Enrichment

FUNCTIONS

1. Given that $f : x \to 4x + m$ and $f^{-1} : x \to nx + \frac{3}{4}$, find the values of m and n.

   Answer:- $m = -3$; $n = \frac{1}{4}$

2. Given that $f : x \to 2x - 1$, $g : x \to 4x$ and $fg : x \to ax + b$, find the values of a and b .

   Answer:- $a = 8$; $b = -1$

3. Given that $f : x \to x + 3$, $g : x \to a + bx^2$ and $gf : x \to 6x^2 + 36x + 56$, find the values of a and b .

   Answer:- $a = 2$; $b = 6$

4. Given that $g : x \to m + 3x$ and $g^{-1} : x \to 2kx - \frac{4}{3}$, find the values of m and k.

   Answer:- $k = \frac{1}{6}$; $m = 4$

5. Given the inverse function $f^{-1}(x) = \frac{2x - 3}{2}$, find
   (a) the value of $f(4)$,
   (b) the value of k if $f^{-1}(2k) = -k - 3$.

   Answer:-(a) $\frac{11}{2}$ (b) $-\frac{1}{2}$

6. Given the function $f : x \to 2x - 1$ and $g : x \to \frac{x}{3} - 2$, find
   (a) $f^{-1}(x)$,
   (b) $f^{-1}g(x)$,
   (c) $h(x)$ such that $hg(x) = 6x - 3$.

   Answer:-(a) $\frac{x + 1}{2}$ *(b)* $\frac{1}{6}x - \frac{1}{2}$ *(c)* $18x + 33$

7. Diagram 1 shows the function $g : x \to \frac{p + 3x}{x - 2}$, $x \neq 2$, where p is a constant.

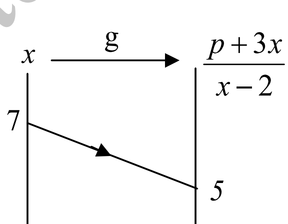

Diagram 1

   Find the value of p.

Answer:- p = 4

8.

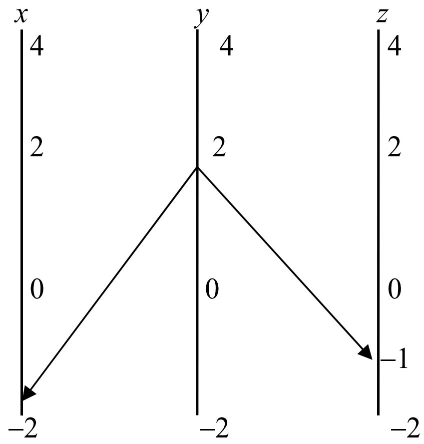

Diagram 2

Diagram 2 shows the mapping of y to x by the function $g : y \to ay + b$ and mapping to z by the function $h : y \to \dfrac{6}{2y-b}, y \neq \dfrac{b}{2}$. Find the,

(a) value of a and value of b,
(b) the function which maps x to y,
(c) the function which maps x to z.

Answer:- (a)a= –6, b=10 (b) $\dfrac{10-y}{6}$ *(c)* $\dfrac{18}{-y-20}$

9. In the Diagram 3, function h mapped x to y and function g mapped y to z.

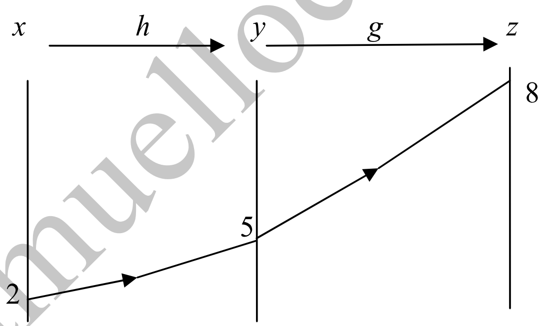

Diagram 3

Determine the values of,
(a) $h^{-1}(5)$,
(b) $gh(2)$

Answer:- (a)2 (b)8

10. Given function $f : x \to 2 - x$ and function $g : x \to kx^2 + n$. If composite function gf is given as $gf : \to 3x^2 - 12x + 8$, find
(a) the value of k and value of n,
(b) the value of $g^2(0)$.

Answer:-(a) k = 3 ,n = –4 (b)44

11. The following information refers to the functions f and g.

$g(x) = 4 - 3x$
$fg(x) = 2x + 5$

Find $f(x)$. *Answer:-* $\frac{23-2x}{3}$

12. (a) Function f, g and h are given as

$f : x \to 2x$

$g : x \to \frac{3}{x-2}, x \neq 2$

$h : x \to 6x^2 - 2.$

(i) Determine the function fh(x). At the same axis, sketch the graphs of $y = g$(x) and $y = fh$(x). Hence, determine the number of solutions for g(x) = fh(x).

(ii) Find the value of $g^{-1}(-2)$.

(b) Function m is defined as $m : x \to 5 - 3x$. If p is another function and mp is defined as $mp : x \to -1 - 3x^2$, determine function p.

Answer:-(a)(i) $12x^2 - 4$ *(b)* $p(x) = 2 + x^2$

13. Given function $f : x \to 4 - 3x$.

(a) Find (i) $f^2(x)$,

(ii) $(f^2)^{-1}(x)$.

(b) Hence, or otherwise, find $(f^{-1})^2(x)$ and show $(f^2)^{-1}(x) = (f^{-1})^2(x)$.

(c) Sketch the graph of $|f^2(x)|$ for the domain $0 \leq x \leq 2$ and find it's corresponding range.

Answer:-(a) $9x - 8$ *(b)* $\frac{x+8}{9}$

14. A function f is defined as $f : x \to \frac{p+x}{3+2x}$, for all values of x except $x = h$ and p are constants.

(a) Determine the value h.

(b) Given value 2 is mapped to itself by the function f. Find the

(i) value p,

(ii) another value of x which is mapped to itself,

(iii) value of $f^{-1}(-1)$.

Answer:-(a) $h = -\frac{3}{2}$ *(b)(i)* $p = 12$ *(ii)* $x = -3$ *(iii)* -5

<u>QUADRATIC EQUATIONS</u>

1. One of the roots of the quadratic equation $x^2 + 8 = (p+1)x$ is twice the other root. Find the possible values of *p*.

   Answer ; p = 5, −7

2. If one of the roots of the quadratic equation $ax^2 + bx + c = 0$ is two times the other root, find an expression that relates a, b and c.

   Answer : $2b^2 = 9ac$

3. Find the possible value of *m* , if the quadratic equation $2(x^2 + 3) = 6mx - 2$ has two equal roots.

   Answer ; $m = \pm\frac{4}{3}$

4. Straight line $y = mx + 1$ is tangent to the curve $x^2 + y^2 - 2x + 4y = 0$. Find the possible values of *m*.

   Answer : $-\frac{1}{2}$ *or 2*

5. Given $\frac{\alpha}{2}$ and $\frac{\beta}{2}$ are roots of the equation $kx(x-1) = 2m - x$.
   If $\alpha + \beta = 6$ and $\alpha\beta = 3$, find the value of *k* and of *m*.

   Answer : $k = -\frac{1}{2}$, $m = \frac{3}{16}$

6. Find the values of λ such that the equation $(3 - \lambda)x^2 - 2(\lambda + 1)x + \lambda + 1 = 0$ has equal roots. Hence, find the roots of the equation base on the values of λ obtained.

   Answer : $\lambda = \pm 1$; *roots:* $\lambda = 1, x = 1$; $\lambda = -1, x = 0$

QUADRATIC FUNCTIONS

1. Diagram 1 shows the graph of the function $y = -2(x-p)^2 + 5$, where p is constant.

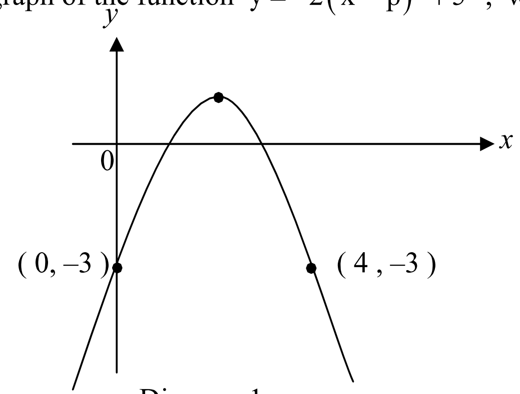

Diagram 1

Find,

(a) the value of *p* ,

(b) the equation of the axis of symmetry,

(c) the coordinate of the maximum point.

Answer:- (a) p = 2 (b) x = 2 (c) (2, 5)

2.

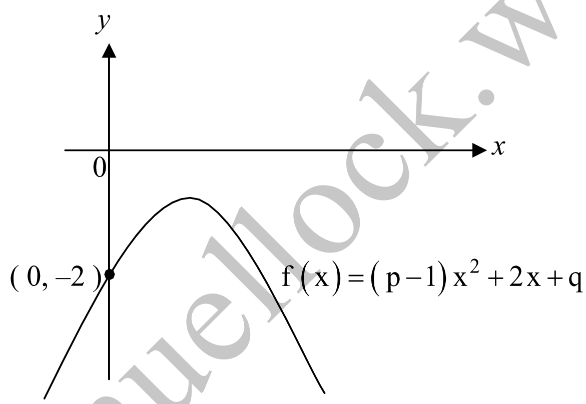

Diagram 2

Diagram 2 shows the graph of the function $f(x) = (p-1)x^2 + 2x + q$.

(a) State the value of *q* .

(b) Find the range of values of *p* .

Answer:-(a) q = – 2 (b) $p < \frac{1}{2}$

3.

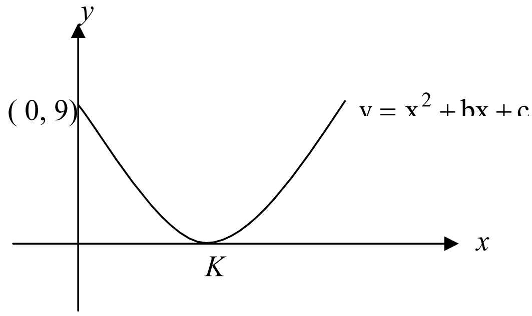

Diagram 3

Diagram 3 shows the graph of the function $y = x^2 + bx + c$ that intersects the *y*- axis at point (0, 9) and touches the *x*- axis at point *K*.
Find,
(a) the value of *b* and *c* ,
(b) the coordinates of point *K*.

Answer:-(a) $b = -6$, $c = 9$ *(b)* $(3,0)$

4.

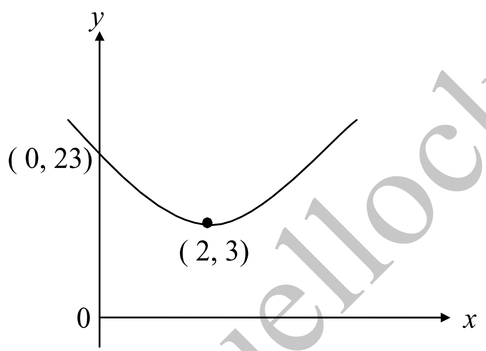

Diagram 4

In Diagram 4 above point (2, 3) is the turning point on the graph which has equation of the form $y = p(x + h)^2 + k$.
Find the,
(a) values of *p*, *h* and *k*,
(b) equation of the curve formed when the graph as shown is reflected at the x–axis.
(c) equation of the curve formed when the graph as shown is reflected at the y–axis.

Answer :- (a) $p = 5$, $h = -2$, $k = 3$ (b) $y = -5(x-2)^2 - 3$ (c) $y = 5(x+2)^2 + 3$

5. Function $f(x) = x^2 - 8kx + 20k^2 + 1$ has a minimum value of $r^2 + 4k$, where *r* and *k* are constants.

   (a) By using the method of completing the square, show that $r = 2k - 1$.
   (b) Hence or otherwise, find the values of *k* and *r* if the graph of the function is symmetrical about $x = r^2 - 13$.

Answer:-(b) k = 3 , –1 and r = –3 , 5

6. The function $f(x) = (6 + x)(2 - x) + h$ has a maximum value of 10 and *h* is a constant.

   (a) Find the value of h.
   (b) Sketch the graph of $f(x) = (6 + x)(2 - x) + h$ for the value of *h* that is determined in (a) above.
   (c) Write the equation of the axis of symmetry.

Answer:- (a) h = --6 (c) x = –2

7. Given $y = x^2 + 2kx + 3k$ has minimum value 2.

   (a) Without using the method of differentiation, find the two possible values of *k*.
   (b) With these values of *k*, sketch on the same axis, two graphs for $y = x^2 + 2kx + 3k$.
   (c) State the coordinates of the minimum point for $y = x^2 + 2kx + 3k$.

Answer:- (a) k =1 , 2 (c) (–1, 2), (–2, 2)

****

SIMULTANEOUS EQUATIONS

1. Solve the simultaneous equations 3x + 2y = 1 and $3x^2 - y^2 = 5x + 3y$.

Answer: x = -7/3 , y = 4 ; x = 1, y = -1

2. Solve the simultaneous equations

$$x + y = \frac{xy - 3}{2} \quad \text{and} \quad \frac{2x - 5}{3} = \frac{y}{5}$$

Answer: x = 41/10, y = 16/3 ; x = 1, y = -5

3. Solve the simultaneous equations $2x - y = 4$ and $2x^2 + xy - 3x = 7$. Give your answers correct to three decimal places.

Answer: x = 2.461, y = 0.922; x = -0.711, y = -5.422

INDICES AND LOGARITHMS

1. Solve the equation $\sqrt{625^{x+2}} = \dfrac{1}{5^x.25^{x-1}}$ *Answer:-* $x = -\dfrac{2}{5}$

2. Solve the equation $2^x.8^x = 4^{5x-3}$

Answer:- $x = 1$

3. Show that $3^{x+2} + 3^{x-1} - 5(3^x)$ is divisible by 13.

Answer : $13(3^{x-1})$

.4. Solve the equation $64^{2x-3} + 2 = 34$

Answer:- $\dfrac{23}{12}$

5. Solve the equation $3^x 2^{2x+1} = 10$

Answer: 0.6477

6. Solve the equation $\log_4[\log_2(2x-3)] = \log_9 3$

Answer:3.5

7. Solve the equation $\log(x+2) - \log(4x-1) = \log\dfrac{1}{x}$

Answer :x =1

8. Solve the equation $\log_2 x - 4\log_x 16 = 0$

Answer: $16, \dfrac{1}{16}$

9. Solve the equation $2(7^{x-1}) = 5^x$

Answer: $x = 3.7232$

10. Solve $3^{\log_2 x} = 81$

Answer : $x = 16$

11. Solve the equation $3^{2x+1} - 2(3^{x+0.5}) - 3 = 0$

Answer: 0.5

12. Solve the equation $10^{2x+1} - 7(10^x) = 26$

*Answer:*0.3010

13. Given that $\log_3 5 = m$ and $\log_9 2 = n$, express $\log_3 50$ in terms of m and n

Answer:- $2m + 2n$

14. Given that $\log_x 2 = k$ and $\log_x 7 = h$, express $\log_{\sqrt{x}} 3.5x$ in terms of *k* and *h*

Answer:- $2h + 2 - 2k$

15. Given that $2\log_2(x+y) = 3 + \log_2 x + \log_2 y$, show that $(x^2 + y^2 = 6xy)$

16. If $\log_2 a + \log_2 b = 4$, show that $\log_4 ab = 2$ and that $\log_8 ab = 4/3$. If $\log_2 a + \log_2 b = 4$, show that $\log_4 ab = 2$ and that $\log_8 ab = 4/3$.

COORDINATE GEOMETRY

1. The following information refers to the equations of two straight lines, AB and CD which are parallel to each other.

$AB : 2y = px + q$
$CD : 3y = (q + 1)x + 2$
Where p and q are constants

Express p in terms of q. *Answer:* $p = \frac{2}{3}(q+1)$

2. The triangle with vertices $A(4,3)$, $B(-1,1)$ and $C(t, -3)$ has an area 11 unit^2.
Find the possible values of t.
Answer: $t = 0, -22$

3. The points $P(3, p)$, $B(-1, 2)$ and $C(9,7)$ lie on a straight line. If P divides BC internally in the ratio $m : n$, find
(a) $m : n$,
(b) the value of p. *Answer:*(a) 2 : 3 (b) $p = 4$

4. (a) A point P moves such that its distance from point A (1,– 4) is always 5 units.
Find the equation of the locus of P. *Answer:* $x^2 + y^2 - 2x + 8y - 8 = 0$

(b) The point A is (-1, 3) and the point B is (4, 6). The point Q moves such that $QA : QB = 2 : 3$. Find the equation of the locus of Q.
Answer: $5x^2 + 5y^2 + 14x + 102y - 54 = 0$

(c) A point R moves along the arc of a circle with centre $A(2, 3)$. The arc passes through $Q(-2, 0)$. Find the equation of the locus of R.
Answer: $x^2 + y^2 - 4x - 6y + 8 = 0$

(d) A point S moves such that its distance from point $A(-3,4)$ is always twice its distance from point $B(6,-2)$. Find the equation of the locus of S.
Answer: $x^2 + y^2 - 18x + 8y + 45 = 0$

(e) The point M is (2, –3) and N is (4, 5). The point T moves such that it is always equidistance from M and from N. Find the equation of locus of T.
Answer : e) $x + 4y = 7$

(f) Given point A (1,2) and point B (4, –5). Find the locus of point W which moves such that $\angle AWB$ is always 90^0.

Answer: $x^2 + y^2 - 5x + 3y - 6 = 0$

Solutions to question no 5, 6 and 7 by scale drawing will not be accepted.

5. In Diagram 1, the straight line *PR* cuts *y*-axis at *Q* such that *PQ : QR* = 1 : 3. The equation of *PS* is $2y = x + 3$.

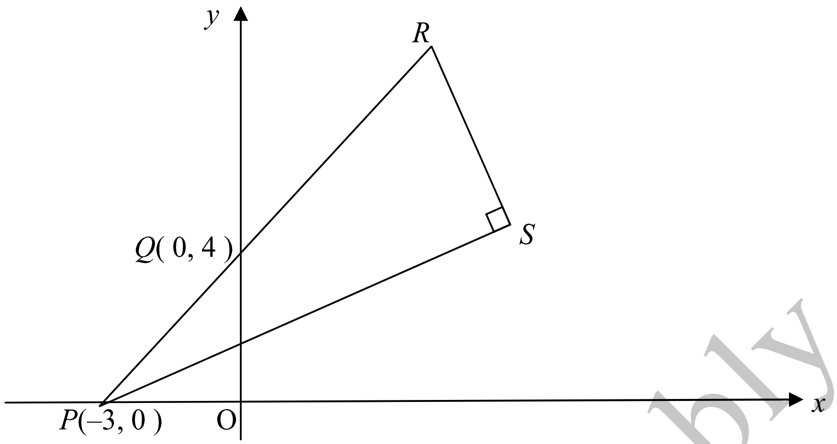

Diagram 1

(a) Find (i) the coordinates of *R*,
(ii) the equation of the straight line *RS*,
(iii) the area ΔPRS.

(b) A point *T* moves such that its locus is a circle which passes through the points *P*, *R* and *S*. Find the equation of the locus of *T*.

Answer: a)(i) $R = (9, 16)$ (ii) $y = -2x + 34$ (iii) 80 unit^2 b) $x^2 + y^2 - 6x - 16y - 27 = 0$

6. Diagram 2 shows the straight line graphs *PQS* and *QRT* in a Cartesian plane.
Point *P* and point *S* lies on the x-axis and y-axis respectively. *Q* is the mid point of *PS*.

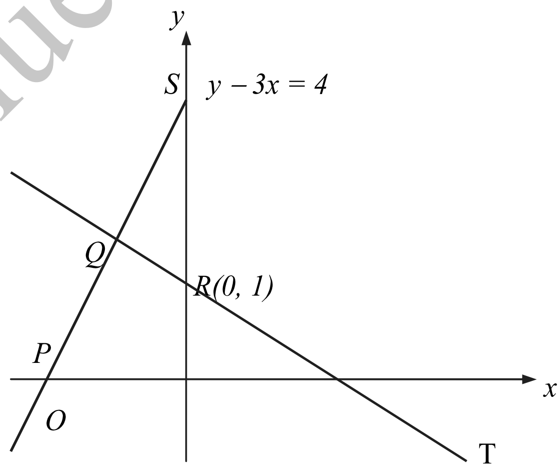

Diagram 2

(a) Find,
   (i) coordinates of the point Q,
   (ii) area of the quadrilateral $OPQR$.
   (iii) The equation of the straight line which is parallel to QT and passes through S.

(b) Given $3QR = RT$, calculate the coordinates of the point T.

(c) A point moves in such a way that it's distance from S is $\frac{1}{2}$ it's distance from the point T.
   (i) Find the equation of locus of the point T.
   (ii) Hence, determine whether the locus cuts the x-axis or not.

.

Answer: (a)(i) $(-\frac{2}{3}, 2)$ *(ii)* $\frac{5}{3}$ *(iii)* $y = -\frac{3}{2}x + 4$ *(b)* $(2, -2)$ *(c)(i)* $3x^2 + 3y^2 + 4x - 36y + 56 = 0$ *(ii) No*

7.

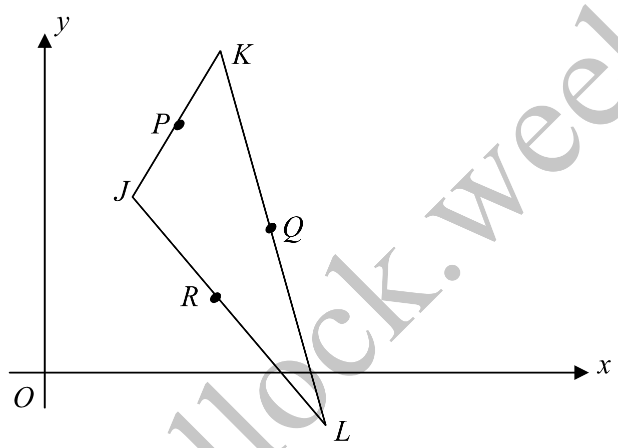

Diagram 3

In Diagram 3, $P(2, 9)$, $Q(5, 7)$ and $R(4\frac{1}{2}, 3)$ are the mid point of the straight line JK, KL and LJ such that $JPQR$ form a parallelogram.

(a) Find,
   (i) the equation of the straight line JK,
   (ii) the equation of the perpendicular bisector of the straight line LJ.

(b) Straight line KJ is extended until it intersects the perpendicular bisector of the straight line LJ at the point S. Find the coordinates of the point S.

(c) Calculate the area of ΔPQR and consequently the area of ΔJKL.

Answer: (a)(i) $y = 8x - 7$ *(ii)* $4y = 6x - 15$ *(b)* $\left(\frac{1}{2}, -3\right)$ *(c)* $6\frac{1}{2}$; 26

STATISTICS

1. Table 1 shows the results obtained by 100 pupils in a test.

Marks	< 20	< 30	< 40	< 50	< 60	< 70	< 80	< 90
Number of pupils	3	8	20	41	65	85	96	100

Table 1

(a) Based on Table 1, complete the table below.

Marks	10 – 19							
Frequency								

(b) Without drawing an ogive, estimate the interquartile range.

Answer:-(b)Interquartile range = 22.62

2. The mean and standard deviation of a set of integers 2 , 4 , 8 , p and q are 5 and 2 respectively.

(a) Find the values of p and of q .
(b) State the mean and variance of the set integers 7, 11, 9 , $2p + 3$ and $2q + 3$

Answer:-(a) $p = 5, q = 6$ or $p = 6, q = 5$ *(b) Mean =13 Variance = 16*

3. The histogram in Diagram 1 shows the marks obtained by 40 students in Mathematics test.

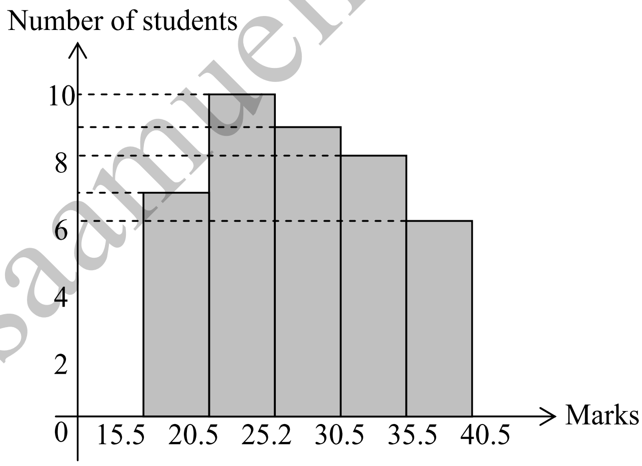

Diagram 1

(a) Without drawing an ogive , calculate the median mark.
(b) Calculate the standard deviation of the marks.

Answer:(a) 27.17 (b) 6.595

4. Table 2 shows the frequency distribution of the Chemistry marks of a group of students.

Marks	Number of students
1 – 10	2 3
11 – 20	5 10
21 – 30	p 2
31 – 40	
41 – 50	
51 – 60	

Table 2

(a) If the median mark is 34.5 , calculate the value of p .

(b) By using a scale of 2 cm to 10 marks on the horizontal axis and 2 cm to 2 students on the vertical axis, draw a histogram to represent the frequency distribution of the marks. Find the modal mark.

(c) What is the modal mark if the mark of each student is increased by 8 ?

Answer:- (a) p = 6 (b) Mode = 36 (c) 44

5. The scores, x , obtained by 32 students of Class 5 Alfa in a test are summarized as $\sum x = 2496$ and $\sum x^2 = 195488$. The mean and the standard deviation of the scores, y , obtained by 40 students and Class 5 Beta in the test are 66 and 6 respectively.

(a) Find (i) $\sum y$ (ii) $\sum y^2$

(b) Calculate the mean and the standard deviation of the scores obtained by all the 72 students.

Answer:-(a)(i) 2640 (ii)175680 (b)Mean = 71.33 , S Deviation= 8.194

6. A set of data consists of 10 numbers. The sum of the numbers is 120 and the sum of the squares of the numbers is 1650.

(a) Find the mean and variance of the set of data,

(b) A number a is added to the set of data and the mean is increased by 2, find
(i) the value of a,
(ii) the standard deviation of the new set of data.

Answer:-(a) Mean = 12 , Variance = 21 (b)(i) a = 34 (ii) S Deviation = 7.687

CIRCULAR MEASURES

1.

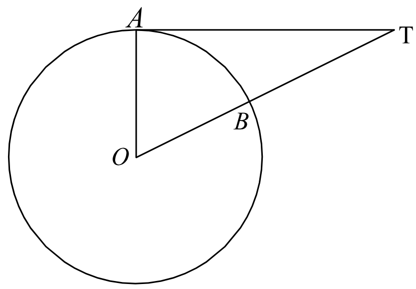

Diagram 1

Diagram 1 shows a circle with centre *O* and *OA* = 10 cm. Straight line *AT* is a tangent to the circle at point *A*, and *AOT* is a triangle. Given that the area of triangle *OAT* = 60 cm^2, find the area of sector *OAB*.

Answer:- 43.80 cm^2

2.

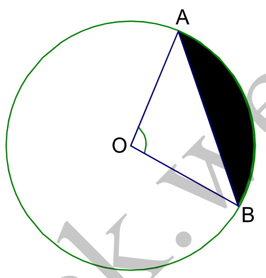

Diagram 2

Given that the area of a sector *OAB* in Diagram 2 with centre *O* and radius 20 cm is 240 cm^2. Calculate

(a) the length of arc *AB*

(b) area of shaded region

Answer:- : (a) 24 cm (b) 53.60

3.

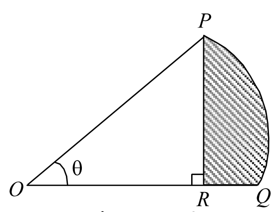

Diagram 3

In the Diagram 3, *POQ* is a circular sector with centre *O* and a radius of 17 cm. Point *R* is on the straight line *ORQ* such that *RQ* = 5 cm. Calculate

(a) the value of θ in radian

(b) the area of the shaded region, in cm^2

Answer:- (a) 0.7871 (b) 41.49

4.

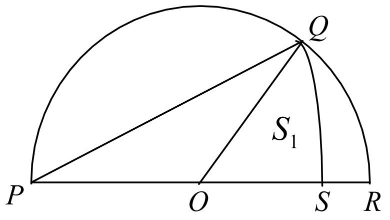

Diagram 4

Diagram 4 shows a semicircle with centre O and radius of 10 cm. Given that QS is the length of arc with centre P and $\angle QPS = \frac{\pi}{6}$ rad. Find

(a) the length of OS.

(b) the area of S_1 *Answer:-(a) 7.32 cm (b) 35.23 cm^2*

5.

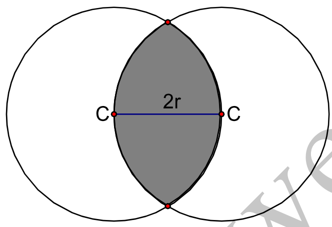

Diagram 5

Two identical circles of radius $2r$ are drawn with their centres, C on the circumference of each circle as shown in the Diagram 5. Show that the area of shaded region $A\ cm^2$, is given by $\frac{2}{3}r^2\left(4\pi - 3\sqrt{3}\right)$.

6.

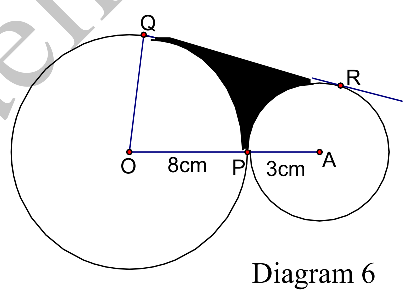

Diagram 6

Diagram 6 shows two circles with centres O and A. The respective radii are 8 cm and 3 cm. A tangent touches the circles at the points Q and R. Given that $\angle QOP = \frac{\pi}{3}$ radians, find

(a) the length of QR

(b) the perimeter of the shaded region

(c) the area of the shaded region

Answer : a) 9.80 cm b) 24.46 cm c) 10.96 cm^2

DIFFERENTIATION

1.(a) (i) Given y = $3x^2 + 5$, find $\frac{dy}{dx}$ by using the first principle.

(ii) Differentiate y = $\frac{4}{x} - 3$ with the first principle.

(b) (i) Find $\frac{d}{dx}\left(\frac{1}{2x+1}\right)$.

(ii) Given f(x) = $4x(2x-1)^5$, find f'(x).

(iii) Differentiate $3x^2(2x-5)^4$ with respect to x.

(iv) Given f(x) = $(2x-3)^5$, find f''(x).

(v) Given f(x) = $\frac{1-2x^3}{x-1}$, find f '(x).

(c) (i) Given h(x) = $\frac{1}{(3x-5)^2}$, find the value h''(1).

(ii) Given f(x) = $\frac{(x^2-2)^5}{1-3x}$, find f '(0).

(d) (i) Find the limit of $\lim_{n \to 2}\left(\frac{n^2-4}{n-2}\right)$.

(ii) Given f (r) = $\frac{4+3r}{5-2r}$. Find the limit of f (r) when r $\to$ 1.

(e) Given y = x(3 – x), express $y\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 12$ in terms of x, in the simplest form.

Hence, find the value of x which satisfy the equation $y\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 12 = 0$.

Answer :

(a) (i) $6x$

(ii) $-\frac{4}{x^2}$

(b)(i) $-\frac{2}{(2x+1)^2}$

(ii) $[4(2x-1)^4)(12x-1)]$

iii) $6x(6x-5)(2x-5)^3$

iv) $40(2x-3)^3$

v) $\frac{-4x^3+6x^2-1}{(x-1)^2}$

c) (i) $\frac{27}{8}$

(ii) –96

d) i) 2

ii) $\frac{7}{3}$

e) 12 – 3x; x = 4

2. (a) Given the function of the graph $f(x) = hx^3 + \frac{k}{x^2}$, which has a gradient function of $f'(x) = 3x^2 - \frac{96}{x^3}$, where h and k are constant. Find,

(i) the value of h and the value of k,
(ii) the coordinate x of the turning point of the graph.

(b) The point P lies on the curve $y = (x-5)^2$. It is given that the gradient of the normal at P is $-\frac{1}{4}$. Find

(i) the coordinates of P.
(ii) the equation of the normal to the curve at point P.

(c) A curve with the gradient function $2x - \frac{2}{x^2}$ has a turning point at $(k, 8)$.

(i) Find the value of k.
(ii) Determine whether the turning point is a maximum or a minimum point.

Answer:
2(a) (i) h=1, k= 48 (ii) 2
(b) (i) (7, 4) (ii) 4y + x = 23
(c) i) k = 1, ii) Minimum

3. (a) Two variables, x and y, are related by the equation $y = 3x + \frac{2}{x}$.

Given that y increases at a constant rate of 4 units per second, find the rate of change of x when $x = 2$.

(b) On a certain day, the rate of increase of temperature, θ°, with respect to time, t s, is given by $\frac{d\theta}{dt} = \frac{1}{2}(12 - t)$.

(i) Find the value of t at the instant when θ is maximum.
(ii) Given θ = 4 when t = 6, find the maximum value of θ.

Answer: (a) 1.6 units^{-1} (b) i) 12 ii) 13

4. (a) Given $y = t - 2t^2$ and $x = 4t + 1$.

(i) Find $\frac{dy}{dx}$ in terms of x.

(ii) If x increases from 3 to 3.01, find the corresponding small increment in t.

(b) Given $y = 2x^3 - 5x^2 + 7$, find the value of $\frac{dy}{dx}$ at the point (2, 3).

Hence, find

(i) the small change in x which causes y to decrease from 3 to 2.98.

(ii) the rate of change of y when $x = 2$ if the rate of change of x is 0.6 unit per second.

(c) Given $y = \frac{16}{x^4}$, find the value of $\frac{dy}{dx}$ when x = 2. Hence, find the approximate value of $\frac{16}{(1.98)^4}$.

Answer: (a) (i) $\frac{2-x}{4}$ *(ii) 0.0025 (b) (i) –0.005 (ii) 2.4 unit* s^{-1} *(c) 1.04*

5. Diagram 1 shows a composite solid made up of a cone resting on a cylinder with radius x cm.

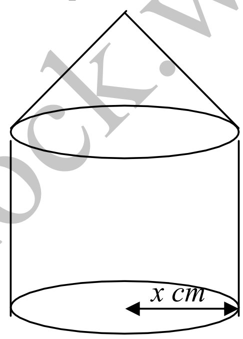

Diagram 1

The total surface area of the solid, A cm^2, is given by the equation $A = 3\pi\left(x^2 + \frac{16}{x}\right)$.

(a) Calculate the minimum value of the surface area of the solid.

(b) Given the surface area of the solid is changing at a rate of 42π cm^2 s^{-1}. Find the rate of change of radius at the instant when the radius is 4 cm.

(c) Given the radius of the cylinder increases from 4 cm to 4.003 cm. Find approximate increment in the surface area of the solid

Answer: b) 36π (c) 2 (d) 0.063π

PROGRESSION

1. Show that log h, log hk, $\log hk^2$, log hk^3,…… is an arithmetic progression. Then find the common difference of this progression.

2. An arithmetic progression has 10 terms. The sum of all these 10 terms is 220. The sum of the odd terms is 100. Find the first term and the common difference.

Answer : $a = 4, d = 4$

3. The sum of the first six terms of an arithmetic progression is 120. The sum of the first six terms is 90 more than the fourth term. Calculate the first term and the common difference.

Answer : $a = -30, d = 20$

4. Given that the sum of n term of an arithmetic progression is $S_n = 2n^2 + 3n$. Find
   (a) the n term in terms of n
   (b) the first term
   (c) the common difference

Answer : (a) $4n+1$ *(b) 5(c) 4*

5. An arithmetic progression has 12 terms. The sum of all these 12 terms is 222. The sum of the odd terms is 102. Find
   (a) the first term and the common difference
   (b) the last term

Answer :(a) a = 2 , d = 3 (b) 35

6. The n term of an arithmetic progression is $5n - 8$. Find the sum of all the terms from the 5^{th} term to the 8^{th} term.

Answer : 98

7. Estimate the sum to infinity of the geometric progression $9 + 3 + 1 + \frac{1}{3} + ...$

Answer : $13\frac{1}{2}$

8. Write $0.7 + 0.07 + 0.007 + 0.007 + ...$ as a fraction. *Answer :* $\frac{7}{9}$

9. The sum of the first n terms of a geometric progression is $S_n = \frac{(2^{2n+1}) - 8}{3}$. Find the least number of terms in the progression that its sum to exceed 60.

Answer : $n = 4$

10. Find the least number of terms of the geometric progression 4,12,36,……which must be taken for its sum to exceed 1 800.

Answer : $n = 7$

11. The sum of the first two terms of a geometric progression is $\frac{3}{4}$ and the sum of the next two terms is $\frac{3}{16}$, where the common ratio is positive. Find the sum to infinity of the progression.

Answer : $S_\infty = 1$

12.

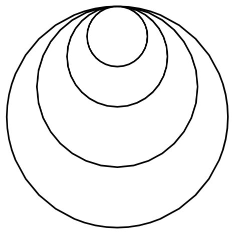

Diagram 1

Diagram 1 shows four circles. Each circle has a radius that is 2 units longer than that of the previous circle. Given that the sum of the perimeters of these four circles is 120π cm,

(i) find the radius of the smallest circle.

(ii) the sum of the perimeters from the fifth term to the tenth term.

Answer :(i) r = 12 cm(ii) 300π *cm*

13. Encik Rahim plans to donate an amount of money to the 'Rumah Penyayang' each year from 2008. The amount in 2008 will be RM50 000, and thereafter, the amount each year will be 90% of the amount for the previous year. Calculate

(a) the year in which the donation falls below RM 20 000 for the first time .

(b) the total donation from 2008 to 2015 inclusive

Answer : a) $n = 10$*, 2017* *b) RM284 766.40*

14.

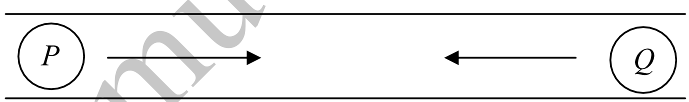

Diagram shows two balls in a tube of length 10 m, moving towards each other. *P* moves from one end traveling 60 cm in the first second, 59 cm in the next second and 58 cm in the third second. *Q* moves from the other end traveling 40 cm in the first second, 39 cm in the next second and 38 cm in the third second. The process continues in this manner until the two balls meet.

(a) Find the shortest time for the two balls to meet.(give your answer to the nearest second)

(b) Calculate the distance traveled by *P*.

(c) Calculate the difference in distance traveled by the two balls.

Answer : a) 11s b) 605 cm c) 220 cm

INTEGRATION

1. (a) Find, (i) $\int \frac{(4-x)(4+x)}{x^2}\,dx$ (ii) $\int \frac{18}{(3x-5)^3}\,dx$

(b) Given $\int_0^3 f(x)\,dx = 8$, find the value of $\int_0^3 \frac{f(x)+2}{2}\,dx$.

(c) Find $\int (2x+7)^3\,dx$

(d) Given $kx^2 - x$ is the gradient function for a curve such that k is a constant.
$y - 5x + 7 = 0$ is the equation of tangent at the point $(1, -2)$ to the curve. Find,
(i) the value of k,
(ii) the equation of the curve.

(e) Given $\frac{d^2y}{dx^2} = 4x^3 + 1$. When $x = -1$, $y = \frac{1}{2}$ and $\frac{dy}{dx} = 3$. Find y in terms of x.

Answer:(a) (i) $-\frac{16}{x} - x + c$ *(ii)* $-\frac{3}{(3x-5)^2} + c$ *(b) 7 (c)* $\frac{(2x+7)^4}{8} + c$

(d(i)) $k = 6$ *(ii)* $y = 2x^3 - \frac{x^2}{2} - \frac{7}{2}$ *(e)* $y = \frac{x^5}{5} + \frac{x^2}{2} + 3x + \frac{16}{5}$

2. Given that $\frac{dy}{dx}$ is directly proportional to $x^2 - 1$, and that $y = 3$ and $\frac{dy}{dx} = 9$, when $x = 2$, find the value of y when $x = 3$.

Answer: 19

3. The rate of change of the area, A cm^2, of a circle is $6t^2 - 2t + 1$. Find A in terms of t if the area of the circle is 11 cm^2 when $t = 2$.

Answer: $A = 2t^3 - t^2 + t - 3$

4. (a) Diagram 1 shows part of the curve of $y = \frac{2}{x^2}$. The straight line $x = k$ divides the shaded region enclosed by the curve $y = \frac{2}{x^2}$, the straight lines $x = 1$ and $x = 5$ and the x-axis into two regions, A and B.

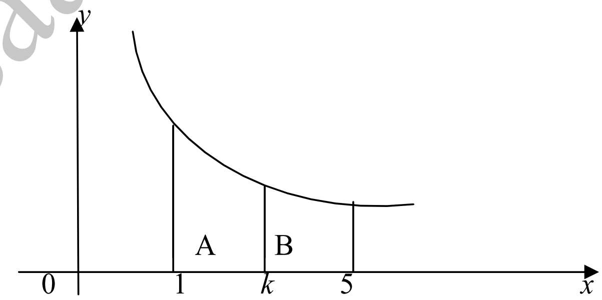

Diagram 1

Given that the area of region A is five times the area of region B, find the value of k.

(b) Diagram 2 shows part of the curve $y=\sqrt{x}(x-2)$.

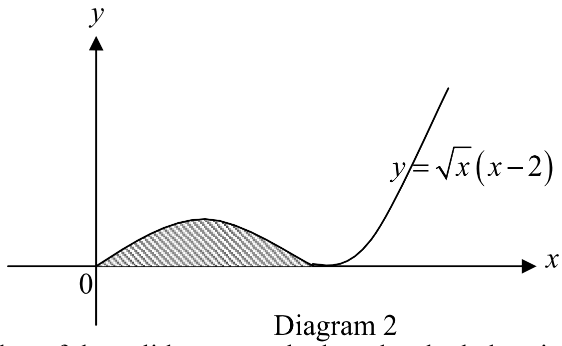

Diagram 2

Find the value of the solid generated when the shaded region is revolved through 360° about the x-axis .

Answer: (a) k = 3 (b) $1\frac{1}{3}\pi$

5. (a) Diagram 3 shows a straight line $y=2x$ and a curve $y=x^2-3x$

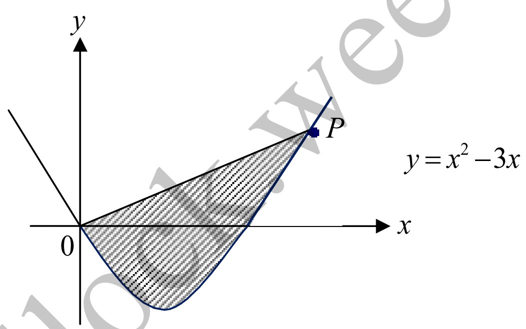

Diagram 3

Find
(i) the coordinate of the point P,
(ii) the area of the shaded region

Answer: (i) (5, 10) (ii) $\frac{125}{6}$

6. Diagram 4 shows part of the curve $y=\frac{4}{(2x-1)^2}$ which passes through $Q(1,4)$

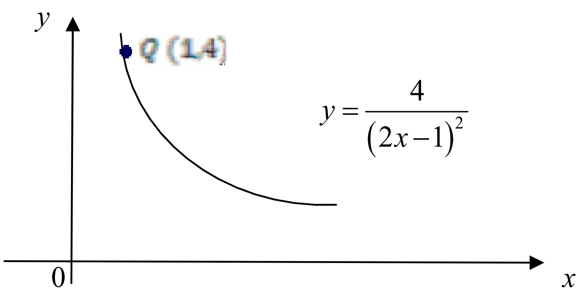

Diagram 4

(a) Find the equation of the tangent to the curve at point Q.

(b) A region is bounded by the curve, the x-axis and the straight lines $x = 2$ and $x = 3$.

(i) Find the area of the region

(ii) The region is revolved through 360° about the x-axis. Find the volume generated, in terms of π

Answer:(a) $y = -6x + 20$ (b) i) $\frac{4}{15}$,ii) $\frac{784}{10125}\pi$

7. (a) Evaluate $\int_0^4 x(4 - x)dx$

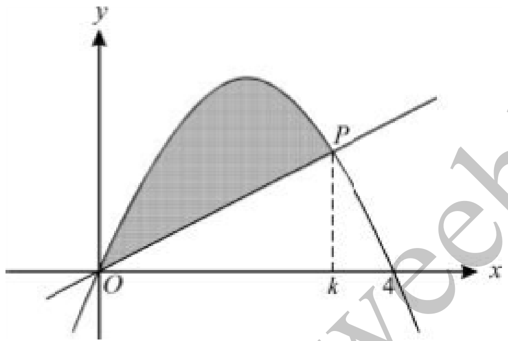

Diagram 5

(b) Diagram 5 shows the curve $y = x(4 - x)$, together with a straight line. This line cuts the curve at the origin O and at the point P with x-coordinate k, where $0 < k < 4$.

(i) Show that the area of the shaded region, bounded by the line and the curve, is $\frac{1}{6}k^3$

(ii) Find, correct to 3 decimal places, the value of k for which the area of the shaded region is half of the total area under the curve between $x = 0$ and $x = 4$.

Answer : k = 3.175

8. Diagram 6 shows, the straight line PQ is normal to the curve $y = \frac{1}{2}x^2 + 1$ at $A(2,3)$.

The straight line AR is parallel to the y – axis.

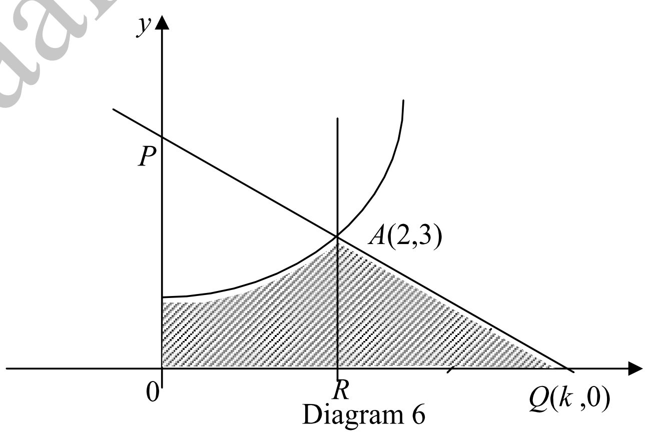

Diagram 6

Find
(a) the value of k,
(b) the area of the shaded region,
(c) the volume generated, in terms of π, when the region bounded by the curve, the y – axis and the straight line $y = 3$ is revolved through 360^o about the y-axis.

Answer : (a) k = 8 (b) area = $12\frac{1}{3}$ *(c) Volume =* 4π

9. Diagram 7 below shows the straight line $y = x + 4$ intersecting the curve $y = (x - 2)^2$ at the points A and B.

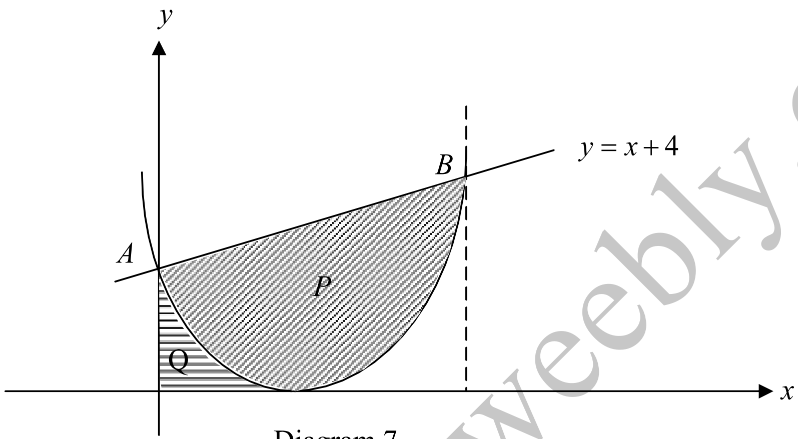

Diagram 7

Find,
(a) the value of k,
(b) the are of the shaded region P
(c) the volume generated, in terms of π, when the shaded region Q is revolved 360^o about the x – axis.

Answer : (a) k = 5 (b) area = 20.83 (c) volume = $\frac{32}{5}\pi$

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LINEAR LAW

1. The data for x and y given in the table below are related by a law of the form

$y = px^2 + x + q$, where p and q are constants.

x	1	2	3	4	5
y	41.5	38.0	31.5	22.0	9.5

(a) Plot $y - x$ againts x^2, using a scale of 2 cm to 4 unit on both axes. Hence , draw the line of best fit.
(b) Use your graph in 1(a) to find the value of
(i) p
(ii) q *Answer:* $p = -1.5$, $q = 42$

2. The variables x and y are known to be connected by the equation $y = Ca^{-x}$. An experiment gave pairs of values of x and y as shown in the table. One of the values of y is subject to an abnormally large area.

x	1	2	3	4	5	6
y	56.20	29.90	25.10	8.91	6.31	3.35

(a) Plot log y against x, using a scale of 2 cm to 1 unit for x-axis and 2 cm to 0.2 unit for y-axis . Hence , draw the line of best fit.
(b) Identify the abnormal reading and estimate its correct value.
(c) Use your graph in 2(b) to find the value of
(i) C
(ii) a

Answer: (b)25.1,17.78 (c) C = 100 , a = 1.78

3. The table shows experimental values of x and y which are known to be related by equation $y = \frac{a}{x} + b\sqrt{x}$.

x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.77	1.86	1.96

(a) Explain how a straight line graph may be drawn to represent the given equation.
(b) Plot xy againts $x\sqrt{x}$, using a scale of 2 cm to 2 unit on both axes . Hence , draw the line of best fit.
(c) Use your graph in 3(b) to find the value of
(i) a
(ii) b

Answer : a=1.5 ; b=0.70

4. Table 1 shows the values of two variables, x and y, obtained from an experiment. Variables x and y are related by the equation $y = p\,k^{x}$, where p and k are constants.

x	2	4	6	8	10	12
y	3.16	5.50	9.12	16.22	28.84	46.77

(a) Plot log y against x by using a scale of 2 cm to 2 units on the x-axis and 2 cm to 0.2 unit on the $\log_{10} y$-axis.
Hence, draw the line of best fit.
(b) Use your graph from (a) to find the value of
(i) p
(ii) k.

Answer : p=1.820 ; k=1.309

5. The variable x and y are related by the equation $y = \dfrac{h}{2x+k}$. Diagram 1 shows the graph of $\dfrac{1}{y}$ againts x .Calculate the values of h and k. The point P lies on the line. Find the value of r.

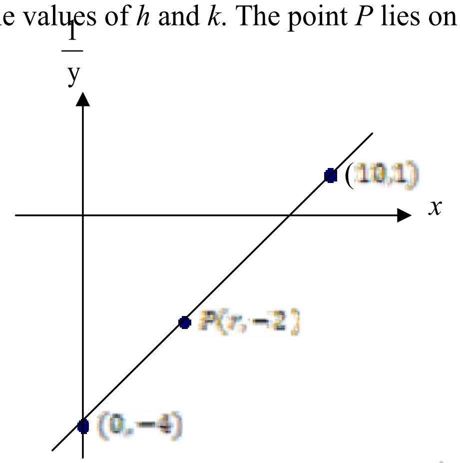


Diagram 1

Answer: $h = 4$, $k = -16$, $r = 4$

6. Variables x and y are related by the equation $\dfrac{a}{x} + \dfrac{b}{y} = 2$, where a and b are constants. When the graph of $\dfrac{1}{y}$ againts $\dfrac{1}{x}$ is drawn, a straight line is obtained. Given that the intercept on the $\dfrac{1}{y}-$axis is -0.5 and that the gradient of the line is 0.75, calculate the value of a and b.

Answer: $a = 3$, $b = -4$

7. Variables x and y are related by the equation $4y = 2(x - 1)^2 + 3k$ where k is a constant.

   (a) When y is plotted against $(x - 1)^2$, a straight line is obtained, which intersects the y-axis at (0, -6). Find the value of k.

   (b) Hence, find the gradient and the y intercept for the straight line obtained by plotting the graph of $(y + x)$ against x^2.

Answer: (a) $y = \frac{1}{2}(x-1)^2 + \dfrac{3k}{4}$, $k = -8$ (b) $y + x = \frac{1}{2}x^2 - \dfrac{11}{2}$, $m = \frac{1}{2}$, y-intercept $= -\dfrac{11}{2}$.

8. (a) Explain how a straight line graph can be drawn from the equation $\frac{y}{x} = \frac{p}{x} + qx$, where p and q are constants.

(b)

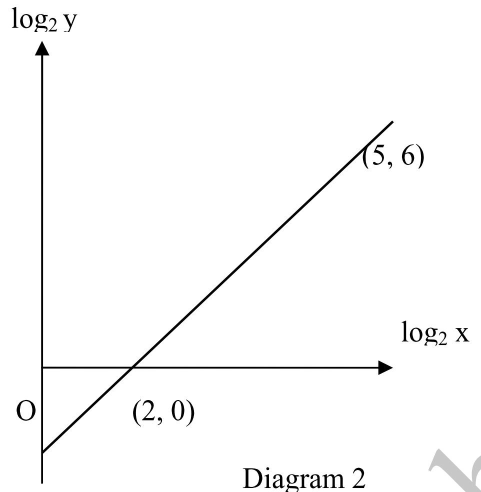


Diagram 2

Diagram 2 shows the graph of $\log_2 y$ against $\log_2 x$. Values of x and values of y are related by the equation $y = \frac{x^{2n}}{k}$, where n and k are constants. Find the value of n and the value of k.

Answer : n = 1, k = 16

9.

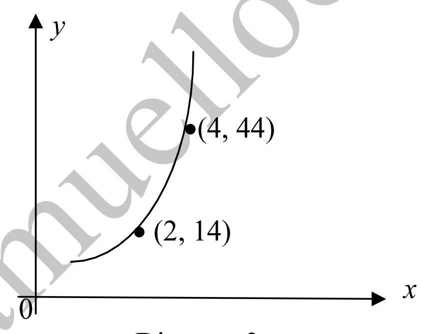


Diagram 3

Diagram 3 shows part of the curve y against x. It is known that x and y are related by the linear equation $\frac{y}{x} = kx + h$, where h and k are constants.

(a) Sketch the straight line graph for the above equation.

(b) Calculate the values of h and k.

Answer : : (a) h = 3, k = 2

10.

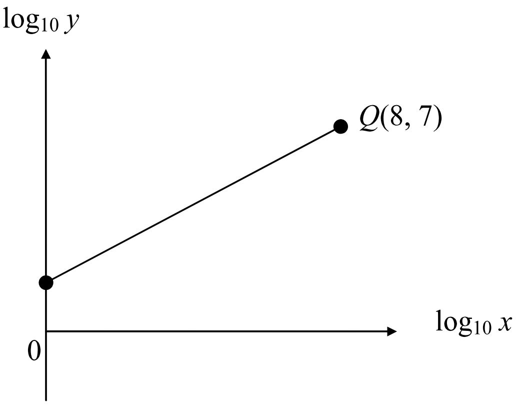


Diagram 4

Diagram 4 shows graph of $\log_{10} y$ against $\log_{10} x$. Given that PQ = 10 units and the point P lies on the $\log_{10} y$-axis.
(a) Find the coordinates of P.
(b) Express y in terms of x.
(c) Find the value of y when x = 16.

Answer: (a) $P(0, 1)$ (b) $y = 10x^{\frac{3}{4}}$ (c) $y = 80$

****

VECTORS

1. Diagram 1 shows a parallelogram, OPQR, drawn on a Cartesan plane.

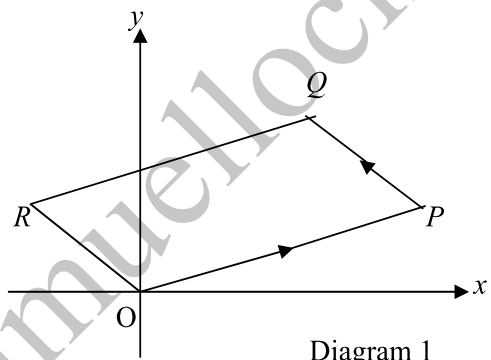


Diagram 1

Given that $\overrightarrow{OP} = 6\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\overrightarrow{PQ} = -4\underset{\sim}{i} + 5\underset{\sim}{j}$. Find $\overrightarrow{PR}$.

Answer: $-10\underline{i} + \underline{j}$

2. Given $O(0, 0)$, $A(-3, 4)$ and $B(2, 16)$, find in terms of unit vector $\underset{\sim}{i}$ and $\underset{\sim}{j}$,

(a) $\overrightarrow{AB}$,
(b) unit vector in the direction of $\overrightarrow{AB}$.

Answer: (a) $5\underline{i} + 12\underline{j}$ (b) $\frac{1}{13}\left(5\underline{i} + 12\underline{j}\right)$

3. Given $A(-2, 6)$, $B(4, 2)$ and $C(m, p)$, find the value of m and the value of p such that $\overrightarrow{AB} + 2\overrightarrow{BC} = 10\underset{\sim}{i} - 12\underset{\sim}{j}$.

Answer: m = 6, p = –4

4. Given the points $A(3,0)$, $B(7,8)$ and $C(1,k)$

(a) Express vector $\overrightarrow{AB}$ in terms of $\underline{i}$ and j,

(b) Find the value of k if vector $\overrightarrow{OC}$ is parallel to vector $\overrightarrow{AB}$.

Answer: (a) $4\underline{i} + 8\underline{j}$ (b) $h = \frac{1}{4}, k = 2$

5. Given that OABC is a rectangle where OA = 6 cm and OC = 5cm. If $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$, find

(a) AC in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

(b) $|\underline{a} + \underline{b}|$

Answer:(a) $-\underline{a} + \underline{b}$ (b) $\sqrt{61}$

6. Diagram 2 shows vector $\underset{\sim}{s}$, vector $\underset{\sim}{t}$ and vector unit $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

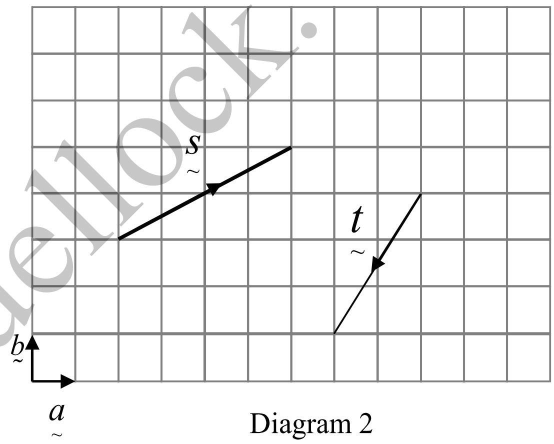


Diagram 2

Given $\underset{\sim}{r} = 2\underset{\sim}{s} - 3\underset{\sim}{t}$, express $\underset{\sim}{r}$ in terms $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

Answer: $14\underline{a} + 13\underline{b}$

7. Given $\overrightarrow{AB} = (k + 1)\underset{\sim}{a}$ and $\overrightarrow{BC} = 2\underset{\sim}{b}$. If A, B and C are collinear, $|\overrightarrow{AB}| = |\overrightarrow{BC}|$ and $\underset{\sim}{b} = 3\underset{\sim}{a}$.

Find the value of k.

Answer:k= 5

8. Given that $\underset{\sim}{a} = -2\underset{\sim}{i} + 2\underset{\sim}{j}$, $\underset{\sim}{b} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{c} = \underset{\sim}{a} - 2\underset{\sim}{b}$. Find

(*a*) $\left|\underset{\sim}{c}\right|$

(*b*) unit vector in the direction of $\underset{\sim}{c}$.

Answer: $(a)10 \quad (b)\frac{1}{5}\left(-3\underline{i} + 4\underline{j}\right)$

9.

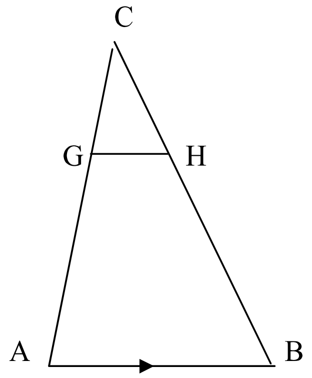


Diagram 3

Diagram 3 shows *GH : AB* = 3 : 10 and *GH* is parallel to $\overrightarrow{AB}$. If $\overrightarrow{AB} = 10\underset{\sim}{a}$, find $\overrightarrow{GH}$ in terms of $\underset{\sim}{a}$.

Answer: $3\underline{a}$

10.

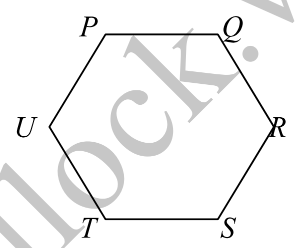


Diagram 4

Diagram 4 shows *PQRSTU* is a regular hexagon. Express $\overrightarrow{PQ} + \overrightarrow{PT} - \overrightarrow{RS}$ as a single vector.

Answer: $\overrightarrow{PR}$

11. In ΔOPQ, $\overrightarrow{OP} = \underset{\sim}{p}$ and $\overrightarrow{OQ} = \underset{\sim}{q}$. *T* is a point on *PQ* where *PT : TQ*=2 : 1. Given that *M* is the midpoint of *OT*, express $\overrightarrow{PM}$ in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$.

Answer: $-\frac{5}{6}\underline{p} + \frac{1}{3}\underline{q}$

12. Diagram 5 shows triangles *OAB*. The straight line *AP* intersects the straight line *OQ* at *R*.
It is given that $OP = \frac{1}{3}OB$, $AQ = \frac{1}{4}AB, \overrightarrow{OP} = 6\underset{\sim}{x}$ and $\overrightarrow{OA} = 2\underset{\sim}{y}$

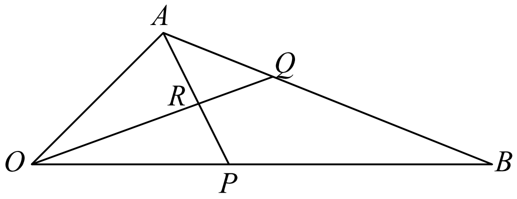


Diagram 5

(a) Express in terms of $\underset{\sim}{x}$ and/or $\underset{\sim}{y}$: (i) $\overrightarrow{AP}$, (ii) $\overrightarrow{OQ}$

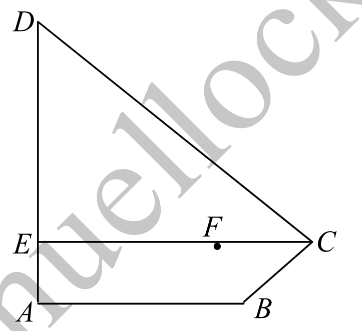
(b) (i) Given that $\overrightarrow{AR} = h\overrightarrow{AP}$, state $\overrightarrow{AR}$ in terms of h, $\underset{\sim}{x}$ and $\underset{\sim}{y}$.

(ii) Given that $\overrightarrow{RQ} = k\overrightarrow{OQ}$, state $\overrightarrow{RQ}$ in terms of k, $\underset{\sim}{x}$ and $\underset{\sim}{y}$.

(c) Using $\overrightarrow{AR}$ and $\overrightarrow{RQ}$ from (b), find the value of h and of k

Answer:(a)(i) $-2\underline{y} + 6\underline{x}$ (ii) $\frac{9}{2}\underline{x} + \frac{3}{2}\underline{y}$ (b)(i) $h\left(-2\underline{y} + 6\underline{x}\right)(ii)k\left(\frac{9}{2}\underline{x} + \frac{3}{2}\underline{y}\right)$(c) $k = \frac{1}{3}, h = \frac{1}{2}$

13. Diagram 6, *ABCD* is a quadrilateral. *AED* and *EFC* are straight lines.



It is given that $\overrightarrow{AB} = 20\underset{\sim}{x}$, $\overrightarrow{AE} = 8\underset{\sim}{y}, \overrightarrow{DC} = 25\underset{\sim}{x} - 24\underset{\sim}{y}, AE = \frac{1}{4}AD$ and $EF = \frac{3}{5}EC$

(a) Express in terms of $\underset{\sim}{x}$ and/or $\underset{\sim}{y}$: (i) $\overrightarrow{BD}$, (ii) $\overrightarrow{EC}$

(b) Show that the points *B, F* and *D* are collinear

(c) If $\left|\underset{\sim}{x}\right| = 2$ and $\left|\underset{\sim}{y}\right| = 3$, find $\left|\overrightarrow{BD}\right|$

Answer:(a)(i) $-20\underline{x} + 32\underline{y}$ (ii) $25\underline{x}$ (c) 104