ • **or** $\sin t = \frac{-8 \pm \sqrt{1984}}{64} = \frac{-1 \pm \sqrt{31}}{8} \Rightarrow t = \sin^{-1}(...)$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1 **[6]**
(c)	**Alternative Method 2:**		
	$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$	M1
	eg. $(4\sin t - 4\cos t)^2 = (-1)^2$	Squaring to give a correct equation. This mark can be implied by a correct equation. **Note:** You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	A1
	So $16\sin^2 t - 32\sin t\cos t + 16\cos^2 t = 1$		
	leading to $16 - 16\sin 2t = 1$	Squares their equation, applies both $\sin^2 t + \cos^2 t = 1$ and $\sin 2t = 2\sin t\cos t$ and then achieves an equation of the form $\pm a \pm b\sin 2t = \pm c$	M1
		$16 - 16\sin 2t = 1$ or equivalent.	A1
	$\left\{\sin 2t = \frac{15}{16} \Rightarrow\right\} t = \frac{\sin^{-1}(....)}{2}$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1 **[6]**

	Question Notes	
(a)	**M1**	Sets $y=1$ to find t and uses their t to find x.
	Note	M1 can be implied by either x or $k = 4-\frac{\pi}{2}$ or 2.429... or $\frac{\pi}{2}-4$ or $-2.429...$
	A1	x or $k = 4-\frac{\pi}{2}$ or $\frac{8-\pi}{2}$
	Note	A decimal answer of 2.429... (without a correct **exact** answer) is A0.
	Note	Allow A1 for a candidate using $t=\frac{\pi}{2}$ to find $x=\frac{\pi}{2}-4$ **and** then stating that k must be $4-\frac{\pi}{2}$ o.e.
(b)	**B1**	At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. **Note:** that this mark can be implied from their working.
	B1	Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct. **Note:** that this mark can be implied from their working.
	M1	Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ **and** attempts to substitute their t into their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	Note	This mark may be implied by their final answer. i.e. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sin t}{1-4\cos t}$ followed by an answer of -2 (from $t=-\frac{\pi}{2}$) or 2 (from $t=\frac{\pi}{2}$)
	Note	**Applying** $\frac{\mathrm{d}x}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}y}{\mathrm{d}t}$ is M0, even if they state $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$.
	A1	Using $t=-\frac{\pi}{2}$ $\left(\text{and not } t=\frac{3\pi}{2}\right)$ to find a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ of -2 *by correct solution only.*

(c)		
	NOTE	If a candidate uses an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression in part (c) then the accuracy marks are not obtainable.
	1st M1	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$
	1st A1	Rearranges to give the correct equation with $\sin t$ and $\cos t$ **on the same side.** eg. $4\sin t-4\cos t=-1$ or $4\cos t-4\sin t=1$ or $\sin t-\cos t=-\frac{1}{4}$ or $\cos t-\sin t=\frac{1}{4}$ or $4\sin t-4\cos t+1=0$ or $4\cos t-4\sin t-1=0$ or $\sin t-\cos t+\frac{1}{4}=0$ etc. are fine for A1.
	2nd M1	Rewrites $\pm\lambda\sin t\pm\mu\cos t$ in the form of either $R\cos(t\pm\alpha)$ or $R\sin(t\pm\alpha)$ where $R\neq 1$ or 0 and $\alpha\neq 0$
	2nd A1	Correct equation. Eg. $4\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)=-1$ or $-4\sqrt{2}\cos\left(t+\frac{\pi}{4}\right)=-1$ or $\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)=-\frac{1}{4}$ or $\sqrt{2}\cos\left(t+\frac{\pi}{4}\right)=\frac{1}{4}$, etc.
	Note	Unless recovered, give A0 for $4\sqrt{2}\sin\left(t-45^\circ\right)=-1$ or $-4\sqrt{2}\cos\left(t+45^\circ\right)=-1$, etc.
	3rd M1	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t=...$
	4th A1	anything that rounds to 0.6077
	Note	**Do not give** the final A1 mark in (c) if there any extra solutions given in the range $-\frac{2\pi}{3}\leqslant t\leqslant\frac{2\pi}{3}$.
	Note	You can give the final A1 mark in (c) if extra solutions are given outside of $-\frac{2\pi}{3}\leqslant t\leqslant\frac{2\pi}{3}$.

Q9.

Question Number	Scheme	Marks
	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$	
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{\mathrm{d}y}{\mathrm{d}t} = -6\sin 2t$	B1 B1
	So, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}$	B1$\sqrt{\ }$ oe
		[3]
(b)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0, \quad x = 4\sin\left(\frac{\pi}{6}\right) = 2, \quad y = 3\cos 0 = 3 \quad \rightarrow (2, 3)$	M1
	@ $t = \frac{\pi}{2}, \quad x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}, \quad y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$	
	@ $t = \pi, \quad x = 4\sin\left(\frac{7\pi}{6}\right) = -2, \quad y = 3\cos 2\pi = 3 \rightarrow (-2, 3)$	
	@ $t = \frac{3\pi}{2}, x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}, \quad y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	A1A1A1
		[5] 8

(a)

B1: Either one of $\frac{\mathrm{d}x}{\mathrm{d}t} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = -6\sin 2t$. They do not have to be simplified.

B1: Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. They do not have to be simplified.

Any or both of the first two marks can be implied.
Don't worry too much about their notation for the first two B1 marks.

B1: Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ or their $\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\text{their}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$. **Note:** This is a follow through mark.

Alternative differentiation in part (a)

$$x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 2\sqrt{3}\cos t - 2\sin t$$

$$y = 3(2\cos^2 t - 1) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 3(-4\cos t\sin t)$$

$$\text{or } y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = -6\cos t\sin t - 6\sin t\cos t$$

$$\text{or } y = 3(1 - 2\sin^2 t) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 3(-4\cos t\sin t)$$

(b)

M1: Candidate sets their numerator from part (a) or their $\frac{\mathrm{d}y}{\mathrm{d}t}$ equal to 0.

Note that their numerator must be a trig function. Ignore $\frac{\mathrm{d}x}{\mathrm{d}t}$ equal to 0 at this stage.

M1: Candidate substitutes a found value of t, to attempt to find either one of x or y.
The first two method marks can be implied by ONE correct set of coordinates for (x, y) or (y, x) interchanged.
A correct point coming from NO WORKING can be awarded M1M1.
A1: At least TWO sets of coordinates.
A1: At least THREE sets of coordinates.
A1: ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0.
Note: Candidate can use the diagram's symmetry to write down some of their coordinates.

Note: When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates.

Also it is fine for candidates to display their coordinates on a table of values.
Note: The coordinates must be exact for the accuracy marks. Ie (3.46..., -3) or (-3.46..., -3) is A0.

Note: $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0.

Note: $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0.

Note: $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \sin t = 0$ & $\cos t = 0$ has the potential to achieve all five marks.

Note: It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a).

(b) An alternative method for finding the coordinates of the two maximum points.
Some candidates may use $y = 3\cos 2t$ to write down that the y-coordinate of a maximum point is 3.
They will then deduce that $t = 0$ or π and proceed to find the x-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the x-coordinates for the maximum point.

M1M1: Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t + \frac{\pi}{6}\right)$.

M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$.
Note: these marks can only be awarded together for a candidate using this method.

A1: For both $(2, 3)$ or $(-2, 3)$.

A0A0: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.

Q10.

Question Number	Scheme	Marks
	(a) $\frac{dx}{dt}=2\sqrt{3}\cos 2t$	B1
	$\frac{dy}{dt}=-8\cos t\sin t$	M1 A1
	$\frac{dy}{dx}=\frac{-8\cos t\sin t}{2\sqrt{3}\cos 2t}$	M1
	$=-\frac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\frac{dy}{dx}=-\frac{2}{3}\sqrt{3}\tan 2t \quad \left(k=-\frac{2}{3}\right)$	A1 (5)
	(b) When $t=\frac{\pi}{3}$ $x=\frac{3}{2},\ y=1$ can be implied	B1
	$m=-\frac{2}{3}\sqrt{3}\tan\left(\frac{2\pi}{3}\right) \quad (=2)$	M1
	$y-1=2\left(x-\frac{3}{2}\right)$	M1
	$y=2x-2$	A1 (4)
	(c) $x=\sqrt{3}\sin 2t=\sqrt{3}\times 2\sin t\cos t$	M1
	$x^2=12\sin^2 t\cos^2 t=12\left(1-\cos^2 t\right)\cos^2 t$	
	$x^2=12\left(1-\frac{y}{4}\right)\frac{y}{4}$ or equivalent	M1 A1 (3)
		[12]
	Alternative to (c)	
	$y=2\cos 2t+2$	M1
	$\sin^2 2t+\cos^2 2t=1$	
	$\frac{x^2}{3}+\frac{(y-2)^2}{4}=1$	M1 A1 (3)

Q11.

Question Number	Scheme		Marks
(a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 (1)
(b)	$x=t^3-8t$, $y=t^2$,		
	$\frac{\mathrm{d}x}{\mathrm{d}t}=3t^2-8$, $\frac{\mathrm{d}y}{\mathrm{d}t}=2t$		
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2t}{3t^2-8}$	Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1
	At A, $\mathrm{m}(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	
	$\mathbf{T}$: $y-(\text{their } 1)=m_T\left(x-(\text{their } 7)\right)$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$ Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+\text{"}c\text{"}$.	dM1
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1 **cso** (5)
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}$ = 55.125 or awrt 55.1 $y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	Candidate uses their value of t to find either the x or y coordinate One of either x or y correct. Both x and y correct.	ddM1 A1 A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt	(6)
			[12]

Q12.

Question Number	Scheme		Marks	
	(a) $\frac{dx}{dt} = 2\sin t\cos t,\ \frac{dy}{dt} = 2\sec^2 t$		B1 B1	
	$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t\cos t}\left(= \frac{1}{\sin t\cos^3 t}\right)$	or equivalent	M1 A1	**(4)**
	(b) At $t = \frac{\pi}{3},\ x = \frac{3}{4},\ y = 2\sqrt{3}$		B1	
	$\frac{dy}{dx} = \frac{\sec^2\frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \frac{16}{\sqrt{3}}$		M1 A1	
	$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$		M1	
	$y = 0 \Rightarrow x = \frac{3}{8}$		M1 A1	**(6)**
				[10]

Q13.

Question Number	Scheme	Marks
	(a) $y=0 \Rightarrow t\left(9-t^2\right)=t(3-t)(3+t)=0$	
	$t=0, 3, -3$ Any one correct value	B1
	At $t=0$, $x=5(0)^2-4=-4$ Method for finding one value of x	M1
	At $t=3$, $x=5(3)^2-4=41$	
	$\left(\text{At } t=-3,\ x=5(-3)^2-4=41\right)$	
	At A, $x=-4$; at B, $x=41$ Both	A1 (3)
	(b) $\frac{dx}{dt}=10t$ Seen or implied	B1
	$\int y\,dx=\int y\frac{dx}{dt}dt=\int t\left(9-t^2\right)10t\,dt$	M1 A1
	$=\int\left(90t^2-10t^4\right)dt$	
	$=\frac{90t^3}{3}-\frac{10t^5}{5}\ (+C) \quad \left(=30t^3-2t^5\ (+C)\right)$	A1
	$\left[\frac{90t^3}{3}-\frac{10t^5}{5}\right]_0^3=30\times3^3-2\times3^5\quad(=324)$	M1
	$A=2\int y\,dx=648\quad\left(\text{units}^2\right)$	A1 (6)
		[9]

Q14.

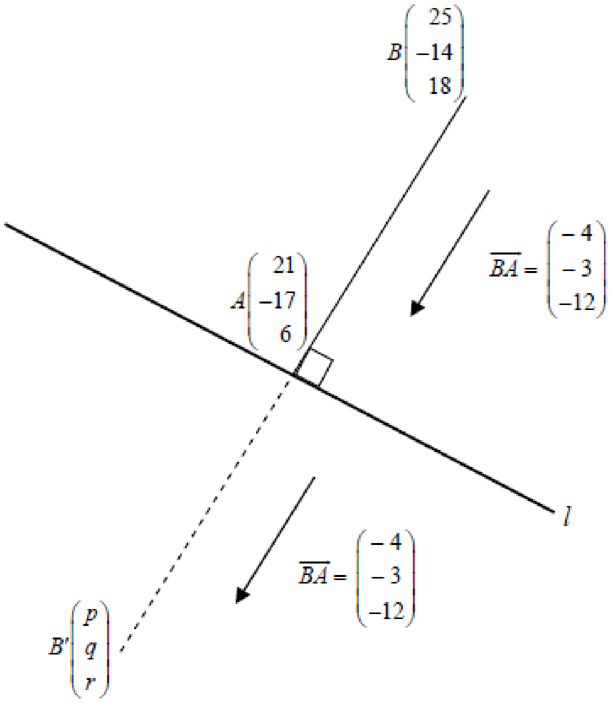
Question Number	Scheme	Marks
Q (a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = -4\sin 2t$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 6\cos t$	B1, B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{6\cos t}{4\sin 2t}$ $\left(= -\frac{3}{4\sin t}\right)$	M1
	At $t = \frac{\pi}{3}$, $m = -\frac{3}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18-9x)}$ $\left(= 3\sqrt{(2-x)}\right)$ cao	A1
	$-2 \le x \le 2$ $k = 2$	B1 (4)
(c)	$0 \le \mathrm{f}(x) \le 6$ either $0 \le \mathrm{f}(x)$ or $\mathrm{f}(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[10]

Q15.

Question Number	Scheme		Marks
	$l: \mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}, \quad \overrightarrow{OA} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$		
(a)	A is on l, so $\begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$		
	$\{\mathbf{k}: \ 10 - \lambda = 6 \Rightarrow\} \ \lambda = 4$	$\lambda = 4$	B1
	$\{\mathbf{i}: \ a + 6\lambda = 21 \Rightarrow\} \ a + 6(4) = 21$	Substitutes their value of λ into $a + 6\lambda = 21$	M1
	$a = -3$	$a = -3$	A1 cao
			[3]
(b)	$\left\{\overrightarrow{AB}\right\} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} - \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}$ | $\left\{\overrightarrow{BA}\right\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling.	M1
	$\left\{\overrightarrow{AB}\right\} = \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$ | $\left\{\overrightarrow{BA}\right\} = \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}$		
	$\left\{\overrightarrow{AB} \perp l \Rightarrow \overrightarrow{AB} \bullet \mathbf{d} = 0\right\} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 24 + 3c - 12 = 0; \Rightarrow c = -4$	See notes.	M1; A1 ft
	$\{\mathbf{j}: \ b + c\lambda = -17 \Rightarrow\} \ b + (-4)(4) = -17 \, ; \Rightarrow \ b = -1$	See notes.	ddM1; A1 cso cao
			[5]
(c)	$\lvert AB \rvert = \sqrt{4^2 + 3^2 + 12^2}$ or $\lvert AB \rvert = \sqrt{(-4)^2 + (-3)^2 + (-12)^2}$	See notes.	M1
	So, $\lvert AB \rvert = 13$		A1 cao
			[2]
(d)	$\overrightarrow{OB'} \left\{= \overrightarrow{OA} + \overrightarrow{BA}\right\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}; = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$	See notes for alternative methods.	M1;A1 cao
			[2] 12

Notes for Question

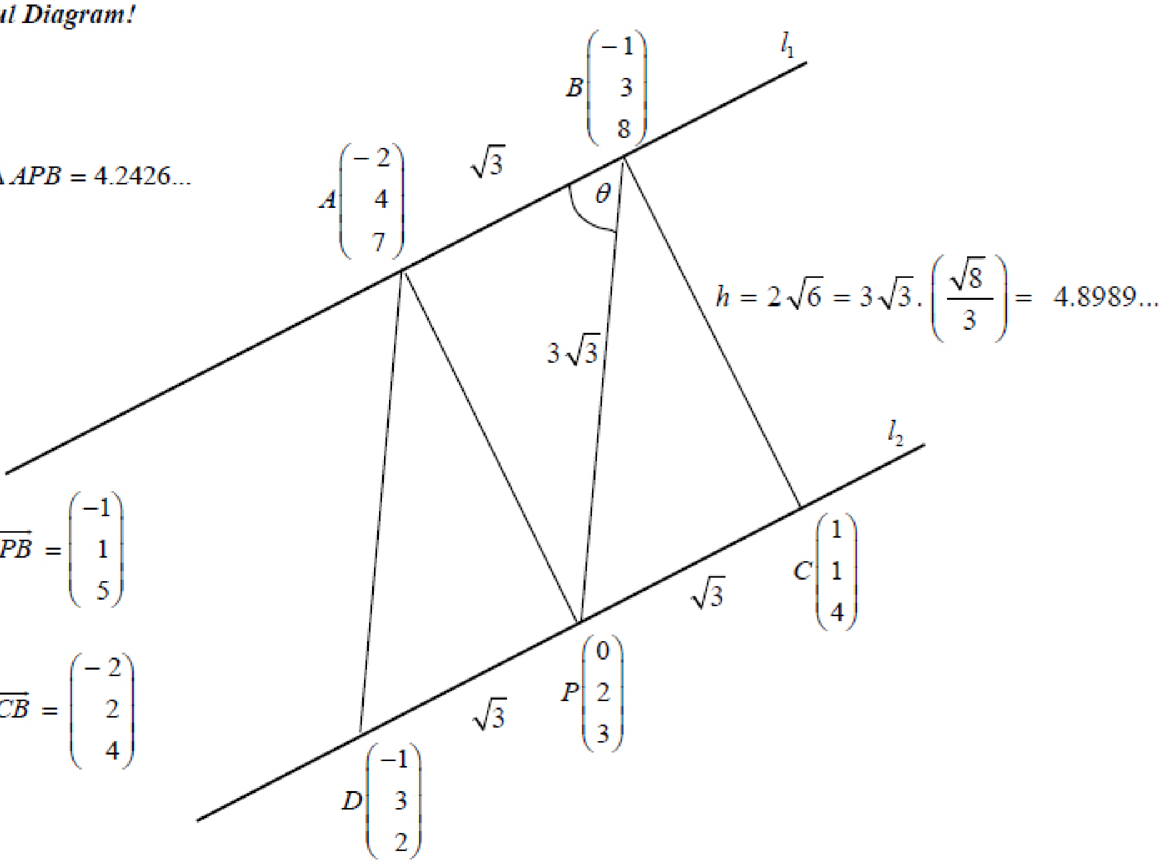
(a)	**B1:** $\lambda = 4$ seen or implied. **M1:** Substitutes their value of λ into $a + 6\lambda = 21$ **A1:** $a = -3$. Note: Award B1M1A1 if the candidate states $a = -3$ from no working. ***Alternative Method Using Simultaneous equations for part (a).*** **B1:** For $60 - 6\lambda = 36$ **M1:** $60 - 6\lambda = 36$ and $a + 6\lambda = 21$ solved simultaneously to give $a = \ldots$ **A1:** $a = -3$, **cao.**

Notes for Question Continued	
(b) ctd	**M1:** Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. **M1:** *Applies* the formula $\overrightarrow{AB}\bullet\begin{pmatrix}6\\c\\-1\end{pmatrix}$ or $\overrightarrow{BA}\bullet\begin{pmatrix}6\\c\\-1\end{pmatrix}$ correctly to give a linear equation in c which is set equal to zero. **Note:** The dot product can also be with $\pm k\begin{pmatrix}6\\c\\-1\end{pmatrix}$. **A1ft:** $c=-4$ **or** for finding a correct follow through c. **ddM1:** Substitutes their value of λ and their value of c into $b+c\lambda=-17$ **Note** that this mark is dependent on the two previous method marks being awarded. **A1:** $b=-1$
(c)	**M1:** An attempt to apply a three term Pythagoras in order to find $\lvert AB\rvert$, so taking the square root is required here. **A1:** 13 **cao** **Note:** Don't recover work for part (b) in part (c).
(d)	**M1:** For a full *applied* method of finding the coordinates of B'. **Note:** You can give M1 for 2 out of 3 correct components of B'. **A1:** For either $\begin{pmatrix}17\\-20\\-6\end{pmatrix}$ or $17\mathbf{i}-20\mathbf{j}-6\mathbf{k}$ or $(17,-20,-6)$ **cao**. ***Helpful diagram!*** 

Notes for Question Continued	
	Acceptable Methods for the Method mark in part (d)
Way 1	$\overrightarrow{OB'}\left\{=\overrightarrow{OA}+\overrightarrow{BA}\right\}=\begin{pmatrix}21\\-17\\6\end{pmatrix}+\begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)
Way 2	$\overrightarrow{OB'}\left\{=\overrightarrow{OA}-\overrightarrow{AB}\right\}=\begin{pmatrix}21\\-17\\6\end{pmatrix}-\begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)
Way 3	$\overrightarrow{OB'}\left\{=\overrightarrow{OB}+2\overrightarrow{BA}\right\}=\begin{pmatrix}25\\-14\\18\end{pmatrix}+2\begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)
Way 4	$\overrightarrow{OB'}\left\{=\overrightarrow{OB}-2\overrightarrow{AB}\right\}=\begin{pmatrix}25\\-14\\18\end{pmatrix}-2\begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)
Way 5	$\begin{pmatrix}25\\-14\\18\end{pmatrix}\rightarrow\begin{pmatrix}\text{Minus }4\\\text{Minus }3\\\text{Minus }12\end{pmatrix}\rightarrow\begin{pmatrix}21\\-17\\6\end{pmatrix}\rightarrow\begin{pmatrix}\text{Minus }4\\\text{Minus }3\\\text{Minus }12\end{pmatrix}\left\{\rightarrow\begin{pmatrix}17\\-20\\-6\end{pmatrix}\right\}$, so $\overrightarrow{OA}$ + their $\overrightarrow{BA}$
Way 6	$\overrightarrow{OB'}\left\{=2\overrightarrow{OA}-\overrightarrow{OB}\right\}=2\begin{pmatrix}21\\-17\\6\end{pmatrix}-\begin{pmatrix}25\\-14\\18\end{pmatrix}$
Way 7	$\overrightarrow{OB}=25\mathbf{i}-14\mathbf{j}+18\mathbf{k}$, $\overrightarrow{OA}=21\mathbf{i}-17\mathbf{j}+6\mathbf{k}$ and $\overrightarrow{OB'}=p\mathbf{i}+q\mathbf{j}+r\mathbf{k}$, $(21,-17,6)=\left(\dfrac{25+p}{2},\dfrac{-14+q}{2},\dfrac{18+r}{2}\right)$ $p=21(2)-25=17$ $q=-17(2)+14=-20$ $r=6(2)-18=-6$ **M1:** Writing down any two equations correctly and an attempt to find at least two of p, q or r.

Q16.

Question Number	Scheme	Marks
	$\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$, $\overrightarrow{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overrightarrow{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$	
(a)	$\overrightarrow{AB} = = \pm((-\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}));\ = \mathbf{i} - \mathbf{j} + \mathbf{k}$	M1; A1
		[2]
(b)	$\{l_1 : \mathbf{r}\ \} = \begin{pmatrix}-2\\4\\7\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix}-1\\3\\8\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	B1ft
		[1]
(c)	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$	M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{PB}\rvert} = \dfrac{\begin{pmatrix}1\\-1\\1\end{pmatrix} \bullet \begin{pmatrix}-1\\1\\5\end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2} . \sqrt{(-1)^2 + (1)^2 + (5)^2}}$ Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{PB}$ or $\overrightarrow{BP})$.	M1
	$\{\cos\theta\} = \dfrac{-1-1+5}{\sqrt{3}.\sqrt{27}} = \dfrac{3}{9} = \underline{\dfrac{1}{3}}$ Correct proof	A1 cso
		[3]
(d)	$\{l_2 : \mathbf{r} =\}\ \begin{pmatrix}0\\2\end{pmatrix} + \mu\begin{pmatrix}1\\-1\end{pmatrix}$ $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$, $\mathbf{p} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{p} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$.	M1
	Correct vector equation.	A1 ft
		[2]
(e)	$\overrightarrow{OC} = \begin{pmatrix}0\\2\\3\end{pmatrix} + \begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\overrightarrow{OD} = \begin{pmatrix}0\\2\\3\end{pmatrix} - \begin{pmatrix}1\\-1\\1\end{pmatrix}$ Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$	M1
	$\{C(1, 1, 4),\ D(-1, 3, 2)\}$ At least one set of coordinates are correct.	A1 ft
	Both sets of coordinates are correct.	A1 ft
		[3]
(f) **Way 1**	$\dfrac{h}{\sqrt{(-1)^2 + (1)^2 + (5)^2}} = \sin\theta$ $\dfrac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$	M1
	$h = \sqrt{27}\sin(70.5...)\ \left\{= \sqrt{27}\dfrac{\sqrt{8}}{3} = 2\sqrt{6} = \text{awrt } 4.9\right\}$ $\sqrt{27}\sin(70.5...)$ or $\sqrt{27}.\dfrac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent	A1 oe
	Area $ABCD = \dfrac{1}{2}2\sqrt{6}\left(\sqrt{3} + 2\sqrt{3}\right)$ $\dfrac{1}{2}(\text{their } h)(\text{their } AB + \text{their } CD)$	dM1
	$\left\{= \dfrac{1}{2}2\sqrt{6}\left(3\sqrt{3}\right) = 3\sqrt{18}\right\} = \underline{9\sqrt{2}}$ $9\sqrt{2}$	A1 cao
		[4] 15

(f)	***Helpful Diagram!*** Area $\Delta APB = 4.2426...$  $\overrightarrow{DA} = \overrightarrow{PB} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ $\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$		
(f) **Way 2**	$\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$, so $BC \perp AB$	Candidates do not need to prove this result for part (f)	
	$h = \lvert\overrightarrow{CB}\rvert = \sqrt{(-2)^2+(2)^2+(4)^2} = \sqrt{24} = 2\sqrt{6} = 4.8989...$	Attempts $\lvert PA\rvert$ or $\lvert CB\rvert$	M1
		$\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$	A1 oe
	Area $ABCD = \frac{1}{2}\sqrt{24}\left(\sqrt{3}+2\sqrt{3}\right)$ or $\frac{1}{2}\sqrt{24}\sqrt{3}+\sqrt{24}\sqrt{3}$	$\frac{1}{2}h($their AB + their $CD)$	dM1 oe
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 cso
			[4]
Way3	**<u>Finds the area of either triangle *APB* or *APD* or *BCP* and triples the result.</u>**		
(f)	Area $\Delta APB = \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\theta$	Attempts $\frac{1}{2}$ (their AB)(their PB)$\sin\theta$	M1
	$= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$	$\frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$ or $3\sqrt{2}$ or awrt 4.24 or equivalent	A1
	Area $ABCD = 3\left(3\sqrt{2}\right)$	$3\times$ Area of ΔAPB	dM1
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 cso
			[4]

	Question	Notes
. (a)	**M1**	Finding the difference (either way) between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	A1	$\mathbf{i}-\mathbf{j}+\mathbf{k}$ or $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $(1, -1, 1)$ or benefit of the doubt $\begin{matrix}1\\-1\\1\end{matrix}$
(b)	**B1ft**	$\{\mathbf{r}\}=\begin{pmatrix}-2\\4\\7\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\{\mathbf{r}\}=\begin{pmatrix}-1\\3\\8\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$, with $\overrightarrow{AB}$ or $\overrightarrow{BA}$ correctly followed through from (a).
	Note	$\mathbf{r}=$ is not needed.
(c)	**M1**	An attempt to find either the vector $\overrightarrow{PB}$ or $\overrightarrow{BP}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	M1	Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB} \text{ or } \overrightarrow{BP}\right)$.
	A1	Obtains $\{\cos\theta\}=\frac{1}{3}$ ***by correct solution only.***
	Note	If candidate starts by applying $\frac{\overrightarrow{AB}\bullet\overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert.\lvert\overrightarrow{PB}\rvert}$ correctly (without reference to $\cos\theta=...$) they can gain both 2nd M1 and A1 mark.
	Note	Award the final A1 mark if candidate achieves $\{\cos\theta\}=\frac{1}{3}$ by either taking the dot product between (i) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ or (ii) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly.
	Note	Award final A0, cso for those candidates who take the dot product between (iii) $\begin{pmatrix}1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\end{pmatrix}$ or (iv) $\begin{pmatrix}-1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\end{pmatrix}$ They will usually find $\{\cos\theta\}=-\frac{1}{3}$ or may fudge $\{\cos\theta\}=\frac{1}{3}$. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso

(c)	**Alternative Method 1: The Cosine Rule**		
	$\overrightarrow{PB}=\overrightarrow{OB}-\overrightarrow{OP}=\begin{pmatrix}-1\\3\\8\end{pmatrix}-\begin{pmatrix}0\\2\\3\end{pmatrix}=\begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP}=\begin{pmatrix}1\\-1\\-5\end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Note $\lvert\overrightarrow{PB}\rvert=\sqrt{27}$, $\lvert\overrightarrow{AB}\rvert=\sqrt{3}$ and $\lvert\overrightarrow{PA}\rvert=\sqrt{24}$		
	$\left(\sqrt{24}\right)^2=\left(\sqrt{27}\right)^2+\left(\sqrt{3}\right)^2-2\left(\sqrt{27}\right)\left(\sqrt{3}\right)\cos\theta$	Applies the cosine rule the correct way round	M1 oe
	$\cos\theta = \frac{27+3-24}{18}=\frac{1}{3}$	Correct proof	A1 cso
			[3]

(c)	**Alternative Method 2: Right-Angled Trigonometry**		
	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Either $\left(\sqrt{24}\right)^2 + \left(\sqrt{3}\right)^2 = \left(\sqrt{27}\right)^2$ or $\overrightarrow{AB} \bullet \overrightarrow{PA} = \begin{pmatrix}1\\-1\\1\end{pmatrix} \bullet \begin{pmatrix}-2\\2\\4\end{pmatrix} = -2-2+4=0$	Confirms ΔPAB is right-angled	M1
	So, $\left\{\cos\theta = \frac{AB}{PB} \Rightarrow\right\} \cos\theta = \frac{\sqrt{3}}{\sqrt{27}} = \frac{1}{3}$	Correct proof	A1 cso
			[3]

(d)	**M1**	Writing down a line in the form $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$ with either $\mathbf{a} = \begin{pmatrix}0\\2\\3\end{pmatrix}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$ $\mathbf{d} =$ their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$ found in part (a).
	A1ft	Writing $\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d} =$ their $\overrightarrow{AB}$ or a multiple of their $\overrightarrow{AB}$ found in part (a).
	Note	$\mathbf{r} =$ is not needed.
	Note	Using the same scalar parameter as in part (b) is fine for A1.
(e)	**M1**	Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$.
	A1ft	At least one set of coordinates are correct. Ignore labelling of C, D
	A1ft	Both sets of coordinates are correct. Ignore labelling of C, D
	Note	You can follow through either or both accuracy marks in this part using their $\overrightarrow{AB}$ from part (a).
(f)	**M1**	Way 1: $\frac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$ Way 2: Attempts $\lvert\overrightarrow{PA}\rvert$ or $\lvert\overrightarrow{CB}\rvert$ Way 3: Attempts $\frac{1}{2}(\text{their } PB)(\text{their } AB)\sin\theta$
	Note	Finding AD by itself is M0.
	A1	Either • $h = \sqrt{27}\sin(70.5...)$ or $\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$ or equivalent. (See Way 1 and Way 2) or • the area of either triangle APB or APD or BDP $= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\left(70.5...\right)$ o.e. (See Way 3).
	dM1	**which is dependent on the 1st M1 mark.** A full method to find the area of trapezium $ABCD$. (See Way 1, Way 2 and Way 3).
	A1	$9\sqrt{2}$ from a correct solution only.
	Note	A decimal answer of 12.7279... (without a correct **exact** answer) is A0.

Q17.

Question Number	Scheme	Marks
	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$	M1
	$\theta = \frac{\pi}{3}$ awrt 1.05	A1 **(2)**
	(b) $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \sec^2\theta, \ \frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos\theta}{\sec^2\theta} \quad \left(= \cos^3\theta\right)$	M1 A1
	At *P*, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$ Can be implied	A1
	Using $mm' = -1$, $m' = -8$	M1
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	M1
	At *Q*, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$ 1.0625	A1 **(6)**
	(c) $\int y^2\,\mathrm{d}x = \int y^2 \frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta = \int \sin^2\theta \sec^2\theta\,\mathrm{d}\theta$	M1 A1
	$= \int \tan^2\theta\,\mathrm{d}\theta$	A1
	$= \int \left(\sec^2\theta - 1\right)\mathrm{d}\theta$	M1
	$= \tan\theta - \theta \quad (+C)$	A1
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\,\mathrm{d}x = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - \left(0 - 0\right)\right]$	M1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2 \qquad \left(p = 1, q = -\frac{1}{3}\right)$	A1 **(7)**
		[15]

Q18.

Question Number	Scheme	Marks
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{t}, \quad \frac{\mathrm{d}y}{\mathrm{d}t}=2t$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2t^2$	M1 A1
	Using $mm'=-1$, at $t=3$	
	$m'=-\frac{1}{18}$	M1 A1
	$y-7=-\frac{1}{18}(x-\ln 3)$	M1 A1 (6)
(b)	$x=\ln t \Rightarrow t=\mathrm{e}^x$	B1
	$y=\mathrm{e}^{2x}-2$	M1 A1 (3)
(c)	$V=\pi\int\left(\mathrm{e}^{2x}-2\right)^2\mathrm{d}x$	M1
	$\int\left(\mathrm{e}^{2x}-2\right)^2\mathrm{d}x=\int\left(\mathrm{e}^{4x}-4\mathrm{e}^{2x}+4\right)\mathrm{d}x$	M1
	$=\frac{\mathrm{e}^{4x}}{4}-\frac{4\mathrm{e}^{2x}}{2}+4x$	M1 A1
	$\pi\left[\frac{\mathrm{e}^{4x}}{4}-\frac{4\mathrm{e}^{2x}}{2}+4x\right]_{\ln 2}^{\ln 4}=\pi\left[(64-32+4\ln 4)-(4-8+4\ln 2)\right]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6) [15]
	Alternative to (c) using parameters	
	$V=\pi\int\left(t^2-2\right)^2\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$	M1
	$\int\left(\left(t^2-2\right)^2\times\frac{1}{t}\right)\mathrm{d}t=\int\left(t^3-4t+\frac{4}{t}\right)\mathrm{d}t$	M1
	$=\frac{t^4}{4}-2t^2+4\ln t$	M1 A1
	The limits are $t=2$ and $t=4$	
	$\pi\left[\frac{t^4}{4}-2t^2+4\ln t\right]_2^4=\pi\left[(64-32+4\ln 4)-(4-8+4\ln 2)\right]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6)

Q19.

Question Number	Scheme	Marks
	Working parametrically: $x=1-\frac{1}{2}t,\quad y=2^t-1$ or $y=\mathrm{e}^{t\ln 2}-1$	
(a)	$\{x=0\Rightarrow\}\ 0=1-\frac{1}{2}t\Rightarrow t=2$ Applies $x=0$ to obtain a value for t.	M1
	When $t=2$, $y=2^2-1=3$ Correct value for y.	A1 **[2]**
(b)	$\{y=0\Rightarrow\}\ 0=2^t-1\Rightarrow t=0$ Applies $y=0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t=0$, $x=1-\frac{1}{2}(0)=1$ $x=1$	A1 **[2]**
(c)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{1}{2}$ and either $\frac{\mathrm{d}y}{\mathrm{d}t}=2^t\ln 2$ or $\frac{\mathrm{d}y}{\mathrm{d}t}=\mathrm{e}^{t\ln 2}\ln 2$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2^t\ln 2}{-\frac{1}{2}}$ Attempts their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$.	M1
	At A, $t="2"$, so $m(\mathbf{T})=-8\ln 2\Rightarrow m(\mathbf{N})=\frac{1}{8\ln 2}$ Applies $t="2"$ and $m(\mathbf{N})=\frac{-1}{m(\mathbf{T})}$	M1
	$y-3=\frac{1}{8\ln 2}(x-0)$ or $y=3+\frac{1}{8\ln 2}x$ or equivalent. See notes.	M1 A1 oe cso **[5]**
(d)	$\text{Area}(R)=\int(2^t-1).\left(-\frac{1}{2}\right)\mathrm{d}t$ Complete substitution for both y and $\mathrm{d}x$	M1
	$x=-1\rightarrow t=4$ and $x=1\rightarrow t=0$	B1
	$=\left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2}-t\right)$ Either $2^t\rightarrow\frac{2^t}{\ln 2}$ or $(2^t-1)\rightarrow\frac{(2^t)}{\pm\alpha(\ln 2)}-t$ or $(2^t-1)\rightarrow\pm\alpha(\ln 2)(2^t)-t$	M1*
	$(2^t-1)\rightarrow\frac{2^t}{\ln 2}-t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2}-t\right]_4^0\right\}=-\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right)-\left(\frac{16}{\ln 2}-4\right)\right)$ **Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$=\frac{15}{2\ln 2}-2$ $\frac{15}{2\ln 2}-2$ or equivalent.	A1 **[6]** **15**
(a)	**M1:** Applies $x=0$ and obtains a value of t. **A1:** For $y=2^2-1=3$ or $y=4-1=3$ ***Alternative Solution 1:*** **M1:** For substituting $t=2$ into either x or y. **A1:** $x=1-\frac{1}{2}(2)=0$ **and** $y=2^2-1=3$ ***Alternative Solution 2:*** **M1:** Applies $y=3$ and obtains a value of t. **A1:** For $x=1-\frac{1}{2}(2)=0$ or $x=1-1=0$. ***Alternative Solution 3:*** **M1:** Applies $y=3$ or $x=0$ and obtains a value of t. **A1:** Shows that $t=2$ for both $y=3$ and $x=0$.	
(b)	**M1:** Applies $y=0$ and obtains a value of t. Working must be seen in part (b). **A1:** For finding $x=1$. **Note:** Award M1A1 for $x=1$.	
(c)	**B1:** Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. This mark can be implied by later working. **M1:** Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ or their $\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{1}{\text{their}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$. **Note:** their $\frac{\mathrm{d}y}{\mathrm{d}t}$ must be a function of t. **M1:** Uses their value of t found in part (a) and applies $m(\mathbf{N})=\frac{-1}{m(\mathbf{T})}$. **M1:** $y-3=(\text{their normal gradient})x$ **or** $y=(\text{their normal gradient})x+3$ or equivalent. **A1:** $y-3=\frac{1}{8\ln 2}(x-0)$ or $y=3+\frac{1}{8\ln 2}x$ or $y-3=\frac{1}{\ln 256}(x-0)$ or $(8\ln 2)y-24\ln 2=x$ or $\frac{y-3}{(x-0)}=\frac{1}{8\ln 2}$. You can apply isw here. **Working in decimals** is ok for the three method marks. B1, A1 require exact values.	
(d)	**M1:** Complete substitution for both y and $\mathrm{d}x$. So candidate should write down $\int(2^t-1).\left(\text{their }\frac{\mathrm{d}x}{\mathrm{d}t}\right)$ **B1:** Changes limits from $x\rightarrow t$. $x=-1\rightarrow t=4$ and $x=1\rightarrow t=0$. Note $t=4$ and $t=0$ seen is B1. **M1*:** Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$ ... or integrates (2^t-1) to give either $\frac{(2^t)}{\pm\alpha(\ln 2)}-t$ or $\pm\alpha(\ln 2)(2^t)-t$. **A1:** Correct integration of (2^t-1) with respect to t to give $\frac{2^t}{\ln 2}-t$. **dM1*: Depends upon the previous method mark.** Substitutes their limits in t and subtracts either way round. **A1:** Exact answer of $\frac{15}{2\ln 2}-2$ or $\frac{15}{\ln 4}-2$ or $\frac{15-4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2}-2$ or $\frac{15}{2}\log_2\mathrm{e}-2$ or equivalent.	
	Alternative: Converting to a Cartesian equation: $t=2-2x\Rightarrow\quad y=2^{2-2x}-1$	
(a)	$\{x=0\Rightarrow\}\ y=2^2-1$ Applies $x=0$ in their Cartesian equation...	M1
	$y=3$ ... to arrive at a correct answer of 3.	A1 **[2]**
(b)	$\{y=0\Rightarrow\}\ 0=2^{2-2x}-1\Rightarrow 0=2-2x\Rightarrow x=\ldots$ Applies $y=0$ to obtain a value for x. (Must be seen in part (b)).	M1
	$x=1$ $x=1$	A1 **[2]**
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\left(2^{2-2x}\right)\ln 2$ $\pm\lambda 2^{2-2x},\ \lambda\neq 1$	M1
	$-2\left(2^{2-2x}\right)\ln 2$ or equivalent	A1
	At A, $x=0$, so $m(\mathbf{T})=-8\ln 2\Rightarrow m(\mathbf{N})=\frac{1}{8\ln 2}$ Applies $x=0$ and $m(\mathbf{N})=\frac{-1}{m(\mathbf{T})}$	M1
	$y-3=\frac{1}{8\ln 2}(x-0)$ or $y=3+\frac{1}{8\ln 2}x$ or equivalent. ***As in the original scheme.***	M1 A1 oe **[5]**
(d)	$\text{Area}(R)=\int\left(2^{2-2x}-1\right)\mathrm{d}x$ Form the integral of their Cartesian equation of C.	M1
	$=\int_{-1}^{1}\left(2^{2-2x}-1\right)\mathrm{d}x$ For $2^{2-2x}-1$ with limits of $x=-1$ and $x=1$. Ie. $\int_{-1}^{1}\left(2^{2-2x}-1\right)$	B1
	$=\left(\frac{2^{2-2x}}{-2\ln 2}-x\right)$ Either $2^{2-2x}\rightarrow\frac{2^{2-2x}}{-2\ln 2}$ or $\left(2^{2-2x}-1\right)\rightarrow\frac{2^{2-2x}}{\pm\alpha(\ln 2)}-x$ or $\left(2^{2-2x}-1\right)\rightarrow\pm\alpha(\ln 2)(2^{2-2x})-x$	M1*
	$\left(2^{2-2x}-1\right)\rightarrow\frac{2^{2-2x}}{-2\ln 2}-x$	A1
	$\left\{\left[\frac{2^{2-2x}}{-2\ln 2}-x\right]_{-1}^{1}\right\}=\left(\left(\frac{1}{-2\ln 2}-1\right)-\left(\frac{16}{-2\ln 2}+1\right)\right)$ **Depends on the previous method mark.** Substitutes limits of -1 and their x_B and subtracts either way round.	dM1*
	$=\frac{15}{2\ln 2}-2$ $\frac{15}{2\ln 2}-2$ or equivalent.	A1 **[6]** **15**
(d)	**Alternative method: In Cartesian and applying $u=2-2x$** $\text{Area}(R)=\int\left(2^u-1\right)\{\mathrm{d}x\}$, where $u=2-2x$ $=\int_4^0\left(2^u-1\right)\left(-\tfrac{1}{2}\right)\{\mathrm{d}u\}$ **M0:** Unless a candidate ***writes*** $\int\left(2^{2-2x}-1\right)\{\mathrm{d}x\}$ Then apply the "working parametrically" mark scheme.	
(d)	***Alternative method: For substitution* u = 2^t** $\text{Area}(R)=\int(2^t-1).\left(-\frac{1}{2}\right)\mathrm{d}t$ Complete substitution for both y and $\mathrm{d}x$	M1
	where $u=2^t\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}t}=2^t\ln 2\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}t}=u\ln 2$ $x=-1\rightarrow t=4\rightarrow u=16$ and $x=1\rightarrow t=0\rightarrow u=1$ Both correct limits in t or both correct limits in u.	B1
	So $\text{area}(R)=-\frac{1}{2}\int\frac{u-1}{u\ln 2}\mathrm{d}u$ $=-\frac{1}{2}\int\frac{1}{\ln 2}-\frac{1}{u\ln 2}\mathrm{d}u$ If not awarded above, you can award M1 for this integral	
	$=\left\{-\frac{1}{2}\right\}\left(\frac{u}{\ln 2}-\frac{\ln u}{\ln 2}\right)$ Either $2^t\rightarrow\frac{u}{\ln 2}$ or $(2^t-1)\rightarrow\frac{u}{\pm\alpha(\ln 2)}-\frac{\ln u}{\ln 2}$ or $(2^t-1)\rightarrow\pm\alpha(\ln 2)(u)-\frac{\ln u}{\ln 2}$	M1*
	$(2^t-1)\rightarrow\frac{u}{\ln 2}-\frac{\ln u}{\ln 2}$	A1
	$\left\{-\frac{1}{2}\left[\frac{u}{\ln 2}-\frac{\ln u}{\ln 2}\right]_{16}^{1}\right\}=-\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right)-\left(\frac{16}{\ln 2}-\frac{\ln 16}{\ln 2}\right)\right)$ **Depends on the previous method mark.** Substitutes their changed limits ***in u*** and subtracts either way round.	dM1*
	$=\frac{15}{2\ln 2}-\frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2}-2$ $\frac{15}{2\ln 2}-\frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2}-2$ or equivalent.	A1 **[6]**

Q20.

Question No	Scheme	Marks
	(a)	
	$\frac{d}{dx}(\ln(3x)) \to \frac{B}{x}$ for any constant B	M1
	Applying vu'+uv' , $\ln(3x) \times 2x + x$	M1, A1 A1
		(4)
	(b)	
	Applying $\frac{vu'-uv\prime}{v^2}$	
	$\frac{x^3 \times 4cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 A1+A1 A1
	$= \frac{4xcos(4x) - 3\sin(4x)}{x^4}$	A1
		(5)
		(9 MARKS)

Q21.

Question No	Scheme	Marks
	$\left(\frac{dx}{dy}\right) = 2sec^2\left(y + \frac{\pi}{12}\right)$	M1,A1
	substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = 2sec^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 8$	M1, A1
	When $y = \frac{\pi}{4}$. $x = 2\sqrt{3}$ awrt 3.46	B1
	$\left(y - \frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$	M1
	$\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$ oe	A1 (7 marks)

Q22.

Question Number	Scheme	Marks
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\frac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\frac{5}{2}x^{2}(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\frac{10}{\sqrt{9}}=12+\frac{10}{3}$	M1
	$=\frac{46}{3}$ Accept awrt 15.3	A1 **(6)**
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin 2x}{x^{2}}\right)=\frac{2x^{2}\cos 2x-2x\sin 2x}{x^{4}}$	M1 $\frac{\text{A1+A1}}{\text{A1}}$ **(4)** **[10]**
	Alternative to (b) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x \quad \left(=\frac{2\cos 2x}{x^{2}}-\frac{2\sin 2x}{x^{3}}\right)$	A1 **(4)**

Q23.

Question Number	Scheme	Marks
	$x=\cos(2y+\pi)$	
	$\frac{\mathrm{d}x}{\mathrm{d}y}=-2\sin(2y+\pi)$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2\sin(2y+\pi)}$ Follow through their $\frac{\mathrm{d}x}{\mathrm{d}y}$ before or after substitution	A1ft
	At $y=\frac{\pi}{4}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2\sin\frac{3\pi}{2}}=\frac{1}{2}$	B1
	$y-\frac{\pi}{4}=\frac{1}{2}x$	M1
	$y=\frac{1}{2}x+\frac{\pi}{4}$	A1 (6)
		[6]

Q24.

Question Number	Scheme		Marks
(a)	$y = \frac{3+\sin 2x}{2+\cos 2x}$		
	Apply quotient rule: $\left\{\begin{array}{ll} u = 3+\sin 2x & v = 2+\cos 2x \\ \frac{du}{dx} = 2\cos 2x & \frac{dv}{dx} = -2\sin 2x \end{array}\right\}$		
	$\frac{dy}{dx} = \frac{2\cos 2x(2+\cos 2x) - -2\sin 2x(3+\sin 2x)}{(2+\cos 2x)^2}$	Applying $\frac{vu' - uv'}{v^2}$ Any one term correct on the numerator Fully correct (unsimplified).	M1 A1 A1
	$= \frac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2+\cos 2x)^2}$		
	$= \frac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2+\cos 2x)^2}$		
	$= \frac{4\cos 2x + 6\sin 2x + 2}{(2+\cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \frac{3+\sin\pi}{2+\cos\pi} = \frac{3}{1} = 3$	$y = 3$	B1
	At $(\frac{\pi}{2}, 3)$, $\mathrm{m(T)} = \frac{6\sin\pi + 4\cos\pi + 2}{(2+\cos\pi)^2} = \frac{-4+2}{1^2} = -2$	$\mathrm{m(T)} = -2$	B1
	Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) **[8]**

Q25.

Question Number	Scheme	Marks
	At P, $y = \underline{3}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \frac{18}{(5-3x)^3}\right\}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{18}{(5-3(2))^3}$ $\{= -18\}$ $\mathrm{m(N)} = \frac{-1}{-18}$ or $\frac{1}{18}$ N: $y-3=\frac{1}{18}(x-2)$ N: $\underline{x-18y+52=0}$	B1 M1A1 M1 M1 M1 A1 **[7]**
	1st M1: $\pm k(5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2nd M1: Substituting $x=2$ into an equation involving their $\frac{\mathrm{d}y}{\mathrm{d}x}$; 3rd M1: Uses $\mathrm{m(N)} = -\frac{1}{\text{their m}(\mathbf{T})}$. 4th M1: $y-y_1=m(x-2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y=mx+c$ with 'their NORMAL ("changed" **numerical**) gradient', their y_1 and $x = 2$. Note: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	

Q26.

Question Number	Scheme		Marks
(i)	$y = \dfrac{\ln(x^2+1)}{x}$		
	$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \to \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \to \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{\begin{array}{ll} u = \ln(x^2+1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 1 \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x) - \ln(x^2+1)}{x^2}$	Applying $\dfrac{xu' - \ln(x^2+1)v'}{x^2}$ correctly.	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2}{(x^2+1)} - \dfrac{1}{x^2}\ln(x^2+1)\right\}$	{Ignore subsequent working.}	
(ii)	$x = \tan y$		
	$\dfrac{dx}{dy} = \sec^2 y$	$\tan y \to \sec^2 y$ or an attempt to differentiate $\dfrac{\sin y}{\cos y}$ using either the quotient rule or product rule.	M1*
		$\dfrac{dx}{dy} = \sec^2 y$	A1
	$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$ $\{= \cos^2 y\}$	Finding $\dfrac{dy}{dx}$ by reciprocating $\dfrac{dx}{dy}$.	dM1*
	$\dfrac{dy}{dx} = \dfrac{1}{1 + \tan^2 y}$	For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y.	dM1*
	Hence, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, (as required)	For the correct proof, leading on from the previous line of working.	A1 AG
			(5)
			[9]

Q27.

Question Number	Scheme		Marks
(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	$\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both $\underline{\underline{\text{underlined steps.}}}$	A1 AG
			(3)
(b)	$y = e^{2x}\sec 3x$		
	$\left\{\begin{array}{ll} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x\tan 3x \end{array}\right\}$	Seen or implied Either $e^{2x} \to 2e^{2x}$ or $\sec 3x \to 3\sec 3x\tan 3x$	M1
		Both $e^{2x} \to 2e^{2x}$ and $\sec 3x \to 3\sec 3x\tan 3x$	A1
	$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x\tan 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v'	M1
		$2e^{2x}\sec 3x + 3e^{2x}\sec 3x\tan 3x$	A1 isw
			(4)
(c)	Turning point $\Rightarrow \dfrac{dy}{dx} = 0$		
	Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$	Sets their $\dfrac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms.	M1
	{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3\tan 3x = 0$, }		
	giving $\tan 3x = -\frac{2}{3}$	$\tan 3x = \pm k$; $k \neq 0$	M1
	$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600...$	Either awrt -0.196^c or awrt -11.2°	A1
	Hence, $y = \{b\} = e^{2(-0.196)}\sec(3 \times -0.196)$		
	$= 0.812093... = 0.812$ (3sf)	0.812	A1 cao
			(4)
			[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Q28.

Question Number	Scheme		Marks
Q (i)(a)	$y = x^2\cos 3x$		
	Apply product rule: $\left\{\begin{matrix} u = x^2 & v = \cos 3x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 2x & \frac{\mathrm{d}v}{\mathrm{d}x} = -3\sin 3x \end{matrix}\right\}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\cos 3x - 3x^2\sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x\cos 3x \pm \beta x^2\sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos\alpha x^2$ or $\sin\beta x^3$.	A1
			(3)
(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \frac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \frac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{\begin{matrix} u = \ln(x^2+1) & v = x^2+1 \\ \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2x}{x^2+1} & \frac{\mathrm{d}v}{\mathrm{d}x} = 2x \end{matrix}\right\}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	Applying $\frac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)

Q29.

Question Number	Scheme	Marks
	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)

(a) M1 Substitute y=-32 into $y=(2w-3)^5$ and proceed to w=….. [Accept positive sign used of y, ie y=+32]

A1 Obtains w or $x=\frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5.

Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y=(2x-3)^5$ using the chain rule.

Sight of $\pm A(2x-3)^4$ where A is a non- zero constant is sufficient for the method mark.

A1 A correct (un simplified) form of the differential.

Accept $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $\frac{dy}{dx}=10(2x-3)^4$

M1 This is awarded for an attempt to find the gradient of the tangent to the curve at P
Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent

dM1 Award for a correct method to find an equation of the tangent to the curve at P. It is dependent upon the previous M mark being awarded.

$$\text{Award for } '\textit{their } 160' = \frac{y-(-32)}{x-\textit{their } '\frac{1}{2}'}$$

If they use $y=mx+c$ it must be a full method, using m= 'their 160', their '$\frac{1}{2}$' and -32.

An attempt must be seen to find c=…

A1 cso $y=160x-112$. The question is specific and requires the answer in this form.

You may isw in this question after a correct answer.

Q30.

Question Number	Scheme		Marks
	(i)(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$ $= 3x^2 \ln 2x + x^2$		M1A1A1
			(3)
	(i)(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3(\boldsymbol{x + \sin 2x})^2 \times (1 + 2\cos 2x)$		**B1**M1A1
			(3)
	(ii) $\frac{\mathrm{d}x}{\mathrm{d}y} = -\mathrm{cosec}^2 y$		M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\mathrm{cosec}^2 y}$		M1
	Uses $\mathrm{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ to get an expression in x		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\mathrm{cosec}^2 y} = -\frac{1}{1 + \cot^2 y} = -\frac{1}{1 + x^2}$	cso	M1, A1*
			(5) **(11 marks)**

(i)(a) M1 Applies the product rule vu'+uv' to $x^3\ln 2x$.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$Ax^2\times\ln 2x + x^3\times\frac{B}{x} \qquad \text{where } A, B \text{ are constants}\neq 0$$

A1 One term correct, either $3x^2\times\ln 2x$ or $x^3\times\frac{1}{2x}\times 2$

A1 Cao. $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2\times\ln 2x+x^3\times\frac{1}{2x}\times 2$. The answer does not need to be simplified.

For reference the simplified answer is $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2\ln 2x+x^2=x^2(3\ln 2x+1)$

(i)(b) B1 Sight of $(x+\sin 2x)^2$

M1 For applying the chain rule to $(x+\sin 2x)^3$. If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of $C(x+\sin 2x)^2\times(1\pm D\cos 2x)$ where C and D are non- zero constants.

Alternatively accept $u=x+\sin 2x$, u'= followed by $Cu^2\times$ their u'

Do not accept $C(x+\sin 2x)^2\times 2\cos 2x$ unless you have evidence that this is their u'

Allow 'invisible' brackets for this mark, ie. $C(x+\sin 2x)^2\times 1\pm D\cos 2x$

A1 Cao $\frac{dy}{dx}=3(x+\sin 2x)^2\times(1+2\cos 2x)$. There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

(ii) M1 Writing the derivative of coty as -cosec^{2}y. It must be in terms of y

A1 $\frac{\mathrm{d}x}{\mathrm{d}y}=-\mathrm{cosec}^2 y$ or $1=-\cos\mathrm{ec}^2 y\frac{\mathrm{d}y}{\mathrm{d}x}$. Both lhs and rhs must be correct.

M1 Using $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\mathrm{d}x/\mathrm{d}y}$

M1 Using $\mathrm{cosec}^2 y=1+\cot^2 y$ **and** $x=\cot y$ to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ just in terms of x.

A1 cso $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{1+x^2}$

Alternative to (a)(i) when ln(2x) is written lnx+ln2

M1 Writes $x^3\ln 2x$ as $x^3\ln 2+x^3\ln x$.

Achieves Ax^2 for differential of $x^3\ln 2$ and applies the product rule vu'+uv' to $x^3\ln x$.

A1 Either $3x^2\times\ln 2+3x^2\ln x$ or $x^3\times\frac{1}{x}$

A1 A correct (un simplified) answer. Eg $3x^2\times\ln 2+3x^2\ln x+x^3\times\frac{1}{x}$

Alternative to (ii) using quotient rule

M1 Writes $\cot y$ as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$)

only accept answers of the form $\frac{\sin y\times\pm\sin y-\cos y\times\pm\cos y}{(\sin y)^2}$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{\sin y\times-\sin y-\cos y\times\cos y}{(\sin y)^2}=\left\{-1-\cot^2 y\right\}$$

M1 Using $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\mathrm{d}x/\mathrm{d}y}$

M1 Using $\sin^2 y+\cos^2 y=1$, $\frac{1}{\sin^2 y}=\mathrm{cosec}^2 y$ and $\mathrm{cosec}^2 y=1+\cot^2 y$ to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in x

A1 cso $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{1+x^2}$

Alternative to (ii) using the chain rule, first two marks

M1 Writes $\cot y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).

Accept answers of the form $-(\tan y)^{-2}\times\sec^2 y$

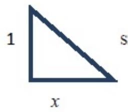
A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{\mathrm{d}x}{\mathrm{d}y}=-(\tan y)^{-2}\times\sec^2 y$$

Alternative to (ii) using a triangle – last M1

M1 Uses triangle with $\tan y=\frac{1}{x}$ to find siny and get $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ just in terms of x

$x=\cot y\Rightarrow\tan y=\frac{1}{x}$



$\sin y=\frac{1}{\sqrt{1+x^2}}$

Q31.

Question Number	Scheme	Marks
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$ Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$ $= \frac{\sqrt{x}\times -2\sin 2x - \cos 2x \times \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2}$ $= \frac{-2\sqrt{x}\sin 2x - \frac{1}{2}x^{-\frac{1}{2}}\cos 2x}{x}$	B1 M1A1 **(3)**
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}(\sec^2 3x) = 2\sec 3x \times 3\sec 3x \tan 3x\,(= 6\sec^2 3x \tan 3x)$ $= 6(1+\tan^2 3x)\tan 3x$ $= 6(\tan 3x + \tan^3 3x)$	M1 dM1 A1 **(3)**
(c)	$x = 2\sin\left(\frac{y}{3}\right) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2}{3}\cos\left(\frac{y}{3}\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{2}{3}\cos\left(\frac{y}{3}\right)} = \frac{1}{\frac{2}{3}\sqrt{\left(1-\sin^2\left(\frac{y}{3}\right)\right)}} = \frac{1}{\frac{2}{3}\sqrt{1-\left(\frac{x}{2}\right)^2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{4-x^2}}$ cao	M1A1 dM1 A1 **(4)** **(10 marks)**
Alt (c)	$y = 3\arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \times \frac{1}{2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{4-x^2}}$ M1 Rearranging to $y = A\arcsin Bx$ and differentiating to $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{\sqrt{1-Bx^2}}$ dM1 As above, but form of the rhs must be correct $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{C}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$ A1 Correct but un simplified answer	M1dM1A1 A1 **(4)**

Notes for Question

(a)

B1 Award for the sight of $\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{\sqrt{x} \times \pm A\sin 2x - \cos 2x \times Bx^{-\frac{1}{2}}}{(\sqrt{x})^2 \text{ or } x^{\frac{1}{4}}}$$

A1 Award for a correct answer. This does not need to be simplified.

Alt (a) using the product rule

B1 Award for the sight of $\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $vu'+uv'$ to $x^{-\frac{1}{2}}\cos 2x$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $vu'+uv'$) then only accept answers of the form

$$\pm Ax^{-\frac{1}{2}}\sin 2x - Bx^{-\frac{3}{2}}\cos 2x$$

A1 Award for a correct answer. This does not need to be simplified.

$$-2x^{-\frac{1}{2}}\sin 2x - \frac{1}{2}x^{-\frac{3}{2}}\cos 2x$$

(b)

M1 Award for a correct application of the chain rule on $\sec^2 3x$

Sight of $C\sec 3x \sec 3x \tan 3x$ is sufficient

dM1 Replacing $\sec^2 3x = 1+\tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using the product rule

M1 Writes $\sec^2 3x$ as $\sec 3x \times \sec 3x$ and uses the product rule with $u' = A\sec 3x \tan 3x$ and $v' = B\sec 3x \tan 3x$ to produce a derivative of the form $A\sec 3x \tan 3x \sec 3x + B\sec 3x \tan 3x \sec 3x$

dM1 Replaces $\sec^2 3x$ with $1+\tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

Notes for Question Continued

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using $\sec 3x = \dfrac{1}{\cos 3x}$ **and proceeding by the chain or quotient rule**

M1 Writes $\sec^2 3x$ as $(\cos 3x)^{-2}$ and differentiates to $A(\cos 3x)^{-3}\sin 3x$

Alternatively writes $\sec^2 3x$ as $\dfrac{1}{(\cos 3x)^2}$ and achieves $\dfrac{(\cos 3x)^2 \times 0 - 1 \times A\cos 3x \sin 3x}{\left(\cos^2 3x\right)^2}$

dM1 Uses $\dfrac{\sin 3x}{\cos 3x} = \tan 3x$ and $\dfrac{1}{\cos^2 3x} = \sec^2 3x$ and $\sec^2 3x = 1 + \tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using $\sec^2 3x = 1 + \tan^2 3x$

M1 Writes $\sec^2 3x$ as $1 + \tan^2 3x$ and
uses chain rule to produce a derivative of the form $A\tan 3x \sec^2 3x$
or the product rule to produce a derivative of the form $C\tan 3x \sec^2 3x + D\tan 3x \sec^2 3x$

dM1 Replaces $\sec^2 3x = 1 + \tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

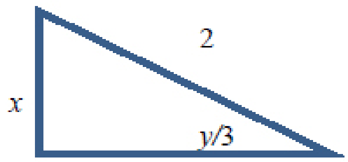
A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

(c)

M1 Award for knowing the method that $\sin\left(\dfrac{y}{3}\right)$ differentiates to $\cos\left(\dfrac{y}{3}\right)$ The lhs does not need to be correct/present. Award for $2\sin\left(\dfrac{y}{3}\right) \to A\cos\left(\dfrac{y}{3}\right)$

A1 $x = 2\sin\left(\dfrac{y}{3}\right) \Rightarrow \dfrac{dx}{dy} = \dfrac{2}{3}\cos\left(\dfrac{y}{3}\right)$. Both sides must be correct

dM1 Award for inverting their $\dfrac{dx}{dy}$ and using $\sin^2\dfrac{y}{3} + \cos^2\dfrac{y}{3} = 1$ to produce an expression for $\dfrac{dy}{dx}$ in terms of x only. It is dependent upon the first M 1 being scored.
An alternative to Pythagoras is a triangle.



$$\sin\left(\frac{y}{3}\right) = \frac{x}{2} \Rightarrow \cos\left(\frac{y}{3}\right) = \frac{\sqrt{4 - x^2}}{2}$$

Notes for Question Continued

Candidates who write $\frac{dy}{dx} = \frac{3}{2\cos\left(\arcsin\left(\frac{x}{2}\right)\right)}$ do not score the mark.

BUT $\frac{dy}{dx} = \frac{3}{2\sqrt{1-\sin^2\left(\arcsin\left(\frac{x}{2}\right)\right)}}$ does score M1 as they clearly use a correct Pythagorean identity as required by the notes.

A1 $\frac{dy}{dx} = \frac{3}{\sqrt{4-x^2}}$. Expression must be in its simplest form.

Do not accept $\frac{dy}{dx} = \frac{3}{2\sqrt{1-\frac{1}{4}x^2}}$ or $\frac{dy}{dx} = \frac{1}{\frac{1}{3}\sqrt{4-x^2}}$ for the final A1

Q32.

Question Number	Scheme	Marks
(a)	$\mathrm{f}(x)=\frac{4x+1}{x-2}, \quad x>2$	
	Applies $\frac{vu'-uv'}{v^2}$ to get $\frac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$	M1A1
	$=\frac{-9}{(x-2)^2}$	A1*
		(3)
(b)	$\frac{-9}{(x-2)^2}=-1\Rightarrow x=..$	M1
	$(5,7)$	A1,A1
		(3)
		6 marks
Alt 1.(a)	$\mathrm{f}(x)=\frac{4x+1}{x-2}=4+\frac{9}{x-2}$	
	Applies chain rule to get $\mathrm{f}'(x)=A(x-2)^{-2}$	M1
	$=-9(x-2)^{-2}=\frac{-9}{(x-2)^2}$	A1, A1*
		(3)

(a)

M1 Applies the quotient rule to $\mathrm{f}(x)=\frac{4x+1}{x-2}$ with $u=4x+1$ and $v=x-2$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=x-2, u'=.., v'=..$ followed by $\frac{vu'-uv'}{v^2}$.

If neither quoted nor implied only accept expressions of the form $\frac{(x-2)\times A-(4x+1)\times B}{(x-2)^2}$ $A,B>0$ allowing for a sign slip inside the brackets.

Condone missing brackets for the method mark but not the final answer mark.

Alternatively they could apply the product rule with $u=4x+1$ and $v=(x-2)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=(x-2)^{-1}, u'=.., v'=..$ followed by $vu'+uv'$.

If it is neither quoted nor implied only accept expressions of the form/ or equivalent to the form

$$(x-2)^{-1}\times C+(4x+1)\times D(x-2)^{-2}$$

A third alternative is to use the Chain rule. For this to score there must have been some attempt to divide first to achieve $\mathrm{f}(x)=\frac{4x+1}{x-2}=..+\frac{..}{x-2}$ before applying the chain rule to get $\mathrm{f}'(x)=A(x-2)^{-2}$

A1 A correct and unsimplified form of the answer.

Accept $\frac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$ from the quotient rule

Accept $\frac{4x-8-4x-1}{(x-2)^2}$ from the quotient rule even if the brackets were missing in line 1

Accept $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}$ or equivalent from the product rule

Accept $9\times -1(x-2)^{-2}$ from the chain rule

A1* Proceeds to achieve the given answer $=\frac{-9}{(x-2)^2}$. Accept $-9(x-2)^{-2}$

All aspects must be correct including the bracketing.

If they differentiated using the product rule the intermediate lines must be seen.

Eg. $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}=\frac{4}{(x-2)}-\frac{4x+1}{(x-2)^2}=\frac{4(x-2)-(4x+1)}{(x-2)^2}=\frac{-9}{(x-2)^2}$

(b)

M1 Sets $\frac{-9}{(x-2)^2}=-1$ and proceeds to $x=\dots$

The minimum expectation is that they multiply by $(x-2)^2$ and then either, divide by -1 before square rooting or multiply out before solving a 3TQ equation.

A correct answer of $x=5$ would also score this mark following $\frac{-9}{(x-2)^2}=-1$ as long as no incorrect work is seen.

A1 $x=5$

A1 (5, 7) or $x=5$, $y=7$. Ignore any reference to $x=-1$ (and $y=1$). Do not accept 21/3 for 7

If there is an extra solution, $x>2$, then withhold this final mark.

Q33.

Question Number	Scheme	Marks
(a)	$x = 8\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi$	B1*
		(1)
(b)	$\frac{\mathrm{d}x}{\mathrm{d}y} = 8\tan 2y + 16y\sec^2(2y)$	M1A1A1
	At P $\frac{\mathrm{d}x}{\mathrm{d}y} = 8\tan 2\frac{\pi}{8} + 16\frac{\pi}{8}\sec^2\left(2\times\frac{\pi}{8}\right) = \{8+4\pi\}$	M1
	$\frac{y-\frac{\pi}{8}}{x-\pi} = \frac{1}{8+4\pi}$, accept $y-\frac{\pi}{8} = 0.049(x-\pi)$	M1A1
	$\Rightarrow (8+4\pi)y = x + \frac{\pi^2}{2}$	A1
		(7)
		(8 marks)

(a)

B1* Either sub $y=\frac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow x = 8\times\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi$

Or sub $x=\pi$, $y=\frac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow \pi = 8\times\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi\times 1 = \pi$

This is a proof and therefore an expectation that at least one intermediate line must be seen, including a term in tangent.

Accept as a minimum $y=\frac{\pi}{8} \Rightarrow x = \pi\tan\left(\frac{\pi}{4}\right) = \pi$

Or $\pi = \pi\times\tan\left(\frac{\pi}{4}\right) = \pi$ ✓

This is a given answer however, and as such there can be no errors.

(b)

M1 Applies the product rule to $8y\tan 2y$ achieving $A\tan 2y + By\sec^2(2y)$

A1 One term correct. Either $8\tan 2y$ or $+16y\sec^2(2y)$. There is no requirement for $\frac{dx}{dy} =$

A1 Both lhs and rhs correct. $\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$

It is an intermediate line and the expression does not need to be simplified.

Accept $\frac{dx}{dy} = \tan 2y \times 8 + 8y \times 2\sec^2(2y)$ or $\frac{dy}{dx} = \frac{1}{\tan 2y \times 8 + 8y \times 2\sec^2(2y)}$ or using implicit differentiation $1 = \tan 2y \times 8\frac{dy}{dx} + 8y \times 2\sec^2(2y)\frac{dy}{dx}$

M1 For fully substituting $y = \frac{\pi}{8}$ into their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ to find a 'numerical' value

Accept $\frac{dx}{dy} =$ awrt 20.6 or $\frac{dy}{dx} =$ awrt 0.05 as evidence

M1 For a correct attempt at an equation of the tangent at the point $\left(\pi, \frac{\pi}{8}\right)$.

The gradient must be an inverted numerical value of their $\frac{dx}{dy}$

Look for $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{\text{numerical } \frac{dx}{dy}}$,

Watch for negative reciprocals which is M0
If the form $y = mx + c$ is used it must be a full method to find a 'numerical' value to c.

A1 A correct equation of the tangent.

Accept $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{8 + 4\pi}$ or if $y = mx + c$ is used accept $m = \frac{1}{8 + 4\pi}$ and $c = \frac{\pi}{8} - \frac{\pi}{8 + 4\pi}$

Watch for answers like this which are correct $x - \pi = (8 + 4\pi)\left(y - \frac{\pi}{8}\right)$

Accept the decimal answers awrt 2sf $y = 0.049x + 0.24$, awrt 2sf $21y = x + 4.9$, $\frac{y - 0.39}{x - 3.1} = 0.049$

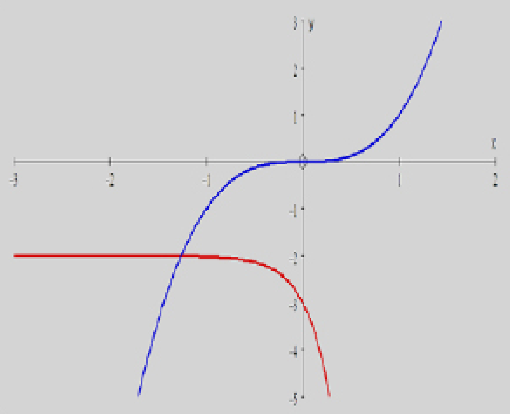
Accept a mixture of decimals and π's for example $20.6\left(y - \frac{\pi}{8}\right) = x - \pi$

A1 Correct answer and solution only. $(8 + 4\pi)y = x + \frac{\pi^2}{2}$

Accept exact alternatives such as $4(2 + \pi)y = x + 0.5\pi^2$ and because the question does not ask for a and b to be simplified in the form $ay = x + b$, accept versions like

$(8 + 4\pi)y = x + \frac{\pi}{8}(8 + 4\pi) - \pi$ and $(8 + 4\pi)y = x + (8 + 4\pi)\left(\frac{\pi}{8} - \frac{\pi}{8 + 4\pi}\right)$

Q34.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$	M1, A1
	Puts $\frac{dy}{dx} = 0$ to give $x^3 = -2 - e^{4x}$	A1 *
		(3)
(b)		
	$y = x^3$	B1
	Shape of $y = -2 - e^{4x}$	B1
	$y = -2 - e^{4x}$ cuts y axis at (0,-3)	B1
	$y = -2 - e^{4x}$ has asymptote at $y = -2$	B1
		(4)
(c)	Only one crossing point	B1 (1)
(d)	-1.26376, **-**1.26126 Accept answers which round to these answers to 5dp	M1 A1 (2)
(e)	α = -1.26 and so turning point is at (-1.26, -2.55)	M1 A1cao (2) **12 marks**

(a)

M1 Two (of the four) terms differentiated correctly

A1 All correct $\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$

A1* States or sets $\frac{dy}{dx} = 0$, and proceeds correctly to achieve printed answer $x^3 = -2 - e^{4x}$.

(b)

B1 Correct shape and position for $y = x^3$. It must appear to go through the origin. It must only appear in Quadrants 1 and 3 and have a gradient that is always $\geqslant 0$. The gradient should appear large at either end. Tolerate slips of the pen.See practice and qualification for acceptable curves.

B1 Correct shape for $y = -2 - e^{4x}$, its position is not important for this mark. The gradient must be approximately zero at the left hand end and increase negatively as you move from left to right along the curve. See practice and qualification for acceptable curves.

B1 Score for $y = -2 - e^{4x}$ cutting or meeting the y axis at (0,-3). Its shape is not important. Accept for the intention of (0,-3), -3 being marked on the y – axis as well as (-3,0) Do not accept 3 being marked on the negative y axis.

B1 Score for $y = -2 - e^{4x}$ having an asymptote stated as $y = -2$. This is dependent upon the curve appearing to have an asymptote there. Do not accept the asymptote marked as '-2' or indeed $x = -2$. See practice and qualification for acceptable solutions.

(c)

B1 Score for a statement to the effect that the graphs cross at one point. Accept minimal statements such as 'one intersection'. Do not award if their diagram shows more than one intersection. They must have a diagram (which may be incorrect)

(d)

M1 Awarded for applying the iteration formula once. Possible ways in which this can be scored are the sight of $\sqrt[3]{-2-e^{-4}}$, $(-2-e^{4x-1})^{\frac{1}{3}}$ or awrt -1.264

A1 Both values correct awrt -1.26376, -1.26126 5dps. The subscripts are unimportant for this mark. Score as the first and second values seen.

(e)

M1 Score for EITHER rounding their value in part (c) to 2 dp OR finding turning point of C by substituting a value of x generated from part (d) into $y = e^{4x} + x^4 + 8x + 5$ in order to find the y value. You may accept the appearance of a y value as evidence of finding the turning point (as long as an x value appears to be generated from part (d) and the correct equation is used.)

A1 (-1.26, -2.55) and correct solution only. It is a deduction and you cannot accept the appearance of a correct answer for two marks.

Q35.

Question Number	Scheme	Marks
(i) (a)	$2\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}=\frac{5}{\sin x}$	B1
	Uses common denominator to give $2\sin^2 x-\cos^2 x=5\cos x$	M1
	Replaces $\sin^2 x$ by $(1-\cos^2 x)$ to give $2(1-\cos^2 x)-\cos^2 x=5\cos x$	M1
	Obtains $3\cos^2 x+5\cos x-2=0$ (a =3, $b=5$, c = -2)	A1 (4)
(b)	Solves $3\cos^2 x+5\cos x-2=0$ to give cos x =	M1
	cos x = $\frac{1}{3}$ only (rejects cos x = -2)	A1
	So x = 1.23 or 5.05	dM1A1 (4)
(ii)	Either Or	
	$\tan\theta+\cot\theta\equiv\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$ $\tan\theta+\cot\theta\equiv\tan\theta+\frac{1}{\tan\theta}$	B1
	$\equiv\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ $\equiv\frac{\tan^2\theta+1}{\tan\theta}$	M1
	$\equiv\frac{2}{\sin 2\theta}$ $\equiv\frac{1}{\cos^2\theta\times\frac{\sin\theta}{\cos\theta}}\equiv\frac{2}{\sin 2\theta}$	M1
	$\equiv 2\operatorname{cosec}2\theta$ (so $\lambda=2$) $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda=2$)	A1 (4) **12 marks**
	Alternatives to Main Scheme	
(i) (a)	$2\tan x-\frac{1}{\tan x}=\frac{5}{\sin x}$ does not score any marks until	
	$\times\tan x\Rightarrow 2\tan^2 x+1=5\sec x$	B1, M1
	Replaces $\tan^2 x$ by $(\sec^2 x-1)$ to give $2(\sec x^2-1)+1=5\sec x$	M1
	Obtains $3\cos^2 x+5\cos x-2=0$ (a =3, $b=5$, c = -2)	A1 (4)
(b)	Solves $3\cos^2 x+5\cos x-2=0$ to give cos x = or $2\sec^2 x-5\sec x-3=0\Rightarrow\sec x=..$	M1
	cos x = $\frac{1}{3}$ only (rejects cos x = -2)	A1
	So x = 1.23 or 5.05	dM1A1 (4)
(ii)	$\tan\theta+\cot\theta=\lambda\operatorname{cosec}2\theta\Rightarrow\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\lambda}{\sin 2\theta}=\cancel{\left(\frac{\lambda}{2\sin\theta\cos\theta}\right)}$	B1
	$\times 2\sin\theta\cos\theta\Rightarrow 2\sin^2\theta+2\cos^2\theta=\lambda$	M1
	Factorises $2(\sin^2\theta+\cos^2\theta)=\lambda\Rightarrow 2=\lambda$	M1
	All above correct + a statement like 'hence true', 'QED'	A1 (4)

(i)(a)

B1 Uses definitions $\tan x=\frac{\sin x}{\cos x}$, $\cot x=\frac{\cos x}{\sin x}$ and $\operatorname{cosec}x=\frac{1}{\sin x}$ to write the equation in terms of $\cos x$ and $\sin x$. Condone $5\operatorname{cosec}x=\frac{1}{5\sin x}$ as the intention is clear.

Alternatively uses $\cot x=\frac{1}{\tan x}$ and $\operatorname{cosec}x=\frac{1}{\sin x}$ to write the equation in terms of $\tan x$ and $\sin x$

This may be implied by later work that achieves $A\tan^2 x\pm B=C\sec x$

M1 Either uses common denominator and cross multiples, or multiplies each term by $\sin x\cos x$ to achieve an equation of the form equivalent to $A\sin^2 x\pm B\cos^2 x=C\cos x$. It may be seen on the numerator of a fraction

Alternatively multiplies by $\tan x$ to achieve $A\tan^2 x\pm B=C\sec x$

M1 Uses a correct Pythagorean relationship, usually $\sin^2 x=1-\cos^2 x$ to form a quadratic equation in terms of $\cos x$. In the alternative uses $\tan^2 x=\sec^2 x-1$ to form a quadratic in sec x, followed by $\sec x=\frac{1}{\cos x}$ to form a quadratic equation in terms of $\cos x$

A1 Obtains $\pm K\left(3\cos^2 x+5\cos x-2\right)=0$ (a =3, $b=5$, c = -2)

(i)(b)

M1 Uses a standard method to solve their quadratic equation in $\cos x$ from **(i)(a) OR** sec$x$ from an earlier line in (a)
See General Principles for Core Mathematics on how to solve quadratics

A1 $\cos x=\frac{1}{3}$ only Do not need to see –2 rejected

dM1 Uses arcos on their value to obtain at least one answer. It is dependent upon the previous M.
It may be implied by one correct answer

A1 Both values correct awrt 3sf x = 1.23 and 5.05.
Ignore any solutions outside the range. Any extra solutions in the range will score A0.
Answers in degrees will score A0.

(ii)

B1 Uses a definition of cot with matching expression for tan. Acceptable answers are

$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$, $\frac{\sin\theta}{\cos\theta}+\frac{1}{\frac{\sin\theta}{\cos\theta}}$, $\tan\theta+\frac{1}{\tan\theta}$. Condone a miscopy on the sign. Eg Allow $\tan\theta-\frac{1}{\tan\theta}$

M1 Uses common denominator, writing their expression as a single fraction. In the examples given above, example 2 would need to be inverted. The denominator has to be correct and one of the terms must be adapted.

M1 Uses identities $\sin^2\theta+\cos^2\theta=1$ and $\sin 2\theta=2\sin\theta\cos\theta$ specifically to achieve an expression of the form $\frac{\lambda}{\sin 2\theta}$

Alternatively uses $1+\tan^2\theta=\sec^2\theta=\frac{1}{\cos^2\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta=2\sin\theta\cos\theta$ specifically to achieve an expression of the form $\frac{\lambda}{\sin 2\theta}$. A line of $\frac{1}{\sin\theta\cos\theta}$ achieved on the lhs followed by $\lambda=\frac{1}{2}$ or 2 would imply this mark

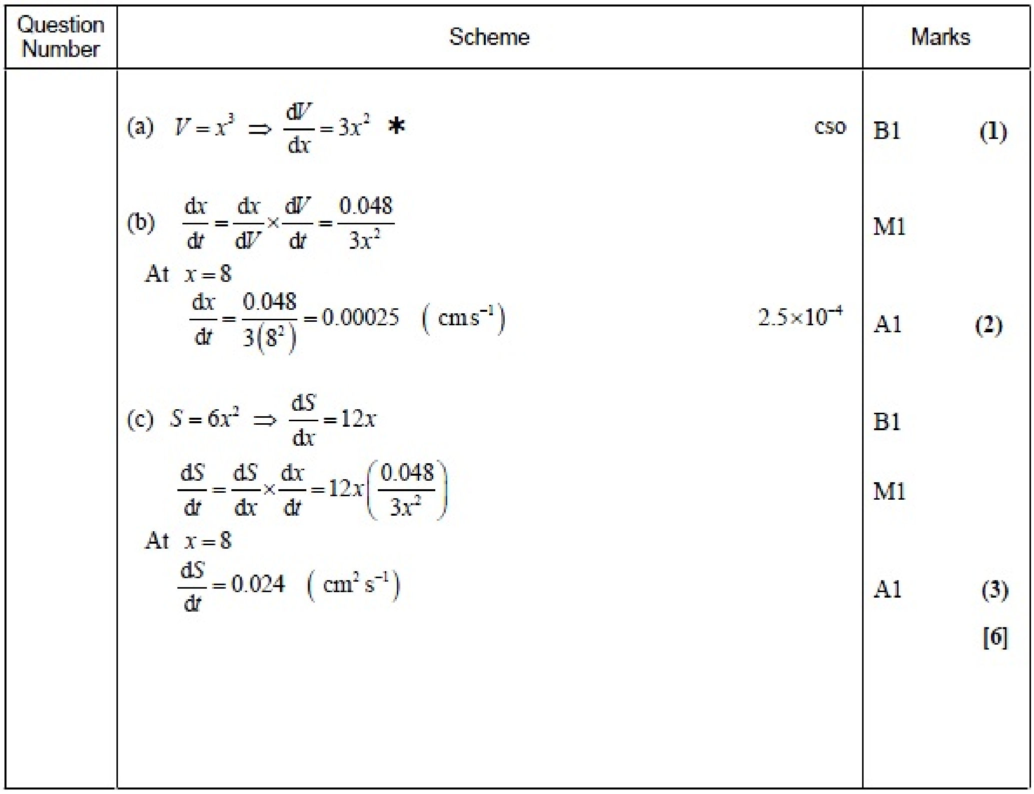
A1 Achieves printed answer with no errors.

Allow for a different variable as long as it is used consistently.

Q36.

Question Number	Scheme	Marks
(a)	$\left\{\frac{\cancel{\mathrm{d}y}}{\cancel{\mathrm{d}x}}\times\right\}\quad \underline{2+6y\frac{\mathrm{d}y}{\mathrm{d}x}}+\left(\underline{\underline{6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}}\right)=\underline{8x}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $\mathrm{m}(\mathbf{T})=\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2}=-\frac{4}{9}$	dM1 A1 cso
		[5]
(b)	So, $\mathrm{m}(\mathbf{N})=\frac{-1}{-\frac{4}{9}}\left\{=\frac{9}{4}\right\}$	M1
	$\mathbf{N}$: $y-1=\frac{9}{4}(x+1)$	M1
	$\mathbf{N}$: $9x-4y+13=0$	A1
		[3] 8
(a)	**M1**: Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$).	
	A1: $(2x+3y^2)\to\left(\underline{2+6y\frac{\mathrm{d}y}{\mathrm{d}x}}\right)$ and $(4x^2\to\underline{8x})$. **Note:** If an extra "sixth" term appears then award A0.	
	B1: $6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$.	
	dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8x-2-6xy}{6y+3x^2}$ is substituted into or evaluated correctly. If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0. Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T})=-4$ Note that this mark is dependent on the previous method mark being awarded.	
	A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44 If the candidate's solution is not completely correct, then do not give this mark.	
(b)	**M1**: Applies $\mathrm{m}(\mathrm{N})=-\frac{1}{\text{their m(T)}}$.	
	M1: Uses $y-1=(m_N)(x--1)$ or finds c using $x=-1$ and $y=1$ and uses $y=(\mathrm{m}_N)x+"c"$, Where $m_N=-\frac{1}{\text{their m(T)}}$ or $m_N=\frac{1}{\text{their m(T)}}$ or $m_N=-\text{their m(T)}$.	
	A1: $9x-4y+13=0$ or $-9x+4y-13=0$ or $4y-9x-13=0$ or $18x-8y+26=0$ etc. Must be "$=0$". So do not allow $9x+13=4y$ etc.	
	Note: $m_N=-\left(\frac{6y+3x^2}{8x-2-6xy}\right)$ is M0M0 unless a numerical value is then found for m_N.	
	Alternative method for part (a): Differentiating with respect to y	
	$\left\{\frac{\cancel{\mathrm{d}x}}{\cancel{\mathrm{d}x}}\times\right\}\quad \underline{2\frac{\mathrm{d}x}{\mathrm{d}y}+6y}+\left(\underline{\underline{6xy\frac{\mathrm{d}x}{\mathrm{d}y}+3x^2}}\right)=\underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}$	
	M1: Differentiates implicitly to include either $2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $6xy\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\pm kx\frac{\mathrm{d}x}{\mathrm{d}y}$. (Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\right)$).	
	A1: $(2x+3y^2)\to\left(\underline{2\frac{\mathrm{d}x}{\mathrm{d}y}+6y}\right)$ and $\left(4x^2\to\underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}\right)$. **Note:** If an extra "sixth" term appears then award A0.	
	B1: $6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$.	
	dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{6y+3x^2}{8x-2-6xy}$ is substituted into or evaluated correctly. If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0. Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T})=-4$ Note that this mark is dependent on the previous method mark being awarded.	
	A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44 If the candidate's solution is not completely correct, then do not give this mark.	

Q37.



Question Number	Scheme	Marks
	(a) $V = x^3 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ * cso	B1 (1)
	(b) $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{0.048}{3x^2}$	M1
	At $x = 8$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{0.048}{3(8^2)} = 0.00025 \quad \left(\mathrm{cm\,s^{-1}}\right)$ 2.5×10^{-4}	A1 (2)
	(c) $S = 6x^2 \Rightarrow \frac{\mathrm{d}S}{\mathrm{d}x} = 12x$	B1
	$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = 12x\left(\frac{0.048}{3x^2}\right)$	M1
	At $x = 8$ $\frac{\mathrm{d}S}{\mathrm{d}t} = 0.024 \quad \left(\mathrm{cm^2\,s^{-1}}\right)$	A1 (3)
		[6]

Q38.

Question Number	Scheme	Marks
	(a) Differentiating implicitly to obtain $\pm ay^2\frac{dy}{dx}$ and/or $\pm bx^2\frac{dy}{dx}$	M1
	$48y^2\frac{dy}{dx} + \ldots - 54 \ldots$	A1
	$9x^2y \rightarrow 9x^2\frac{dy}{dx} + 18xy$ or equivalent	B1
	$\left(48y^2 + 9x^2\right)\frac{dy}{dx} + 18xy - 54 = 0$	M1
	$\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2}$ $\left(= \frac{18 - 6xy}{16y^2 + 3x^2}\right)$	A1 (5)
	(b) $18 - 6xy = 0$	M1
	Using $x = \frac{3}{y}$ or $y = \frac{3}{x}$	
	$16y^3 + 9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right) = 0$ or $16\left(\frac{3}{x}\right)^3 + 9x^2\left(\frac{3}{x}\right) - 54x = 0$	M1
	Leading to	
	$16y^4 + 81 - 162 = 0$ or $16 + x^4 - 2x^4 = 0$	M1
	$y^4 = \frac{81}{16}$ or $x^4 = 16$	
	$y = \frac{3}{2}, -\frac{3}{2}$ or $x = 2, -2$	A1 A1
	Substituting either of their values into $xy = 3$ to obtain a value of the other variable.	M1
	$\left(2, \frac{3}{2}\right), \left(-2, -\frac{3}{2}\right)$ both	A1 (7)
		[12]

Q39.

Question Number	Scheme	Marks	
	(a) $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{2}\pi h - \pi h^2$ or equivalent	M1 A1	
	At $h = 0.1$, $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$ $\frac{\pi}{25}$	M1 A1	**(4)**
	(b) $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$ or $\frac{\pi}{800} \div$ their (a)	M1	
	At $h = 0.1$, $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$ awrt 0.031	A1	**(2)**
			[6]

Q40.

Question Number	Scheme	Marks
	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ \dots$	B1
	$\dots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1
	At $x = 2$, $\quad \ln y = 2(2)\ln 2$	M1
	leading to $\quad y = 16 \quad$ Accept $y = \mathrm{e}^{4\ln 2}$	A1
	At $(2, 16) \quad \frac{1}{16}\frac{\mathrm{d}y}{\mathrm{d}x} = 2\ln 2 + 2$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 16(2 + 2\ln 2)$	A1 (7)
		[7]
	Alternative	
	$y = \mathrm{e}^{2x\ln x}$	B1
	$\frac{\mathrm{d}}{\mathrm{d}x}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)\mathrm{e}^{2x\ln x}$	M1 A1
	At $x = 2$, $\quad \frac{\mathrm{d}y}{\mathrm{d}x} = (2\ln 2 + 2)\mathrm{e}^{4\ln 2}$	M1
	$= 16(2 + 2\ln 2)$	A1 (7)

Q41.

Question Number	Scheme		Marks
(a)	$C: \ y^2 - 3y = x^3 + 8$		
	$\left\{\cancel{\frac{\mathrm{d}y}{\mathrm{d}x}} \cancel{=}\right\} \quad 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm 3\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x}$'. Can be implied.	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe
			(4)
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2, 3) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(4)}{6-3} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
			(3)
	1(b) final A1 $\sqrt{}$. Note if the candidate inserts their x value and $y = 3$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$, then an answer of $\frac{\mathrm{d}y}{\mathrm{d}x} =$ their x^2, *may* indicate a correct follow through.		
			[7]

Q42.

Question Number	Scheme	Marks
(a)	Similar triangles $\Rightarrow \frac{r}{h} = \frac{16}{24} \Rightarrow r = \frac{2h}{3}$ Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h = \frac{4\pi h^3}{27}$ **AG** Substitutes $r = \frac{2h}{3}$ into the formula for the volume of water V.	A1
		(2)
(b)	From the question, $\frac{dV}{dt} = 8$ $\frac{dV}{dt} = 8$	B1
	$\frac{dV}{dh} = \frac{12\pi h^2}{27} = \frac{4\pi h^2}{9}$ $\frac{dV}{dh} = \frac{12\pi h^2}{27}$ or $\frac{4\pi h^2}{9}$	B1
	$\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = 8 \times \frac{9}{4\pi h^2} = \frac{18}{\pi h^2}$ Candidate's $\frac{dV}{dt} \div \frac{dV}{dh}$; $8 \div \left(\frac{12\pi h^2}{27}\right)$ or $8 \times \frac{9}{4\pi h^2}$ or $\frac{18}{\pi h^2}$ oe	M1; A1
	When $h = 12$, $\frac{dh}{dt} = \frac{18}{144\pi} = \frac{1}{8\pi}$ $\frac{18}{144\pi}$ or $\frac{1}{8\pi}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\frac{18}{144\pi}$.	(5)
		[7]

Q43.

Question Number	Scheme	Marks
	$\frac{dI}{dt} = -16\ln(0.5)0.5^t$	M1 A1
	At $t = 3$ $\frac{dI}{dt} = -16\ln(0.5)0.5^3$	M1
	$= -2\ln 0.5 = \ln 4$	M1 A1
		[5]

Q44.

Question Number	Scheme	Marks
	$\frac{d}{dx}\left(2^x\right) = \ln 2.2^x$	B1
	$\ln 2.2^x + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$	M1 A1= A1
	Substituting $(3, 2)$	
	$8\ln 2 + 4\frac{dy}{dx} = 4 + 6\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = 4\ln 2 - 2$ Accept exact equivalents	M1 A1 (7)
		[7]

Q45.

Question Number	Scheme	Marks
	(a) $-2\sin 2x - 3\sin 3y\frac{dy}{dx} = 0$	M1 A1
	$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}, \frac{-2\sin 2x}{3\sin 3y}$	A1 (3)
	(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$	M1
	$\cos 3y = \frac{1}{2}$	A1
	$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349	A1 (3)
	(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{dy}{dx} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$	M1
	$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$	M1
	Leading to $6x + 9y - 2\pi = 0$	A1 (3) **[9]**

Q46.

Question Number	Scheme	Marks
	$\frac{dA}{dt} = 1.5$	B1
	$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$	B1
	When $A = 2$	
	$2 = \pi r^2 \Rightarrow r = \sqrt{\frac{2}{\pi}}$ $(= 0.797\,884\ ...\ )$	M1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$	
	$1.5 = 2\pi r \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ awrt 0.299	A1
		[5]

Q47.

Question Number	Scheme	Marks
Q (a)	$e^{-2x}\frac{dy}{dx} - 2y\,e^{-2x} = 2 + 2y\frac{dy}{dx}$ A1 correct RHS	M1 A1
	$\frac{d}{dx}\left(y\,e^{-2x}\right) = e^{-2x}\frac{dy}{dx} - 2y\,e^{-2x}$	B1
	$\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2 + 2y\,e^{-2x}$	M1
	$\frac{dy}{dx} = \frac{2 + 2y\,e^{-2x}}{e^{-2x} - 2y}$	A1 (5)
(b)	At P , $\frac{dy}{dx} = \frac{2 + 2e^{0}}{e^{0} - 2} = -4$	M1
	Using $mm' = -1$	
	$m' = \frac{1}{4}$	M1
	$y - 1 = \frac{1}{4}(x - 0)$	M1
	$x - 4y + 4 = 0$ or any integer multiple	A1 (4)
		[9]
	Alternative for (a) *differentiating implicitly with respect to y.*	
	$e^{-2x} - 2y\,e^{-2x}\frac{dx}{dy} = 2\frac{dx}{dy} + 2y$ A1 correct RHS	M1 A1
	$\frac{d}{dy}\left(y\,e^{-2x}\right) = e^{-2x} - 2y\,e^{-2x}\frac{dx}{dy}$	B1
	$\left(2 + 2y\,e^{-2x}\right)\frac{dx}{dy} = e^{-2x} - 2y$	M1
	$\frac{dx}{dy} = \frac{e^{-2x} - 2y}{2 + 2y\,e^{-2x}}$	
	$\frac{dy}{dx} = \frac{2 + 2y\,e^{-2x}}{e^{-2x} - 2y}$	A1 (5)

Q48.

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td></td><td colspan="2">$x^2 + 4xy + y^2 + 27 = 0$</td><td></td></tr>
<tr><td>(a)</td><td colspan="2">$\left\{ \cancel{\frac{\mathrm{d}y}{\mathrm{d}x}} \cancel{=} \right\} \quad \underline{2x} + \left(\underline{\underline{4y + 4x\frac{\mathrm{d}y}{\mathrm{d}x}}} \right) \underline{+ 2y\frac{\mathrm{d}y}{\mathrm{d}x}} = \underline{0}$</td><td>M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$</td></tr>
<tr><td></td><td colspan="2">$2x + 4y + (4x + 2y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$</td><td>dM1</td></tr>
<tr><td></td><td colspan="2">$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2x - 4y}{4x + 2y} \left\{ = \frac{-x - 2y}{2x + y} \right\}$</td><td>A1 cso oe</td></tr>
<tr><td></td><td colspan="2"></td><td>[5]</td></tr>
<tr><td>(b)</td><td colspan="2">$4x + 2y = 0$</td><td>M1</td></tr>
<tr><td></td><td>$y = -2x$</td><td>$x = -\frac{1}{2}y$</td><td>A1</td></tr>
<tr><td></td><td>$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$</td><td>$\left(-\frac{1}{2}y\right)^2 + 4\left(-\frac{1}{2}y\right)y + y^2 + 27 = 0$</td><td>M1*</td></tr>
<tr><td></td><td>$-3x^2 + 27 = 0$</td><td>$-\frac{3}{4}y^2 + 27 = 0$</td><td></td></tr>
<tr><td></td><td>$x^2 = 9$</td><td>$y^2 = 36$</td><td>dM1*</td></tr>
<tr><td></td><td>$x = -3$</td><td>$y = 6$</td><td>A1</td></tr>
<tr><td></td><td>When $x = -3$, $y = -2(-3)$</td><td>When $y = 6$, $x = -\frac{1}{2}(6)$</td><td>ddM1*</td></tr>
<tr><td></td><td>$y = 6$</td><td>$x = -3$</td><td>A1 cso</td></tr>
<tr><td></td><td colspan="2"></td><td>[7]
12</td></tr>
<tr><th colspan="4">Notes for Question</th></tr>
<tr><td>(a)</td><td colspan="3">M1: Differentiates implicitly to include either $4x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).
A1: $(x^2) \to (\underline{2x})$ and $\left(... + y^2 + 27 = 0 \to \underline{+ 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0} \right)$.
Note: If an extra term appears then award A0.
Note: The "$= 0$" can be implied by rearrangement of their equation.
i.e.: $2x + 4y + 4x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ leading to $4x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = -2x - 4y$ will get A1 (implied).
B1: $4y + 4x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $4\left(y + x\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ or equivalent
dM1: An attempt to factorise out $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are at least two terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$.
ie. $... + (4x + 2y)\frac{\mathrm{d}y}{\mathrm{d}x} = ...$ or $... + 2(2x + y)\frac{\mathrm{d}y}{\mathrm{d}x} = ...$
Note: This mark is dependent on the previous method mark being awarded.
A1: For $\frac{-2x - 4y}{4x + 2y}$ or equivalent. Eg: $\frac{+2x + 4y}{-4x - 2y}$ or $\frac{-2(x + 2y)}{4x + 2y}$ or $\frac{-x - 2y}{2x + y}$
cso: If the candidate's solution is not completely correct, then do not give this mark.</td></tr>
</table>

Notes for Question Continued

(b)

M1: Sets the denominator of their $\frac{dy}{dx}$ equal to zero (or the numerator of their $\frac{dx}{dy}$ equal to zero) oe.

A1: Rearranges to give either $y = -2x$ or $x = -\frac{1}{2}y$. (correct solution only).

The first two marks can be implied from later working, i.e. for a correct substitution of either $y = -2x$ into y^2 or for $x = -\frac{1}{2}y$ into $4xy$.

M1*: Substitutes $y = \pm\lambda x$ or or $x = \pm\mu y$ or $y = \pm\lambda x \pm a$ or $x = \pm\mu y \pm b$ $(\lambda \neq 0,\ \mu \neq 0)$ into $x^2 + 4xy + y^2 + 27 = 0$ to form an equation in one variable.

dM1*: leading to at least either $x^2 = A,\ A > 0$ or $y^2 = B,\ B > 0$

Note: This mark is dependent on the previous method mark (M1*) being awarded.

A1: For $x = -3$ (ignore $x = 3$) or if y was found first, $y = 6$ (ignore $y = -6$) (correct solution only).

ddM1* Substitutes their value of x into $y = \pm\lambda x$ to give $y =$ value

or substitutes their value of x into $x^2 + 4xy + y^2 + 27 = 0$ to give $y =$ value.

Alternatively, substitutes their value of y into $x = \pm\mu y$ to give $x =$ value

or substitutes their value of y into $x^2 + 4xy + y^2 + 27 = 0$ to give $x =$ value

Note: This mark is dependent on the two previous method marks (M1* and dM1*) being awarded.

A1: $(-3, 6)$ **cso.**

Note: If a candidate offers two sets of coordinates without either rejecting the incorrect set or accepting the correct set then award A0. **DO NOT APPLY ISW ON THIS OCCASION.**

Note: $x = -3$ followed later in working by $y = 6$ is fine for A1.

Note: $y = 6$ followed later in working by $x = -3$ is fine for A1.

Note: $x = -3, 3$ followed later in working by $y = 6$ is A0, unless candidate indicates that they are rejecting $x = 3$

Note: Candidates who set the numerator of $\frac{dy}{dx}$ equal to 0 (or the denominator of their $\frac{dx}{dy}$ equal to zero) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a maximum marking profile of M0A0M1M1A0M1A0. They will usually find $(-6, 3)$ { or even $(6, -3)$ }.

Note: Candidates who set ***the numerator*** or ***the denominator*** of $\frac{dy}{dx}$ equal to $\pm k$ (usually $k = 1$) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a marking profile of M0A0M1M1A0M1A0.

Special Case: It is possible for a candidate who does not achieve full marks in part (a), (but has a correct denominator for $\frac{dy}{dx}$) to gain all 7 marks in part (b).

Eg: An incorrect part (a) answer of $\frac{dy}{dx} = \frac{2x - 4y}{4x + 2y}$ can lead to a correct $(-3, 6)$ in part (b) and 7 marks.

Q49.

Question Number	Scheme		Marks
	$3^{x-1}+xy-y^2+5=0$		
		$3^{x-1}\to 3^{x-1}\ln 3$	B1 oe
	$\left\{\cancel{\frac{\mathrm{d}y}{\mathrm{d}x}}\cancel{=}\right\}$ (ignore) $3^{x-1}\ln 3+\left(y+x\frac{\mathrm{d}y}{\mathrm{d}x}\right)-2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	Differentiates implicitly to include either $\pm\lambda x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$.	M1*
		$xy\to +y+x\frac{\mathrm{d}y}{\mathrm{d}x}$	B1
		$\ldots+y+x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1
	$\{(1,3)\Rightarrow\}\ 3^{(1-1)}\ln 3+3+(1)\frac{\mathrm{d}y}{\mathrm{d}x}-2(3)\frac{\mathrm{d}y}{\mathrm{d}x}=0$	Substitutes $x=1,\ y=3$ into their differentiated equation or expression.	dM1*
	$\ln 3+3+\frac{\mathrm{d}y}{\mathrm{d}x}-6\frac{\mathrm{d}y}{\mathrm{d}x}=0\ \Rightarrow\ 3+\ln 3=5\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3+\ln 3}{5}$		dM1*
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}\left(\ln e^3+\ln 3\right)=\frac{1}{5}\ln\left(3e^3\right)$	Uses $3=\ln e^3$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}\ln\left(3e^3\right)$	A1 cso
			[7] 7

Notes for Question

B1: Correct differentiation of 3^{x-1}. I.e. $3^{x-1}\to 3^{x-1}\ln 3$ or $3^{x-1}=\frac{1}{3}\left(3^x\right)\to\frac{1}{3}\left(3^x\right)\ln 3$

or $3^{x-1}=e^{(x-1)\ln 3}\to\ln 3\,e^{(x-1)\ln 3}$ or $3^{x-1}=\frac{1}{3}\left(3^x\right)=\frac{1}{3}e^{x\ln 3}\to\frac{1}{3}(\ln 3)e^{x\ln 3}$

M1: Differentiates implicitly to include either $\pm\lambda x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$).

B1: $xy\to +y+x\frac{\mathrm{d}y}{\mathrm{d}x}$

1st A1: $\ldots+y+x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ **Note:** The 1st A0 follows from an award of the 2nd B0.

Note: The "$=0$" can be implied by rearrangement of their equation.

ie: $3^{x-1}\ln 3+y+x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}y}{\mathrm{d}x}$ leading to $3^{x-1}\ln 3+y=2y\frac{\mathrm{d}y}{\mathrm{d}x}-x\frac{\mathrm{d}y}{\mathrm{d}x}$ will get A1 (implied).

2nd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.
Substitutes $x=1,\ y=3$ into their differentiated equation or expression. Allow one slip.

3rd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.

Candidate has two differentiated terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$ and rearranges to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject.

Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.

Eg: Candidate may write $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y+3^{x-1}\ln 3}{2y-x}$ before substituting in $x=1$ and $y=3$

2nd A1: cso. Uses $3=\ln e^3$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}\ln\left(3e^3\right)$, $\left(=\frac{1}{\lambda}\ln\left(\mu e^3\right),\ \lambda=5\text{ and }\mu=3\right)$

Note: $3=\ln e^3$ needs to be seen in their proof.

Notes for Question Continued			
***Aliter* Way 2**	**<u>Alternative Method: Multiplying both sides by 3</u>** $3^{x-1}+xy-y^2+5=0$ $3^x+3xy-3y^2+15=0$		
		$3^x \to 3^x\ln 3$	B1
	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\cancel{=}\right\}$ $3^x\ln3+\left(3y+3x\frac{dy}{dx}\right)-6y\frac{dy}{dx}=0$ (ignore)	Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
		$3xy \to +3y+3x\frac{dy}{dx}$	B1
		$\ldots+3y+3x\frac{dy}{dx}-6y\frac{dy}{dx}=0$	A1
	$\{(1,3)\Rightarrow\}\ 3^1\ln3+3(3)+(3)(1)\frac{dy}{dx}-6(3)\frac{dy}{dx}=0$	Substitutes $x=1$, $y=3$ into their differentiated equation or expression.	dM1*
	$3\ln3+9+3\frac{dy}{dx}-18\frac{dy}{dx}=0 \Rightarrow 9+3\ln3=15\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{9+3\ln3}{15}\left\{=\frac{3+\ln3}{5}\right\}$		dM1*
	$\frac{dy}{dx}=\frac{1}{5}\left(\ln e^3+\ln3\right)$ $\frac{dy}{dx}=\frac{1}{5}\left(\ln e^3+\ln3\right)=\frac{1}{5}\ln\left(3e^3\right)$	Uses $3=\ln e^3$ to achieve $\frac{dy}{dx}=\frac{1}{5}\ln\left(3e^3\right)$	A1 cso [7] 7
	NOTE: Only apply this scheme if the candidate has multiplied both sides of their equation by 3. **NOTE:** For reference, $\frac{dy}{dx}=\frac{3y+3^x\ln3}{6y-3x}$ **NOTE:** If the candidate applies this method then $3xy \to +3y+3x\frac{dy}{dx}$ must be seen for the 2nd B1 mark.		

Q50.

Question Number	Scheme	Marks
	$x^3 + 2xy - x - y^3 - 20 = 0$	
(a)	$\left\{\cancel{\frac{dy}{dx}}\times\right\}$ $\underline{3x^2} + \underline{\underline{\left(2y + 2x\frac{dy}{dx}\right)}}\,\underline{-1 - 3y^2\frac{dy}{dx} = 0}$ $3x^2 + 2y - 1 + \left(2x - 3y^2\right)\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$ dM1 A1 cso [5]
(b)	At $P(3, -2)$, $\text{m}(\mathbf{T}) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \frac{22}{6}$ or $\frac{11}{3}$ **and either T**: $y - -2 = \frac{"11"}{3}(x - 3)$ **see notes** **or** $(-2) = \left(\frac{11}{3}\right)(3) + c \Rightarrow c = \ldots,$	M1
	T: $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$	A1 cso [2]
(a)	**Alternative method for part (a)** $\left\{\cancel{\frac{dx}{dy}}\times\right\}$ $\underline{3x^2\frac{dx}{dy}} + \underline{\underline{\left(2y\frac{dx}{dy} + 2x\right)}}\,\underline{-\frac{dx}{dy} - 3y^2 = 0}$ $2x - 3y^2 + \left(3x^2 + 2y - 1\right)\frac{dx}{dy} = 0$ $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$ dM1 A1 cso [5]

		Question Notes
(a) **General**	**Note**	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$ **from no working is full marks.**
	Note	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{2x - 3y^2}$ or $\frac{1 - 3x^2 - 2y}{3y^2 - 2x}$ **from no working is M1A0B0M1A0**
	Note	Few candidates will write $3x^2 + 2y + 2x\,dy - 1 - 3y^2 dy = 0$ leading to $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$, o.e. This should get full marks.
(a)	**M1**	Differentiates implicitly to include either $2x\frac{dy}{dx}$ or $-y^3 \to \pm ky^2\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).
	A1	$x^3 \to 3x^2$ **and** $-x - y^3 - 20 = 0 \to -1 - 3y^2\frac{dy}{dx} = 0$
	B1	$2xy \to 2y + 2x\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.

(a) ctd	**Note**	$3x^2 + 2y + 2x\frac{dy}{dx} - 1 - 3y^2\frac{dy}{dx} \rightarrow 3x^2 + 2y - 1 = 3y^2\frac{dy}{dx} - 2x\frac{dy}{dx}$ will get 1st A1 (implied) as the "= 0" can be implied by rearrangement of their equation.
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are *at least two terms* in $\frac{dy}{dx}$. ie. $... + (2x - 3y^2)\frac{dy}{dx} = ...$
	Note	Placing an extra $\frac{dy}{dx}$ at the beginning and then including it in their factorisation is fine for dM1.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$ cso: If the candidate's solution is not completely correct, then do not give this mark. isw: You can, however, ignore subsequent working following on from correct solution.
(b)	**M1**	**Some attempt to substitute both** $x = 3$ **and** $y = -2$ into their $\frac{dy}{dx}$ which contains both x and y to find m_T **and** • **either** applies $y - -2 = (\text{their } m_T)(x - 3)$, where m_T is a numerical value. • **or** finds c by solving $(-2) = (\text{their } m_T)(3) + c$, where m_T is a numerical value.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ is M0).
	A1	Accept any integer multiple of $11x - 3y - 39 = 0$ or $11x - 39 - 3y = 0$ or $-11x + 3y + 39 = 0$, where their tangent equation is equal to 0.
	cso	A correct solution is required from a correct $\frac{dy}{dx}$
	isw	You can ignore subsequent working following a correct solution.
	Alternative method for part (a): Differentiating with respect to *y*	
(a)	**M1**	Differentiates implicitly to include either $2y\frac{dx}{dy}$ or $x^3 \rightarrow \pm kx^2\frac{dx}{dy}$ or $-x \rightarrow -\frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy} = \right)$).
	A1	$x^3 \rightarrow 3x^2\frac{dx}{dy}$ **and** $-x - y^3 - 20 = 0 \rightarrow -\frac{dx}{dy} - 3y^2 = 0$
	B1	$2xy \rightarrow 2y\frac{dx}{dy} + 2x$
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dx}{dy}$ as long as there are *at least two terms* in $\frac{dx}{dy}$.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$ cso: If the candidate's solution is not completely correct, then do not give this mark.

Q51.

Question Number	Scheme	Marks
	$\frac{\mathrm{d}V}{\mathrm{d}t}=80\pi,\ \ V=4\pi h(h+4)=4\pi h^2+16\pi h,$	
	$\frac{\mathrm{d}V}{\mathrm{d}h}=8\pi h+16\pi$ $\pm\alpha h\pm\beta,\ \alpha\neq 0,\ \beta\neq 0$ $8\pi h+16\pi$	M1 A1
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\Rightarrow\right\}\ (8\pi h+16\pi)\frac{\mathrm{d}h}{\mathrm{d}t}=80\pi$ $\left(\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}h}\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}=80\pi$ $\left\{\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}h}\Rightarrow\right\}\ \frac{\mathrm{d}h}{\mathrm{d}t}=\ 80\pi\times\frac{1}{8\pi h+16\pi}$ or $80\pi\div\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}h}$	M1 oe
	When $h=6$, $\left\{\frac{\mathrm{d}h}{\mathrm{d}t}=\right\}\frac{1}{8\pi(6)+16\pi}\times 80\pi\ \left\{=\frac{80\pi}{64\pi}\right\}$ **dependent on the previous M1 see notes**	dM1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\underline{1.25}\ (\text{cms}^{-1})$ 1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	A1 oe
		[5] **5**
	<u>Alternative Method for the first M1A1</u>	
	Product rule: $\left\{\begin{matrix} u=4\pi h & v=h+4 \\ \frac{\mathrm{d}u}{\mathrm{d}h}=4\pi & \frac{\mathrm{d}v}{\mathrm{d}h}=1\end{matrix}\right\}$	
	$\frac{\mathrm{d}V}{\mathrm{d}h}=4\pi(h+4)+4\pi h$ $\pm\alpha h\pm\beta,\ \alpha\neq 0,\ \beta\neq 0$ $4\pi(h+4)+4\pi h$	M1 A1

Question Notes

M1	An expression of the form $\pm\alpha h\pm\beta,\ \alpha\neq 0,\ \beta\neq 0$. **Can be simplified or un-simplified.**
A1	Correct simplified or un-simplified differentiation of V. eg. $8\pi h+16\pi$ or $4\pi(h+4)+4\pi h$ or $8\pi(h+2)$ or equivalent.
Note	Some candidates will use the product rule to differentiate V with respect to h. **(See Alt Method 1).**
Note	$\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.
M1	$\left(\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}h}\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}=80\pi$ or $80\pi\div\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}h}$
Note	Also allow 2nd M1 for $\left(\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}h}\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}=\mathbf{80}$ or $\mathbf{80}\div\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}h}$
Note	Give 2nd M0 for $\left(\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}h}\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}=\mathbf{80\pi t}$ **or 80k** or **$80\pi t$ or 80k** $\div$ Candidate's $\frac{\mathrm{d}V}{\mathrm{d}h}$
dM1	**which is dependent on the previous M1 mark.** Substitutes $h=6$ into an expression which is a result of a quotient of their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and 80π (or 80)
A1	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
Note	$\frac{80\pi}{64\pi}$ as a final answer is A0.
Note	Substituting $h=6$ into a correct $\frac{\mathrm{d}V}{\mathrm{d}h}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)

Q52.

Question Number	Scheme		Marks
	$x^2 + y^2 + 10x + 2y - 4xy = 10$		
(a)	$\left\{ \cancel{\frac{dy}{dx}} \times \right\} \quad \underline{2x + 2y\frac{dy}{dx} + 10 + 2\frac{dy}{dx}} - \left(\underline{\underline{4y + 4x\frac{dy}{dx}}} \right) = \underline{0}$	**See notes**	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$2x + 10 - 4y + (2y + 2 - 4x)\frac{dy}{dx} = 0$	**Dependent on the first M1 mark.**	dM1
	$\frac{dy}{dx} = \frac{2x + 10 - 4y}{4x - 2y - 2}$		
	Simplifying gives $\frac{dy}{dx} = \frac{x + 5 - 2y}{2x - y - 1}$ $\left\{ = \frac{-x - 5 + 2y}{-2x + y + 1} \right\}$		A1 **cso oe**
			[5]
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \quad x + 5 - 2y = 0$		M1
	So $x = 2y - 5$,		
	$(2y - 5)^2 + y^2 + 10(2y - 5) + 2y - 4(2y - 5)y = 10$		M1
	$4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$		
	gives $-3y^2 + 22y - 35 = 0$ or $3y^2 - 22y + 35 = 0$	$3y^2 - 22y + 35 \{= 0\}$ **see notes**	A1 oe
	$(3y - 7)(y - 5) = 0$ and $y = \ldots$	Method mark for solving a quadratic equation.	ddM1
	$y = \frac{\;}{3}, 5$	$\{y =\} \frac{\;}{3}, 5$	A1 **cao**
			[5]
	<u>Alternative method for part (b)</u>		
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \quad x + 5 - 2y = 0$		M1
	So $y = \frac{x + 5}{2}$,		
	$x^2 + \left(\frac{x+5}{2}\right)^2 + 10x + 2\left(\frac{x+5}{2}\right) - 4x\left(\frac{x+5}{2}\right) = 10$		M1
	$x^2 + \frac{x^2 + 10x + 25}{4} + 10x + x + 5 - 2x^2 - 10x = 10$		
	$4x^2 + x^2 + 10x + 25 + 40x + 4x + 20 - 8x^2 - 40x = 40$		
	gives $-3x^2 + 14x + 5 = 0$ or $3x^2 - 14x - 5 = 0$	$3x^2 - 14x - 5 \{= 0\}$ **see notes**	A1 oe
	$(3x + 1)(x - 5) = 0, \; x = \ldots$ $y = \frac{-\frac{1}{3} + 5}{2}, \frac{5 + 5}{2}$	**Solves a quadratic and finds at least one value for** y**.**	ddM1
	$y = \frac{7}{3}, 5$	$\{y =\} \frac{7}{3}, 5$	A1 **cao**
			[5]
			10

	Question	Notes
(a)	**M1**	Differentiates implicitly to include either $\pm 4x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $y^2 \to 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $2y \to 2\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).
	A1	$x^2 + y^2 + 10x + 2y \to 2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 10 + 2\frac{\mathrm{d}y}{\mathrm{d}x}$ **and** $10 \to 0$
	M1	$-4xy \to \pm 4y \pm 4x\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	If an extra term appears then award 1st A0.
	Note	$2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 10 + 2\frac{\mathrm{d}y}{\mathrm{d}x} - 4y - 4x\frac{\mathrm{d}y}{\mathrm{d}x} \to 2x + 10 - 4y = -2y\frac{\mathrm{d}y}{\mathrm{d}x} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + 4x\frac{\mathrm{d}y}{\mathrm{d}x}$ will get 1st A1 (implied) as the "$= 0$" can be implied by rearrangement of their equation.
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are *at least two terms* in $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	A1	$\frac{x + 5 - 2y}{2x - y - 1}$ or $\frac{-x - 5 + 2y}{-2x + y + 1}$ (must be simplified).
	cso:	If the candidate's solution is not completely correct, then do not give this mark.
(b)	**M1**	Sets the numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or the denominator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) oe.
	NOTE	**If the numerator involves one variable only then *only* the 1st M1 mark is possible in part (b).**
	M1	Substitutes their x or their y into the printed equation to give an equation in one variable only.
	A1	For obtaining either $-3y^2 + 22y - 35 \{= 0\}$ or $3y^2 - 22y + 35 \{= 0\}$
	Note	This mark can also awarded for a correct three term equation, eg. either $-3y^2 + 22y = 35$ $3y^2 - 22y = -35$ or $3y^2 + 35 = 22y$ are all fine for A1.
	ddM1	**Dependent on the previous 2 M marks.** **See notes at the beginning of the mark scheme: Method mark for solving a 3 term quadratic** • $(3y - 7)(y - 5) = 0 \Rightarrow y = ...$ • $y = \frac{22 \pm \sqrt{(-22)^2 - 4(3)(35)}}{2(3)}$ • $y^2 - \frac{22}{3}y - \frac{35}{3} = 0 \Rightarrow \left(y - \frac{11}{3}\right)^2 - \frac{121}{9} + \frac{35}{3} = 0 \Rightarrow y = ...$ • Or writes down at least one correct y- root from their quadratic equation. This is usually found from their calculator.
	Note	If a candidate applies *the alternative method* then they also need to use their $y = \frac{x + 5}{2}$ in order to find **at least one value** for y in order to gain the final M1.
	A1	$y = \frac{7}{3}, 5$. **cao.** (2.33 or 2.3 without reference to $\frac{7}{3}$ or $2\frac{1}{3}$ is not allowed for this mark.)
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{\mathrm{d}y}{\mathrm{d}x}$) to gain all 5 marks in part (b).

Q53.

Question Number	Scheme	Marks
(a)	From question, $V=\frac{4}{3}\pi r^3$, $S=4\pi r^2$, $\frac{\mathrm{d}V}{\mathrm{d}t}=3$ $\left\{V=\frac{4}{3}\pi r^3 \Rightarrow\right\}\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ (Can be implied)	B1 oe
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\Rightarrow\right\}\left(4\pi r^2\right)\frac{\mathrm{d}r}{\mathrm{d}t}=3$ $\left(\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}r}\right)\times\frac{\mathrm{d}r}{\mathrm{d}t}=3$ $\left\{\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}r}\Rightarrow\right\}\frac{\mathrm{d}r}{\mathrm{d}t}=(3)\frac{1}{4\pi r^2};\left\{=\frac{3}{4\pi r^2}\right\}$ or $3\div\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}r}$;	M1 oe
	When $r=4\text{cm}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{3}{4\pi(4)^2}\left\{=\frac{3}{64\pi}\right\}$ **dependent on previous M1.** **see notes**	dM1
	Hence, $\frac{\mathrm{d}r}{\mathrm{d}t}=0.01492077591...(\text{cm}^2\text{ s}^{-1})$ anything that rounds to 0.0149	A1 **[4]**
(b)	$\left\{\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{\mathrm{d}S}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=\right\}\Rightarrow\frac{\mathrm{d}S}{\mathrm{d}t}=8\pi r\times\frac{3}{4\pi r^2}$ $\left\{\text{or }\frac{6}{r}\text{ or }8\pi r\times 0.0149...\right\}$ $8\pi r\times\text{Candidate's }\frac{\mathrm{d}r}{\mathrm{d}t}$ When $r=4\text{cm}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=8\pi(4)\times\frac{3}{4\pi(4)^2}$ or $\frac{6}{4}$ or $8\pi(4)\times 0.0149...$	M1; oe
	Hence, $\frac{\mathrm{d}S}{\mathrm{d}t}=1.5\ (\text{cm}^2\text{ s}^{-1})$ anything that rounds to 1.5	A1 cso **[2]** **6**

	Question Notes	
(a)	**B1**	$\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ Can be implied by later working.
	M1	$\left(\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}r}\right)\times\frac{\mathrm{d}r}{\mathrm{d}t}=3$ or $3\div\text{Candidate's }\frac{\mathrm{d}V}{\mathrm{d}r}$
	dM1	**(dependent on the previous method mark)** Substitutes $r=4$ into an expression which is a result of a quotient of "3" and their $\frac{\mathrm{d}V}{\mathrm{d}r}$.
	A1	anything that rounds to 0.0149 (units are not required)
(b)	**M1**	$8\pi r\times\text{Candidate's }\frac{\mathrm{d}r}{\mathrm{d}t}$
	A1	anything that rounds to 1.5 (units are not required). **Correct solution only.**
	Note	Using $\frac{\mathrm{d}r}{\mathrm{d}t}=0.0149$ gives $\frac{\mathrm{d}S}{\mathrm{d}t}=1.4979...$ which is fine for A1.

Q54.

Question Number	Scheme	Marks
	(a) $\frac{\mathrm{d}y}{\mathrm{d}x}=3^x\ln 3$	B1 (1)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=3^x\ln 3+3^{-x}\ln 3$	M1 A1
	At $x=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2\ln 3$	A1
	$y=2x\ln 3+2$ Accept equivalent equations	M1 A1 (5) **(6)**

Q55.

Question Number	Scheme	Marks
	(a) $V=\pi r^2 h$ $\left(\text{or base }\left(\pi r^2\right)\times\text{ height}\right)$	B1 (1)
	As $h=r$, $V=\pi r^3$ ✱	
	(b) $\frac{\mathrm{d}V}{\mathrm{d}r}=3\pi r^2$	B1 (1)
	(c) $V=\int\frac{2t}{2+t^2}\,\mathrm{d}t=\ln\left(2+t^2\right)+C$ Require C for the A	M1 A1
	$t=0, V=3\Rightarrow 3=\ln 2+C$	M1
	$V=\ln\left(2+t^2\right)-\ln 2+3$	A1 (4)
	(d) $V=\ln 3-\ln 2+3$ $(=3.405\,46\,...)$	M1 A1
	$r=\sqrt[3]{\frac{1}{\pi}(\ln 3-\ln 2+3)}\approx 1.03$ awrt 1.03	M1 A1 (4)
	(e) $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$	M1
	$=\frac{2t}{3\pi r^2\left(2+t^2\right)}$	A1 (2)
	(f) $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{2}{9\pi r^2}\approx 0.0670$ awrt 0.067	M1 A1 (2)
		(14)

Q56.

Question Number	Scheme	Marks
(a)	p=7.5	B1
		(1)
(b)	$2.5 = 7.5e^{-4k}$	M1
	$e^{-4k} = \frac{1}{3}$	M1
	$-4k = \ln(\frac{1}{3})$	dM1
	$-4k = -\ln(3)$	
	$k = \frac{1}{4}\ln(3)$	A1*
	See notes for additional correct solutions and the last A1	
		(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$	
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1
	t=4.1486…. 4.15 or awrt 4.1	A1
		(6)
		11Marks

Q57.

Question Number	Scheme	Marks
(a)	Either $y = 2$ or $(0, 2)$	B1
		(1)
(b)	When $x = 2$, $y = (8 - 10 + 2)\mathrm{e}^{-2} = 0\mathrm{e}^{-2} = 0$	B1
	$(2x^2 - 5x + 2) = 0 \Rightarrow (x - 2)(2x - 1) = 0$	M1
	Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.	A1
		(3)
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = (4x - 5)\mathrm{e}^{-x} - (2x^2 - 5x + 2)\mathrm{e}^{-x}$	M1A1A1
		(3)
(d)	$(4x - 5)\mathrm{e}^{-x} - (2x^2 - 5x + 2)\mathrm{e}^{-x} = 0$	M1
	$2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$	M1
	$x = \frac{7}{2}, 1$	A1
	When $x = \frac{7}{2}$, $y = 9\mathrm{e}^{-\frac{7}{2}}$, when $x = 1$, $y = -\mathrm{e}^{-1}$	ddM1A1
		(5) **[12]**
	(b) If the candidate believes that $\mathrm{e}^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark. (c) M1: (their u')e^{-x} + $(2x^2 - 5x + 2)$(their v') A1: Any one term correct. A1: Both terms correct. (d) 1st M1: For setting their $\frac{\mathrm{d}y}{\mathrm{d}x}$ found in part (c) equal to 0. 2nd M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$. See rules for solving a three term quadratic equation on page 1 of this Appendix. 3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)\mathrm{e}^{-x}$. Note that this method mark is dependent on the award of the two previous method marks in this part. Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf. Final A1: Both $\{x = 1\}$, $y = -\mathrm{e}^{-1}$ and $\{x = \frac{7}{2}\}$, $y = 9\mathrm{e}^{-\frac{7}{2}}$. **cao** Note that both exact values of y are required.	

Q58.

Question Number	Scheme	Marks
(a)	$P = \frac{800e^0}{1+3e^0}, = \frac{800}{1+3} = 200$	M1,A1
		(2)
(b)	$250 = \frac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t}) = 800e^{0.1t} \Rightarrow 50e^{0.1t}=250, \Rightarrow e^{0.1t} = 5$	M1,A1
	$t = \frac{1}{0.1}\ln(5)$	M1
	$t = 10\ln(5)$	A1
		(4)
(c)	$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow \frac{dP}{dt} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$	M1,A1
	At t=10	
	$\frac{dP}{dt} = \frac{(1+3e)\times 80e - 240e^2}{(1+3e)^2} = \frac{80e}{(1+3e)^2}$	M1,A1
		(4)
(d)	$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow P_{\max} = \frac{800}{3} = 266$. Hence P cannot be 270	B1
		(1) **(11 marks)**

(a)

M1 Sub $t = 0$ into P **and** use $e^0 = 1$ in at least one of the two cases. Accept $P = \frac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P = \frac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t} = B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t} = D$

A1 $e^{0.1t} = 5$ or $e^{-0.1t} = 0.2$

M1 Dependent upon gaining $e^{0.1t} = E$, for taking ln's of both sides and proceeding to t=…

Accept $e^{0.1t} = E \Rightarrow 0.1t = \ln E \Rightarrow t = ...$It could be implied by $t =$ awrt 16.1

A1 $t = 10\ln(5)$

Accept exact equivalents of this as long as a and b are integers. Eg. $t = 5\ln(25)$ is fine.

(c)

M1 Scored for a full application of the quotient rule and knowing that $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$ and NOT $kte^{0.1t}$

If the rule is quoted it must be correct.

It may be implied by their $u = 800e^{0.1t}, v = 1+3e^{0.1t}, u' = pe^{0.1t}, v' = qe^{0.1t}$ followed by $\frac{vu'-uv'}{v^2}$.

If it is neither quoted nor implied only accept expressions of the form

$$\frac{(1+3e^{0.1t})\times pe^{0.1t} - 800e^{0.1t}\times qe^{0.1t}}{(1+3e^{0.1t})^2}$$

Condone missing brackets.

You may see the chain or product rule applied to

For applying the product rule see question 1 but still insist on $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$

For the chain rule look for

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow \frac{dP}{dt} = 800\times\left(e^{-0.1t}+3\right)^{-2}\times -0.1e^{-0.1t}$$

A1 A correct unsimplified answer to

$$\frac{dP}{dt} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$$

M1 For substituting $t = 10$ into their $\frac{dP}{dt}$, NOT P

Accept numerical answers for this. 2.59 is the numerical value if $\frac{dP}{dt}$ was correct

A1 $\frac{dP}{dt} = \frac{80e}{(1+3e)^2}$ or equivalent such as $\frac{dP}{dt} = 80e(1+3e)^{-2}, \frac{80e}{1+6e+9e^2}$

Note that candidates who substitute $t = 10$ before differentiation will score 0 marks

(d)

B1 Accept solutions from substituting P=270 and showing that you get an unsolvable equation

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow 0.1t = \ln(-27)$ which has no answers.

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow e^{0.1t} / e^x$ is never negative

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment 'so it cannot reach 270', or a large value of t $(t > 99)$ being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment

Q59.

Question Number	Scheme	Marks
	(a) $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$	M1 A1
	$V = 9\pi h \quad \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = 9\pi \frac{\mathrm{d}h}{\mathrm{d}t}$	B1
	$9\pi \frac{\mathrm{d}h}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$	M1
	Leading to $75\frac{\mathrm{d}h}{\mathrm{d}t} = 4 - 5h$ $\ast$ cso	A1 **(5)**
	(b) $\int \frac{75}{4-5h}\,\mathrm{d}h = \int 1\,\mathrm{d}t$ separating variables	M1
	$-15\ln(4-5h) = t \quad (+C)$	M1 A1
	$-15\ln(4-5h) = t + C$	
	When $t = 0$, $h = 0.2$	
	$-15\ln 3 = C$	M1
	$t = 15\ln 3 - 15\ln(4-5h)$	
	When $h = 0.5$	
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	M1 A1
	Alternative for last 3 marks	
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$	
	$= -15\ln 1.5 + 15\ln 3$	M1 M1
	$= 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	A1 **(6)**

Q60.

Question Number	Scheme	Marks
(a)	$f'(x)=50x^2e^{2x}+50xe^{2x}$ oe.	M1A1
	Puts $f'(x)=0$ to give $x=-1$ and $x=0$ or one coordinate	dM1A1
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$ CSO	A1
		(5)
(b)	Puts $25x^2e^{2x}-16=0 \Rightarrow x^2=\frac{16}{25}e^{-2x} \Rightarrow x=\pm\frac{4}{5}e^{-x}$	B1*
		(1)
(c)	Subs $x_0=0.5$ into $x=\frac{4}{5}e^{-x} \Rightarrow x_1=$ awrt 0.485	M1A1
	$\Rightarrow x_2=$ awrt 0.492, $x_3=$ awrt 0.489	A1
		(3)
(d)	$\alpha=0.49$	B1
	$f(0.485)=-0.487$, $f(0.495)=(+)0.485$, sign change and deduction	B1
		(2)
		(11 marks)

Notes for Question

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1	Uses $vu'+uv'$. If the rule is quoted it must be correct. It can be implied by their $u=..,v=...,u'=...,v'=...$ followed by their $vu'+uv'$ If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x}+Bxe^{2x}$
A1	$f'(x)=50x^2e^{2x}+50xe^{2x}$. Allow un simplified forms such as $f'(x)=25x^2\times 2e^{2x}+50x\times e^{2x}$
dM1	Sets $f'(x)=0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x This is dependent upon the first M1 being scored.
A1	Both $x=-1$ and $x=0$ or one complete coordinate . Accept $(0,-16)$ and $(-1, 25e^{-2}-16)$ or $(-1, awrt -12.6)$
A1	CSO. Obtains both solutions from differentiation. Coordinates can be given in any way. $x=-1, 0$ $y=\frac{25}{e^2}-16, -16$ or linked together by coordinate pairs $(0,-16)$ and $(-1, 25e^{-2}-16)$ but the 'pairs' must be correct and exact.

Notes for Question Continued

(b)

B1 This is a show that question and all elements must be seen

Candidates must 1) State that f(x)=0 or writes $25x^2e^{2x}-16=0$ or $25x^2e^{2x}=16$

2) Show at least one intermediate (correct) line with either

x^2 or x the subject. Eg $x^2=\frac{16}{25}e^{-2x}$, $x=\sqrt{\frac{16}{25}e^{-2x}}$ oe

or square rooting $25x^2e^{2x}=16\Rightarrow 5xe^x=\pm 4$

or factorising by DOTS to give $(5xe^x+4)(5xe^x-4)=0$

3) Show the given answer $x=\pm\frac{4}{5}e^{-x}$.

Condone the minus sign just appearing on the final line.

A 'reverse' proof is acceptable as long as there is a statement that f(x)=0

(c)

M1 Substitutes $x_0=0.5$ into $x=\frac{4}{5}e^{-x}\Rightarrow x_1=....$

This can be implied by $x_1=\frac{4}{5}e^{-0.5}$, or awrt 0.49

A1 $x_1=$ awrt 0.485 3dp. Mark as the first value given. Don't be concerned by the subscript.

A1 $x_2=$ awrt 0.492, $x_3=$ awrt 0.489 3dp. Mark as the second and third values given.

(d)

B1 States $\alpha=0.49$

B1 Justifies by

either calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp,

$f(0.485)=-0.5, f(0.495)=(+)0.5$ rounded

$f(0.485)=-0.4, f(0.495)=(+)0.4$ truncated

giving a reason – accept change of sign, >0 <0 or $f(0.485)\times f(0.495)<0$

and giving a minimal conclusion. Eg. Accept hence root or $\alpha=0.49$

A smaller interval containing the root may be used, eg $f(0.49)$ and $f(0.495)$. Root = 0.49007

or by stating that the iteration is oscillating

or by calculating by continued iteration to at least the value of $x_4=$ awrt 0.491 and stating (or seeing each value round to) 0.49

Q61.

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx}=0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\tan 3x=-\sqrt{3}$	A1
	$3x=\frac{2\pi}{3}\Rightarrow x=\frac{2\pi}{9}$	M1A1
		(6)
	(b) At x=0 $\quad \frac{dy}{dx}=3$	B1
	Equation of normal is $-\frac{1}{3}=\frac{y-0}{x-0}$ or any equivalent $y=-\frac{1}{3}x$	M1A1
		(3) **(9 marks)**

(a) M1 Applies the product rule vu'+uv' to $e^{x\sqrt{3}}\sin 3x$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=….,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form $\frac{dy}{dx}=Ae^{x\sqrt{3}}\sin 3x+e^{x\sqrt{3}}\times\pm B\cos 3x$

A1 Correct expression for $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$

M1 Sets **their** $\frac{dy}{dx}=0$, factorises out or divides by $e^{x\sqrt{3}}$ producing an equation in sin3x and cos3x

A1 Achieves either $\tan 3x=-\sqrt{3}$ or $\tan 3x=-\frac{3}{\sqrt{3}}$

M1 Correct order of arctan, followed by ÷3.

Accept $3x=\frac{5\pi}{3}\Rightarrow x=\frac{5\pi}{9}$ or $3x=\frac{-\pi}{3}\Rightarrow x=\frac{-\pi}{9}$ but not $x=\arctan(\frac{-\sqrt{3}}{3})$

A1 CS0 $x=\frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

(b) B1 Sight of **3** for the gradient

M1 A full method for finding an equation of the normal.

Their tangent gradient m must be modified to $-\frac{1}{m}$ and used together with (0, 0).

Eg $-\frac{1}{their\ 'm'}=\frac{y-0}{x-0}$ or equivalent is acceptable

A1 $y=-\frac{1}{3}x$ or any correct equivalent including $-\frac{1}{3}=\frac{y-0}{x-0}$.

Alternative in part (a) using the form $R\sin(3x+\alpha)$ JUST LAST 3 MARKS

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx}=0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$(\sqrt{12})\sin(3x+\frac{\pi}{3})=0$	A1
	$3x=\frac{2\pi}{3}\Rightarrow x=\frac{2\pi}{9}$	M1A1
		(6)

A1 Achieves either $(\sqrt{12})\sin(3x+\frac{\pi}{3})=0$ or $(\sqrt{12})\cos(3x-\frac{\pi}{6})=0$

M1 Correct order of arcsin or arcos, etc to produce a value of x

Eg accept $3x+\frac{\pi}{3}=0$ or π or $2\pi\Rightarrow x=....$

A1 Cao $x=\frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

Alternative to part (a) squaring both sides JUST LAST 3 MARKS

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx}=0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\sqrt{3}\sin 3x=-3\cos 3x\Rightarrow\cos^2(3x)=\frac{1}{4}$ or $\sin^2(3x)=\frac{3}{4}$	A1
	$x=\frac{1}{3}\arccos(\pm\sqrt{\frac{1}{4}})$ oe	M1
	$x=\frac{2\pi}{9}$	A1

Q62.

Question Number	Scheme	Marks
	(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{\mathrm{d}}{\mathrm{d}x}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x}$	M1A1
		(3)
	(ii)	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{(2x-1)^{10}}$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{80x}{(2x-1)^6}$	A1
		(3)
	(b) $x = 3\tan 2y \;\; \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = 6\sec^2 2y$	M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y = 1+\tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$	
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6(1+(\frac{x}{3})^2)} = (\frac{3}{18+2x^2})$	M1A1
		(5) **(11 marks)**

Note that this is marked B1M1A1 on EPEN

(a)(i) M1 Attempts to differentiate $\ln(3x)$ to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine.

M1 Attempts the product rule for $x^{\frac{1}{2}}\ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$\ln(3x)\times Ax^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{B}{x}, \quad A, B > 0$

A1 Any correct (un simplified) form of the answer. Remember to isw any incorrect further work

$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x} = (\frac{\ln(3x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}) = x^{-\frac{1}{2}}(\frac{1}{2}\ln 3x + 1)$

Note that this part does not require the answer to be in its simplest form

(ii) M1 Applies the quotient rule, a version of which appears in the formula booklet. If the formula is quoted it must be correct. There must have been an attempt to differentiate both terms. If the formula is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\frac{(2x-1)^5\times\pm 10-(1-10x)\times C(2x-1)^4}{(2x-1)^{10 \text{ or } 7 \text{ or } 25}}$$

A1 Any un simplified form of the answer. Eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{((2x-1)^5)^2}$

A1 Cao. It must be simplified as required in the question $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{80x}{(2x-1)^6}$

(b) M1 Knows that $3\tan 2y$ differentiates to $C\sec^2 2y$. The lhs can be ignored for this mark. If they write $3\tan 2y$ as $\frac{3\sin 2y}{\cos 2y}$ this mark is awarded for a correct attempt of the quotient rule.

A1 Writes down $\frac{\mathrm{d}x}{\mathrm{d}y} = 6\sec^2 2y$ or implicitly to get $1 = 6\sec^2 2y\frac{\mathrm{d}y}{\mathrm{d}x}$

Accept from the quotient rule $\frac{6}{\cos^2 2y}$ or even $\frac{\cos 2y\times 6\cos 2y - 3\sin 2y\times -2\sin 2y}{\cos^2 2y}$

M1 An attempt to invert 'their' $\frac{\mathrm{d}x}{\mathrm{d}y}$ to reach $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(y)$, or changes the subject of their implicit

. differential to achieve a similar result $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(y)$

M1 Replaces an expression for $\sec^2 2y$ in their $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ with x by attempting to use

$\sec^2 2y = 1+\tan^2 2y$. Alternatively, replaces an expression for y in $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ with $\frac{1}{2}\arctan(\frac{x}{3})$

A1 Any correct form of $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6(1+(\frac{x}{3})^2)}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{18+2x^2}$ *or* $\frac{1}{6\sec^2(\arctan(\frac{x}{3}))}$

Question Number	Scheme	Marks
	(a)(ii) Alt using the product rule	
	Writes $\frac{1-10x}{(2x-1)^5}$ as $(1-10x)(2x-1)^{-5}$ and applies vu'+uv'.	
	See (a)(i) for rules on how to apply $(2x-1)^{-5}\times -10+(1-10x)\times -5(2x-1)^{-6}\times 2$	M1A1
	Simplifies as main scheme to $80x(2x-1)^{-6}$ or equivalent	A1
		(3)
	(b) Alternative using arctan. They must attempt to differentiate to score any marks. Technically this is M1A1M1A2	
	Rearrange $x = 3\tan 2y$ to $y = \frac{1}{2}\arctan(\frac{x}{3})$ **and attempt to differentiate**	M1A1
	Differentiates to a form $\frac{A}{1+(\frac{x}{3})^2}$, $= \frac{1}{2}\times\frac{1}{(1+(\frac{x}{3})^2)}\times\frac{1}{3}$ or $\frac{1}{6(1+(\frac{x}{3})^2)}$ oe	M1, A2
		(5)

Q63.

Question Number	Scheme	Marks
(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1 (2)
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1* (4)
(c)	$\frac{d^2y}{dx^2} - \frac{0-[6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1
		(4)
		(10 marks)
Alt 1 to (a)	$x = (\cos 3y)^{-2} \Rightarrow \frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3\sin 3y$	M1A1
Alt 2 to (a)	$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3\sec 3y \tan 3y + \sec 3y \times 3\sec 3y \tan 3y$	M1A1
Alt 1 To (c)	$\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$	M1A1
	$= \frac{1}{6}x^{-2}(x-1)^{-\frac{3}{2}}[x(-\frac{1}{2}) + (-1)(x-1)]$	dM1
	$= \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe	A1
		(4)

Notes for Question

(a)

M1 Uses the chain rule to get $A\sec 3y\sec 3y\tan 3y = \left(A\sec^2 3y\tan 3y\right)$.

There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3}\sin 3y$

or the quotient rule on $\dfrac{1}{(\cos 3y)^2} \Rightarrow \dfrac{\pm A\cos 3y\sin 3y}{(\cos 3y)^4}$

A1 $\dfrac{dx}{dy} = 2\times 3\sec 3y\sec 3y\tan 3y$ or equivalent. There is no need to simplify the rhs but both sides must be correct.

(b)

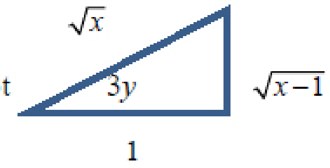
M1 Uses $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ to get an expression for $\dfrac{dy}{dx}$. Follow through on their $\dfrac{dx}{dy}$

Allow slips on the coefficient but not trig expression.

B1 Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = (x-1)^{\frac{1}{2}}$

All elements **must be present.**

Accept



$\cos 3y = \dfrac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$

If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \dfrac{\sqrt{x-1}}{\sqrt{x}}$

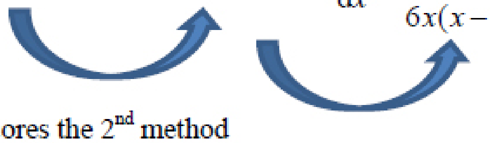
M1 Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\dfrac{dy}{dx}$ in just x. Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$.

It may be implied- see below

A1* CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

Eg. $\dfrac{dx}{dy} = 6\sec^2 3y\tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6x(x-1)^{\frac{1}{2}}}$



Scores the 2nd method

Scores the 1st method

The above solution will score M1, B0, M1, A0

Notes for Question Continued

Example 1- Scores 0 marks in part (b)

$$\frac{dx}{dy} = 6\sec^2 3y \tan 3y \Rightarrow \frac{dy}{dx} = \frac{1}{6\sec^2 3x \tan 3x} = \frac{1}{6\sec^2 3x\sqrt{\sec^2 3x - 1}} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{dx}{dy} = 2\sec^2 3y \tan 3y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 3y \tan 3y} = \frac{1}{2\sec^2 3y\sqrt{\sec^2 3y - 1}} = \frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) **Using Quotient and Product Rules**

M1 Uses the quotient rule $\frac{vu' - uv'}{v^2}$ with $u = 1$ and $v = 6x(x-1)^{\frac{1}{2}}$ **and** achieving

$u' = 0$ and $v' = A(x-1)^{\frac{1}{2}} + Bx(x-1)^{-\frac{1}{2}}$.

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\xcancel{\left(\frac{d^2y}{dx^2}\right)} = \frac{0 - [A(x-1)^{\frac{1}{2}} + Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}}\right)^2} \quad \text{or} \quad \xcancel{\left(\frac{d^2y}{dx^2}\right)} = \frac{0 - A(x-1)^{\frac{1}{2}} \pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{d^2y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe

dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2y}{dx^2} = \frac{2 - 3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question   Continued

(c) Using Product and Chain Rules

M1 Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and

v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct.

If the rules are not quoted nor implied then award if you see an expression of the form

$(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$

A1 $\xcancel{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} = \tfrac{1}{6}[x^{-1}(-\tfrac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$

dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} == \tfrac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ *oe*

(c) Using Quotient and Chain rules Rules

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ and achieving

$u' = A(x-1)^{-\frac{3}{2}}$ and $v' = B$.

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

$$\xcancel{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)} = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$$

A1 Correct un simplified expression $\xcancel{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$

dM1 Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question Continued	
(c)	**Using just the chain rule**
M1	Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = \frac{1}{(36x^3-36x^2)^{\frac{1}{2}}} = (36x^3-36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to $A(36x^3-36x^2)^{-\frac{3}{2}}(Bx^2-Cx)$.
M1	Would automatically follow under this method if the first M has been scored

Q64.

Question Number	Scheme		Marks
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *)		
	$\{t=0, \theta=90 \Rightarrow\}$ $90 = 20 + Ae^{-k(0)}$	Substitutes $t=0$ and $\theta=90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70e^{-kt}$		
	$\{t=5, \theta=55 \Rightarrow\}$ $55 = 20 + 70e^{-k(5)}$ $\frac{35}{70} = e^{-5k}$	Substitutes $t=5$ and $\theta=55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$		
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{1}{5}\ln 2.(70)e^{-\frac{1}{5}t\ln 2}$	$\pm\alpha e^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2e^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t = 10$, $\frac{\mathrm{d}\theta}{\mathrm{d}t} = -14\ln 2e^{-2\ln 2}$		
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426$ °C/min (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**

Q65.

Question Number	Scheme		Marks
(a)			
	Crosses x-axis $\Rightarrow \text{f}(x) = 0 \Rightarrow (8-x)\ln x = 0$		
	Either $(8-x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of {x}=1 OR x={8}	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1, \{0\})$ and $B(8, \{0\})$	B1
			(2)
(b)	Apply product rule: $\left\{\begin{matrix} u = (8-x) & v = \ln x \\ \frac{\text{d}u}{\text{d}x} = -1 & \frac{\text{d}v}{\text{d}x} = \frac{1}{x} \end{matrix}\right\}$	$vu' + uv'$	M1
	$\text{f}'(x) = -\ln x + \frac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
(c)	$\text{f}'(3.5) = 0.032951317...$ $\text{f}'(3.6) = -0.058711623...$ Sign change (and as $\text{f}'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $\text{f}'(3.5)$ and $\text{f}'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
(d)	At Q, $\text{f}'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$	Setting $\text{f}'(x) = 0$.	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)
Question Number	Scheme		Marks
(e)	Iterative formula: $x_{n+1} = \frac{8}{\ln x_n + 1}$		
	$x_1 = \frac{8}{\ln(3.55) + 1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$	Both $x_1 = \text{awrt } 3.529$ and $x_2 = \text{awrt } 3.538$	A1
	$x_1 = 3.529$, $x_2 = 3.538$, $x_3 = 3.534$, to 3 dp.	x_1, x_2, x_3 all stated correctly to 3 dp	A1
			(3) **[13]**

Q66.

Question Number	Scheme		Marks
	$x = 27\sec^3 t, \quad y = 3\tan t, \quad 0 \leqslant t \leqslant \frac{\pi}{3}$		
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 81\sec^2 t\sec t\tan t, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = 3\sec^2 t$	At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct.	B1
		Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct.	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2 t}{81\sec^3 t\tan t} \left\{= \frac{1}{27\sec t\tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t}\right\}$	Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72}\left\{= \frac{3}{54} = \frac{1}{18}\right\}$	$\frac{4}{72}$	A1 cao cso
			[4]
(b)	$\left\{1 + \tan^2 t = \sec^2 t\right\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\frac{x}{27}\right)}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * cso
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1
			[3]
(c)	$V = \pi\int_{27}^{125}\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2 \mathrm{d}x$ or $\pi\int_{27}^{125}\left(x^{\frac{2}{3}} - 9\right)\mathrm{d}x$	For $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\}\left[\frac{3}{5}x^{\frac{5}{3}} - 9x\right]_{27}^{125}$	Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\frac{3}{5}x^{\frac{5}{3}}$ oe	M1
		$\frac{3}{5}x^{\frac{5}{3}} - 9x$ oe	A1
	$= \{\pi\}\left(\left(\frac{3}{5}(125)^{\frac{5}{3}} - 9(125)\right) - \left(\frac{3}{5}(27)^{\frac{5}{3}} - 9(27)\right)\right)$	Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.	dM1
	$= \{\pi\}((1875 - 1125) - (145.8 - 243))$		
	$= \frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1
			[5] 12

Notes for Question

(a)	**B1:** At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. **Note:** that this mark can be implied from their working. **B1:** Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct. **Note:** that this mark can be implied from their working. **M1:** Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$, where both $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$ are trigonometric functions of *t*. **A1:** $\frac{4}{72}$ or any equivalent correct rational answer not involving surds. Allow $0.0\dot{5}$ with the recurring symbol.

Notes for Question Continued

Note: Please check that their $\frac{\mathrm{d}x}{\mathrm{d}t}$ is differentiated correctly.

Eg. Note that $x = 27\sec^3 t = 27(\cos t)^{-3} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = -81(\cos t)^{-2}(-\sin t)$ is correct.

(b)

M1: Either:

- Applying a correct trigonometric identity (usually $1 + \tan^2 t = \sec^2 t$) to give a Cartesian equation in x and y only.
- Starting from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.
- Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity.

A1*: For a correct proof of $y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$.

Note this result is printed on the Question Paper, so no incorrect working is allowed.

B1: Both $a = 27$ and $b = 216$. **Note** that $27 \leqslant x \leqslant 216$ is also fine for B1.

(c)

B1: For a correct statement of $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$. Ignore limits and $\mathrm{d}x$. Can be implied.

M1: Either integrates to give $\pm Ax^{\frac{5}{3}} \pm Bx$, $A \neq 0$, $B \neq 0$ or integrates $x^{\frac{2}{3}}$ correctly to give $\frac{3}{5}x^{\frac{5}{3}}$ oe

A1: $\frac{3}{5}x^{\frac{5}{3}} - 9x$ or. $\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} - 9x$ oe.

dM1: Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.
Note: that this mark is dependent upon the previous method mark being awarded.

A1: A correct exact answer of $\frac{4236\pi}{5}$ or 847.2π.

Note: The π in the volume formula is only required for the B1 mark and the final A1 mark.
Note: A decimal answer of 2661.557... without a correct exact answer is A0.
Note: If a candidate gains the first B1M1A1 and then writes down 2661 or awrt 2662 with no method for substituting limits of 125 and 27, then award the final M1A0.

(a)

Way 2

Alternative response using the Cartesian equation in part (a)

$\left\{y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \Rightarrow\right\} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm Kx^{-\frac{1}{3}}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ oe	A1
At $t = \frac{\pi}{6}$, $x = 27\sec^3\left(\frac{\pi}{6}\right) = 24\sqrt{3}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(\left(24\sqrt{3}\right)^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}\left(24\sqrt{3}\right)^{-\frac{1}{3}}\right)$	Uses $t = \frac{\pi}{6}$ to find x and substitutes their x into an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.	dM1
So, $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{3\sqrt{3}}\right) = \frac{1}{18}$	$\frac{1}{18}$	A1 cao cso

Note: Way 2 is marked as M1 A1 dM1 A1
Note: For way 2 the second M1 mark is dependent on the first M1 being gained.

Notes for Question Continued			
(b) **Way 2**	*Alternative responses for M1A1 in part (b)*: STARTING FROM THE RHS		
	{RHS =} $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}} = \sqrt{\left(27\sec^3 t\right)^{\frac{2}{3}}-9} = \sqrt{9\sec^2 t-9} = \sqrt{9\tan^2 t}$	For applying $1+\tan^2 t=\sec^2 t$ oe to achieve $\sqrt{9\tan^2 t}$	M1
	$=3\tan t = y$ {= LHS} **cso**	Correct proof from $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$ to y.	A1*
	M1: Starts from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.		
(b) **Way 3**	*Alternative responses for M1A1 in part (b)*: STARTING FROM THE LHS		
	{LHS =} $y = 3\tan t = \sqrt{\left(9\tan^2 t\right)} = \sqrt{9\sec^2 t-9}$	For applying $1+\tan^2 t=\sec^2 t$ oe to achieve $\sqrt{9\sec^2 t-9}$	M1
	$= \sqrt{9\left(\frac{x}{27}\right)^{\frac{2}{3}}-9} = \sqrt{9\left(\frac{x^{\frac{2}{3}}}{9}\right)-9} = \left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$ **cso**	Correct proof from y to $\left(x^{\frac{2}{3}}-9\right)^{\frac{1}{2}}$.	A1*
	M1: Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t-9}$ by using a correct trigonometric identity.		
(c) **Way 2**	*Alternative response for part (c) using parametric integration*		
	$V=\pi\int 9\tan^2 t\left(81\sec^2 t\sec t\tan t\right)\,dt$	$\pi\int 3\tan t\left(81\sec^2 t\sec t\tan t\right)\,dt$ Ignore limits and dx. Can be implied.	B1
	$=\{\pi\}\int 729\sec^2 t\tan^2 t\sec t\tan t\,dt$		
	$=\{\pi\}\int 729\sec^2 t\left(\sec^2 t-1\right)\sec t\tan t\,dt$		
	$=\{\pi\}\int 729\left(\sec^4 t-\sec^2 t\right)\sec t\tan t\,dt$		
	$=\{\pi\}\int 729\left(\sec^4 t-\sec^2 t\right)\sec t\tan t\,dt$		
	$=\{\pi\}\left[729\left(\frac{1}{5}\sec^5 t-\frac{1}{3}\sec^3 t\right)\right]$	$\pm A\sec^5 t\pm B\sec^3 t$	M1
		$729\left(\frac{1}{5}\sec^5 t-\frac{1}{3}\sec^3 t\right)$	A1
	$V=\{\pi\}\left[729\left(\frac{1}{5}\left(\frac{5}{3}\right)^5-\frac{1}{3}\left(\frac{5}{3}\right)^3\right)-729\left(\frac{1}{5}1^5-\frac{1}{3}1^3\right)\right]$	Substitutes $\sec t=\frac{5}{3}$ and $\sec t=1$ into an integrated function and subtracts the correct way round.	dM1
	$=729\pi\left[\left(\frac{250}{243}\right)-\left(-\frac{2}{15}\right)\right]$		
	$=\frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1
			[5]

Q67.

<table>
<tr><th>Question Number</th><th colspan="3">Scheme</th><th></th><th>Marks</th></tr>
<tr><td>(a)</td><td colspan="3">$\frac{dN}{dt} = \frac{(kt-1)(5000-N)}{t}, \quad t > 0, \;\; 0 < N < 5000$
$\int \frac{1}{5000-N}\,dN = \int \frac{(kt-1)}{t}\,dt \quad \left\{\text{or } = \int\left(k - \frac{1}{t}\right)dt\right\}$
$-\ln(5000-N) = kt - \ln t; + c$</td><td>
See notes
See notes</td><td>
B1
M1 A1; A1</td></tr>
<tr><td></td><td><i>then eg either...</i></td><td><i>or...</i></td><td><i>or...</i></td><td></td><td></td></tr>
<tr><td></td><td>$-kt + c = \ln(5000-N) - \ln t$
$-kt + c = \ln\left(\frac{5000-N}{t}\right)$
$e^{-kt+c} = \frac{5000-N}{t}$</td><td>$kt + c = \ln t - \ln(5000-N)$
$kt + c = \ln\left(\frac{t}{5000-N}\right)$
$e^{kt+c} = \frac{t}{5000-N}$</td><td>$\ln(5000-N) = -kt + \ln t + c$
$5000 - N = e^{-kt + \ln t + c}$
$5000 - N = t\,e^{-kt + c}$</td><td></td><td></td></tr>
<tr><td></td><td colspan="4">leading to $N = 5000 - Ate^{-kt}$ with no incorrect working/statements. See notes</td><td>A1 * cso
[5]</td></tr>
<tr><td>(b)</td><td colspan="3">$\{t = 1, N = 1200 \Rightarrow\}\;\; 1200 = 5000 - Ae^{-k}$
$\{t = 2, N = 1800 \Rightarrow\}\;\; 1800 = 5000 - 2Ae^{-2k}$
So $Ae^{-k} = 3800$
and $2Ae^{-2k} = 3200$ or $Ae^{-2k} = 1600$
Eg. $\frac{e^{-k}}{2e^{-2k}} = \frac{3800}{3200}$ or $\frac{2e^{-2k}}{e^{-k}} = \frac{3200}{3800}$
$\frac{1}{2}$ · $\frac{3800}{3200}$ · $\frac{3200}{3800}$</td><td>At least one correct statement written down using the boundary conditions
An attempt to eliminate A by producing an equation in only k.</td><td>B1
M1</td></tr>
<tr><td></td><td colspan="3">$k = \ln\left(\frac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\frac{19}{8}\right)\right\}$</td><td>At least one of $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or exact equivalent</td><td>A1</td></tr>
<tr><td></td><td colspan="3">$\left\{A = 3800(e^{k}) = 3800\left(\frac{19}{8}\right) \Rightarrow\right\}\;\; A = 9025$</td><td>Both $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or exact equivalent</td><td>A1
[4]</td></tr>
<tr><td></td><td colspan="3"><u>Alternative Method for the M1 mark in (b)</u>
$e^{-k} = \frac{3800}{A}$
$2A\left(\frac{3800}{A}\right)^2 = 3200$</td><td>An attempt to eliminate k by producing an equation in only A</td><td>M1</td></tr>
<tr><td>(c)</td><td colspan="3">$\left\{t = 5,\;\; N = 5000 - 9025(5)e^{-5\ln\left(\frac{19}{8}\right)}\right\}$
$N = 4402.828401... = 4400$ (fish) (nearest 100)</td><td>anything that rounds to 4400</td><td>B1
[1]
10</td></tr>
</table>

	Question Notes	
(a)		
	B1	Separates variables as shown. dN and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	M1	**Either** $\pm\lambda\ln(5000-N)$ or $\pm\lambda\ln(N-5000)$ **or** $kt-\ln t$ where $\lambda \neq 0$ is a constant.
	A1	For $-\ln(5000-N) = kt - \ln t$ or $\ln(5000-N) = -kt + \ln t$ or $-\frac{1}{k}\ln(5000-N) = t - \frac{1}{k}\ln t$ oe
	A1	**which is dependent on the 1st M1 mark being awarded.** For applying a constant of integration, eg. $+c$ or $+\ln e^c$ or $+\ln c$ or A to their integrated equation
	Note	$+c$ can be on either side of their equation for the 2nd A1 mark.
	A1	Uses a constant of integration eg. "c" or " $\ln e^c$ " "$\ln c$" or and applies a fully correct method to prove the result $N = 5000 - Ate^{-kt}$ with no incorrect working seen. (Correct solution only.)
	NOTE	**IMPORTANT** There needs to be an intermediate stage of justifying the A **and** the e^{-kt} in Ate^{-kt} by for example • **either** $5000 - N = e^{\ln t - kt + c}$ • **or** $5000 - N = te^{-kt + c}$ • **or** $5000 - N = te^{-kt}e^c$ or equivalent needs to be stated before achieving $N = 5000 - Ate^{-kt}$
(b)	**B1**	At least one of **either** $1200 = 5000 - Ae^{-k}$ (or equivalent) **or** $1800 = 5000 - 2Ae^{-2k}$ (or equivalent)
	M1	• **Either** an attempt to eliminate A by producing an equation in only k. • or an attempt to eliminate k by producing an equation in only A
	A1	At least one of $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or equivalent
	A1	Both $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or equivalent
	Note	Alternative correct values for k are $k = \ln\left(\frac{19}{8}\right)$ or $k = -\ln\left(\frac{8}{19}\right)$ or $k = \ln 7600 - \ln 3200$ or $k = -\ln\left(\frac{3800}{9025}\right)$ or equivalent.
	Note	$k = 0.8649...$ without a correct exact equivalent is A0.
(c)	**B1**	anything that rounds to 4400

Q68.

Question Number	Scheme	Marks
(a)	$y_{2.1} = -0.224$, $y_{2.2} = (+)0.546$	M1
	Change of sign⇒Q lies between	A1
		(2)
(b)	At R $\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow$ $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
(c)	$x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$	M1
	$x_1 =$ awrt 1.284 $x_2 =$ awrt 1.276	A1
		(2)
		(8 marks)

(a)

M1 Sub both $x = 2.1$ and $x = 2.2$ into y and achieve at least one correct to 1 sig fig

In radians $y_{2.1} = \text{awrt} -0.2 \quad y_{2.2} = \text{awrt/truncating to } 0.5$

In degrees $y_{2.1} = \text{awrt } 3 \quad y_{2.2} = \text{awrt } 4$

A1 Both values correct to 1 sf with a reason and a minimal conclusion.

$y_{2.1} = \text{awrt} -0.2 \quad y_{2.2} = \text{awrt/truncating to } 0.5$

Accept change of sign, positive and negative, $y_{2.1} \times y_{2.2} = -1$ as reasons and hence root, Q lies between 2.1 and 2.2, QED as a minimal conclusion.

Accept a smaller interval spanning the root of 2.131528, say 2.13 and 2.14, but the A1 can only be scored when the candidate refers back to the question, stating that as root lies between 2.13 and 2.14 it lies between 2.1 and 2.2

(b)

M1 Differentiating to get $\frac{dy}{dx} = \ldots \sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$ where ... is a constant, or a linear function in x.

A1 $\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$

M1 Sets their $\frac{dy}{dx} = 0$ and proceeds to make the x of their $3x^2$ the subject of the formula

Alternatively they could state $\frac{dy}{dx} = 0$ and write a line such as

$2x\sin\left(\frac{1}{2}x^2\right) = 3x^2 - 3$, before making the x of $3x^2$ the subject of the formula

A1* Correct given solution. $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$

Watch for missing x's in their formula

(c)

M1 Subs $x = 1.3$ into the iterative formula to find at least x_1.

This can be implied by $x_1 = \text{awrt } 1.3$ (not just 1.3)

or $x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$ or $x_1 = \text{awrt } 1.006$ (degrees)

A1 Both answers correct (awrt 3 decimal places). The subscripts are not important. Mark as the first and second values seen. $x_1 = \text{awrt } 1.284 \quad x_2 = \text{awrt } 1.276$

Q69.

Question Number	**Scheme**	**Marks**
(a)	$f(x)=0\Rightarrow x^2+3x+1=0$	
	$\Rightarrow x=\dfrac{-3\pm\sqrt{5}}{2}$ = awrt -0.382, -2.618	M1A1
		(2)
(b)	Uses $vu'+uv'$ $\quad f'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$	M1A1A1
		(3)
(c)	$e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x=0$	
	$\Rightarrow e^{x^2}\left\{2x^3+6x^2+4x+3\right\}=0$	M1
	$\Rightarrow x(2x^2+4)=-3(2x^2+1)$	M1
	$\Rightarrow x=-\dfrac{3(2x^2+1)}{2(x^2+2)}$	A1*
		(3)
(d)	Sub $x_0=-2.4$ into $\quad x_{n+1}=-\dfrac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$	
	$x_1=awrt-2.420,\ x_2=awrt-2.427\ \ x_3=awrt-2.430$	M1A1,A1
		(3)
(e)	Sub x= - 2.425 and -2.435 into $f'(x)$ and start to compare signs	
	$f'(-2.425)=+22.4, f'(-2.435)=-15.02$	**M1**
	Change in sign, hence $f'(x)=0$ in between. Therefore α = - 2.43 (2dp)	A1
		(2)
		(13 marks)
Alt (c)	$x=-\dfrac{3(2x^2+1)}{2(x^2+2)}\quad\Rightarrow 2x(x^2+2)=-3(2x^2+1)\quad\Rightarrow 2x^3+6x^2+4x+3=0$	M1
	$f'(x)=e^{x^2}\left\{2x^3+6x^2+4x+3\right\}=0$ when $2x^3+6x^2+4x+3=0$	M1
	Hence the minimum point occurs when $x=-\dfrac{3(2x^2+1)}{(2x^2+4)}$	**A1**

Question Number	Scheme	Marks
Alt 1 (e)	Sub x= - 2.425 and -2.435 into cubic part of $f'(x) = 2x^3+6x^2+4x+3$ and start to compare signs	
	Adapted $f'(-2.425)=+0.06, f'(-2.435)=-0.04$	**M1**
	Change in sign, hence $f'(x)=0$ in between. Therefore $\alpha = -2.43$ (2dp)	A1
		(2)
Alt 2 (e)	Sub x= - 2.425, -2.43 and -2.435 into $f(x)=(x^2+3x+1)e^{x^2}$ and start to compare sizes	
	$f(-2.425)=-141.2, f(-2.435)=-141.2, f(-2.43)=-141.3$	**M1**
	$f(-2.43)<f(-2.425), f(-2.43)<f(-2.435)$. Therefore $\alpha = -2.43$ (2dp)	A1
		(2)

Notes for Question

(a)

M1 Solves $x^2+3x+1=0$ by completing the square or the formula, producing two 'non integer answers. **Do not accept factorisation here** . Accept awrt -0.4 and -2.6 for this mark

A1 Answers correct. Accept awrt -0.382, -2.618.

Accept just the answers for both marks. Don't withhold the marks for incorrect labelling.

(b)

M1 Applies the product rule $vu'+uv'$ to $(x^2+3x+1)e^{x^2}$.

If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form

$$\left(\frac{dy}{dx}\right)=f'(x)=e^{x^2}(Ax+B)+(x^2+3x+1)Cxe^{x^2}$$

A1 One term of $f'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$ correct.

There is no need to simplify

A1 A fully correct (un simplified) answer $f'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$

(c)

M1 Sets their $f'(x)=0$ and either factorises out, or cancels by e^{x^2} to produce a polynomial equation in x

M1 Rearranges the cubic polynomial to $Ax^3+Bx=Cx^2+D$ and factorises to reach $x(Ax^2+B)=Cx^2+D$ or equivalent

A1* Correctly proceeds to $x=-\dfrac{3(2x^2+1)}{2(x^2+2)}$. This is a given answer

Notes on Question Continued

(c) Alternative to (c) working backwards

M1 Moves correctly from $x = -\dfrac{3(2x^2+1)}{2(x^2+2)}$ to $2x^3+6x^2+4x+3=0$

M1 States or implies that $\mathrm{f}'(x)=0$

A1 Makes a conclusion to tie up the argument

For example, hence the minimum point occurs when $x = -\dfrac{3(2x^2+1)}{(2x^2+4)}$

(d)

M1 Sub $x_0 = -2.4$ into $x_{n+1} = -\dfrac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$

This may be implied by awrt -2.42, or $x_{n+1} = -\dfrac{3(2\times -2.4^2+1)}{2(-2.4^2+2)}$

A1 Awrt. $x_1 = -2.420$.

The subscript is not important. Mark as the first value given

A1 awrt $x_2 = -2.427$ awrt $x_3 = -2.430$

The subscripts are not important. Mark as the second and third values given

(e)

Note that continued iteration is not allowed

M1 Sub x= - 2.425 and -2.435 into $\mathrm{f}'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $\mathrm{f}'(a)\times\mathrm{f}'(b)<0$

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Alt 1 using adapted $\mathrm{f}'(x)$

(e)

M1 Sub x= - 2.425 and -2.435 into cubic part of $\mathrm{f}'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct of adapted $\mathrm{f}'(x)$ correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $\mathrm{f}'(a)\times\mathrm{f}'(b)<0$

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Alt 2 using $\mathrm{f}(x)$

(e)

M1 Sub x= - 2.425, -2.43 and -2.435 into $\mathrm{f}(x)$, starts to compare sizes and gets at least one correct to 4sf rounded

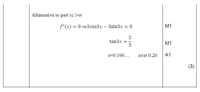
A1 All three values correct of $\mathrm{f}(x)$ correct (4sf rounded), a reason and a minimal conclusion

Acceptable reasons are $\mathrm{f}(-2.43)<\mathrm{f}(-2.425)$, $\mathrm{f}(-2.43)<\mathrm{f}(-2.435)$, a sketch

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Q70.

Question Number	Scheme	Marks
(a)	$R^2 = 2^2 + 3^2$	M1
	$R = \sqrt{13}$ or 3.61 …..	A1
	$\tan\alpha = \frac{3}{2}$	M1
	$\alpha = 0.983 \ldots$	A1
		(4)
(b)	$f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$	M1A1A1
	$= e^{2x}(2\cos 3x - 3\sin 3x)$	M1
	$= e^{2x}(R\cos(3x+\alpha)$	
	$= Re^{2x}\cos(3x+\alpha)$	A1* cso
		(5)
(c)	$f'(x) = 0 \Rightarrow \cos(3x+\alpha) = 0$	M1
	$3x + \alpha = \frac{\pi}{2}$	M1
	x=0.196… awrt 0.20	A1
		(3)
		12 Marks



	Alternative to part (c)⇒	
	$f'(x) = 0 \Rightarrow 2\cos 3x - 3\sin 3x = 0$	M1
	$\tan 3x = \frac{2}{3}$	M1
	x=0.196… awrt 0.20	A1
		(3)

Q71.

Question Number	Scheme	Marks
(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$	
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
	$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$ Convincing proof. Must see both $\underline{\underline{\text{underlined steps}}}$.	A1 AG
		(3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}.$	
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$ $K\sec 2y \tan 2y$	M1
	$2\sec 2y \tan 2y$	A1
		(2)
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$ Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x\tan 2y}$ Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$ Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$	
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$ $\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
		(4)
		[9]