TRIGONOMETRIC FUNCTIONS

1. Prove that $\text{cosec}^2 x - 2\sin^2 x - \cot^2 x = \cos 2x$

2. Prove that $\tan^2 A \sin^2 A \equiv \tan^2 A - \sin^2 A$

3. Prove that $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$.

4. Find all the angles between 0° and 360° which satisfy

   (a) $3\cos 2\alpha - 5 = 8\cos\alpha$
   (b) $\tan 2\alpha \tan\alpha = 1$

$Answer\left(\text{a}\right)131.81^{\circ}, 228.19^{\circ}$ (b)$30^{\circ}, 150^{\circ}, 210^{\circ}, 330^{\circ}$:

5. Solve the equation $4\sin\theta + 3\cos\theta = 0$ for $0^0 \le \theta \le 360^0$

$Answer\,36.87^{\circ}, 216.87^{\circ}$:

6. Find all the angles between 0° and 360° which satisfy
   (a) $3\sin 2\text{A} = 4\sin A$
   (b) $5\sin^2\text{A} = 5 - \cos 2\text{A}$

$Answer\left(\text{a}\right)0^{\circ}, 180^{\circ}, 360^{\circ}, 48.19^{\circ}, 311.81^{\circ}\left(\text{b}\right)35.27^{\circ}, 215.27^{\circ}, 144.73^{\circ}, 324.73^{\circ}$:

7. Given that $\sin\alpha = \dfrac{8}{17}$, $90^{\circ} < \alpha < 270^{\circ}$ and $\sin\beta = -\dfrac{12}{13}$, $90^{\circ} < \beta < 270^{\circ}$.
   Calculate the value of
   (a) $\sin(\alpha + \beta)$ (b) $\cos(\beta - \alpha)$

$Answer\left(\text{a}\right)\dfrac{140}{221} \quad \left(\text{b}\right)-\dfrac{21}{221}$:

8. Given that $\cos x = \dfrac{3}{5}$ and $0^{o} \le x \le 180^{o}$, find $\sec x + \text{cosec}\, x$.

$Answer\,\dfrac{35}{12}$:

9. Given that $\sin\Theta = k$ and Θ is acute angle, express in term of k:
   (a) $\tan\Theta$
   (b) $\text{cosec}\,\Theta$

$Answer\left(\text{a}\right)\dfrac{\text{k}}{\sqrt{1-\text{k}^2}} \quad \left(\text{b}\right)\dfrac{1}{\text{k}}$:

10. Solve the equation $5\sin^2 A + 2cos^2 A - 3 = 0, 0^o \le A \le 360^o$

Answer $35.27^{\circ}, 144.73^{\circ}, 215.27^{\circ}, 324.73^{\circ}$ *:*

11. Solve the equation $\frac{1}{\cot^2 x} + sek^2 x = 3$ for $0° \le x \le 360°$.

Answer $45^{\circ}, 135^{\circ}, 225^{\circ}, 315^{\circ}$ *:*

12. (a) Prove that $\frac{\sin 2x}{1 - \cos 2x} = \cot x$.

(b) Given $\cos\frac{\theta}{2} = \frac{1}{\sqrt{1+p^2}}$,

i. prove that $\tan\theta = \frac{2p}{1-p^2}$.

ii. hence, find $\sin 2\theta$, when $p = 2$.

Answer $(\text{b})(\text{i}) - \frac{24}{25}$ *:*

13. (a) Prove that $\tan\theta + \cot\theta = 2 \operatorname{cosec} 2\theta$.

(b)(i) Sketch the graph $y = 2\cos\frac{3}{2}x$ for $0^o \le x \le 2\pi$.

(ii) Find the equation of a suitable straight line for solving the equation $\cos\frac{3}{2}x = \frac{3}{4\pi}x - 1$. Hence, using the same axes, sketch the straight line and state the number of solutions to the equation $\cos\frac{3}{2}x = \frac{3}{4\pi}x - 1$ for $0^o \le x \le 2\pi$.

Answer:(b)(ii)3

14. (a) Sketch the graph of $y = \cos 2x$ for $0^o \le x \le 180^o$.

(b) Hence, by drawing a suitable straight line on the same axes, find the number of solutions satisfying the equation $2\sin^2 x = 2 - \frac{x}{180}$ for $0^o \le x \le 180^o$.

Answer:(b)2

15. (a) Prove that $\operatorname{cosec}^2 x - 2\sin^2 x - \cot^2 x = \cos 2x$.
    (b)(i) Sketch the graph of $y = \cos 2x$ for $0 \le x \le 2\pi$.
    (ii) Hence , using the same axes , draw a suitable straight line to find the number of solutions to the equation $3(\operatorname{cosec}^2 x - 2\sin^2 x - \cot^2 x\ ) = \frac{x}{\pi} - 1$ for $0 \le x \le 2\pi$. State the number of solutions .

Answer:(b)(ii)4

16. (a) Sketch the graph of $y = -2\cos x$ for $0 \le x \le 2\pi$.
    (b) Hence , using the same axis , sketch a suitable graph to find the number of solutions to the equation $\frac{\pi}{x} + 2\cos x = 0$ for $0 \le x \le 2\pi$.

Answer:2

17. (a) Given $\tan A = \frac{3}{4}$,and A is an acute angle. Find the value of $\cos 2A$.
    (b)(i) Sketch the curve $y = \sin 2x$ for $0 \le x \le 2\pi$.
    (ii) Hence, by drawing a suitable straight line on the same axes, find the number of solutions satisfying the equation $\sin x \text{ kos } x = \frac{x}{4\pi} - \frac{1}{2}$ for $0 \le x \le 2\pi$.

Answer: $(a)\frac{7}{25}(b)(i)4$

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PERMUTATION AND COMBINATION

1. Four girls and three boys are to be seated in a row. Calculate the number of possible arrangements
   (a) if all the three boys have to be seated together
   (b) a boy has to be seated at the centre

Answer:(a)720 (b)2880

2. Find the number of the arrangement of all nine letters of word SELECTION in which the two letters E are not next to each other

Answer:282240

3. Calculate the number of four digit even number can be formed from the digits 3, 4, 5, 6 and 9 without repetitions. *Answer:48*

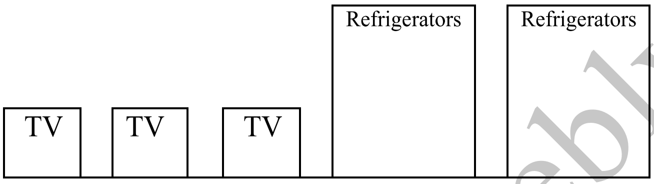
4. Three alphabets are chosen from the word WALID. Find the number of possible choice if (a) the alphabet *A* is chosen
   (b) the alphabet *A* and *D* are chosen

Answer:(a) 6 (b) 3

5. A bowling team consists of 8 person. The team will be chosen from a group of 7 boys and 6 girls. Find the number of team that can be formed such that each team consists of
   (a) 3 boys
   (b) not more than 1 girl

Answer:(a) 210 (b) 6

6.



Pak Adam's shop has 5 televisions *P, Q, R, S* and *T* and 4 refrigerators *W, X, Y* and *Z*
(a) If a televisions and a refrigerator is chosen randomly, calculate the probability that television *P* or *Q* and refrigerator *W* are chosen.
(b) Pak Adam wish to display his goods as shown in the diagram above. Calculate the number of ways the goods can be displayed.

Answer: (a) $\frac{1}{10}$ (*b*) 720

7.. Diagram 1 shows 5 letter and 3 digits.

A	*B*	*C*	*D*	*E*	6	7	8

Diagram 1

A code is to be formed using those letters and digits. The code must consist of 3 letters followed by 2 digits. How many codes can be formed if no letter or digit is repeated in each code ?

Answer:144

8. A debating team consists of 5 students. These 5 students are chosen from 4 monitors, 2 assistant monitors and 6 prefects. Calculate the number of different ways the team can be formed if (a) there is no restriction
   (b) the team contains only one monitor and exactly 3 prefects

Answer:(a)792 (b)160

PROBABILITY

1. At the place where Lam stays, rain falls in any two days of a week. Out of 75% of the raining days, Lam goes to school in his father's car. If there is no rain, Lam cycles to to school. For every 5 days Lam goes to school in his father's car, for 3 days Lam is able to keep his pocket money. In a certain day, find the probability that
   (a) Lam does not goes to school in his father's car,
   (b) Lam keeps his pocket money because he goes to school in his father's car.

   Answer:- (a) $\frac{11}{14}$ *(b)* $\frac{9}{70}$

2. Rashid and Rudi compete in a badminton game. The game will end when any of the players has won two sets. The probability that Rashid will win any one set is $\frac{3}{5}$.
   Calculate the probability that
   (a) the game will end in only two set,
   (b) Rashid will win the competition after playing 3 sets.

   Answer:- (a) $\frac{13}{25}$ (b) $\frac{36}{125}$

3. A container consists of 4 soya beans, 3 coffee beans and 2 cocoa beans.

   (a) If a bean is drawn at random from the container, calculate the probability that the bean is not a cocoa bean.
   (b) Two beans are drawn at random from the container, one after the other, without replacement. Find the probability that only one bean out of the two beans is a cocoa bean.

   .Answer:- (a) 7/9 , (b) 7/18

4. Bag *P* contains five card numbered 5, 6, 7, 8 and 9. Bag *Q* contains three cards numbered 5, 7 and 9. A card is drawn at random from bag *P* and at the same time, another card is drawn from bag *Q*. Find the probability that the two numbers drawn have the same value or their product is an even number.

   Answer: $\frac{3}{5}$

Box	Number of marbles		
	Green	Red	Yellow
P	6	7	2
Q	3	5	8

Table 1

5. Table 1 shows the number of marbles of different colours in boxes *P* and *Q*. A marble is picked at random from each box. Find the probability that
   (a) both are of the same colour,
   (b) both are of different colours,
   (b) a yellow marble is picked from box *Q*.

   Answer: (a) 23/80 (b) 57/80 (c) ½

6. Diagram 1 shows a board with a grid of 20 squares, of which a few squares are shaded. A dart is thrown at the board.

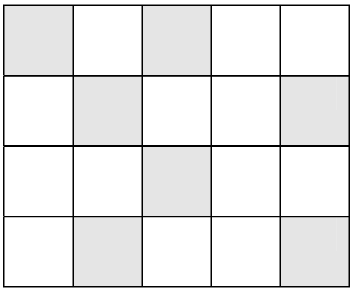


Diagram 1

(a) Find the probability that it will hit a shaded square.
(b) Find how many additional squares need to be shaded if the probability is increased to $\frac{3}{5}$.

Answer: (a) 7/20 , (b) 5

7. A box contains 4 blue balls, x white balls and y red balls. A ball is drawn at random from the box. If the probability of getting a white ball is $\frac{1}{6}$ and the probability of getting a red ball is $\frac{1}{2}$, find the values of x and y.

Answer: $x = 2, y = 6$

8. The letters of the word *G R O U P S* are arranged in a row. Find the probability that an arrangement chosen at random
(a) begins with the letter *P*,
(b) begins with the letter *P* and ends with a vowels.

Answer: (a) 1/6 , (b) 1/ 15

9. A bag contains 6 red balls and 5 green balls. A ball is chosen from the bag and returned. Another ball is chosen and returned again. Find the probability that

(a) both balls are red
(b) both balls have same color,
(c) both balls have different color.

Answer: (a) 36/121 , (b) 61/121 , (c) 60/121

10. There are 7 ribbons in a bag. 1 yellow, 3 black and 3 blue ribbons.

(a) If a ribbon is taken out and not returned back, find the probability for the ribbon to be black.
(b) If two ribbons are taken, find the probability first one to be blue followed by a black if none of the ribbons are returned.
(c) If three ribbons are taken, find the probability for first one to be blue, followed by yellow and a black ribbon if none of the ribbons are returned.

Answer: (a) 3/7 , (b) 3/14 , (c) 3/70

PROBABILITY OF DISTRIBUTION

1. (a) A study in a district shows that one out of three teenagers in the district join the `Rakan Muda' program.

   (i) If 5 teenagers are chosen randomly from the district, find the probability that 2 or more of them join the `Rakan Muda' program.
   (ii) If they are 2 490 teenagers in the district, calculate the mean and the standard deviation of the number of teenagers who join the `Rakan Muda' program.

   (b) From a study, it is found that the mass of a deer from a certain jungle shows a normal distribution with mean 55 kg and variance 25 kg^2.

   (i) If a deer is caught randomly from the jungle, find the probability that the deer has a mass more than 60 kg.
   (ii) Find the percentage number of dears with mass between 45 kg and 60 kg.

   Answer: (a)(i) $\frac{131}{243}$ or 0.5391 (ii) 830, 23.52 (b)(i) 0.1587 (ii) 81.85%

2. (a) Usually, when fishing, Wan will get fish as many as 60% from the total number of his throws..
   Calculate,
   (i) the probability Wan will get at least 4 fishes in 5 throws,
   (ii) the minimum number throws made by Wan so that the probability of getting at least a fish is greater than 0.87.

   (b) The mass of students in a school has a normal distribution with a mean of μ kg and a standard deviation σ kg. It is known that 10.56% of the above students have mass more than 50 kg and 15.87% of them have mass less than 32 kg. Find the value of μ and the value σ.

   Answer: (a)(i) 0.3370 or $\frac{1053}{3125}$ (ii) 3 (b) $\sigma = 8$, $\mu = 40$

3. (a) In a game of guessing, the probability of guessing correctly is p.
   (i) Find the number of trials required and the value of p, such that the mean and the standard deviation of success are 15 and $\frac{3}{2}\sqrt{5}$ respectively.
   (ii) If 10 trials are done, find the probability of guessing exactly 3 correct.

   (b) The volume of 600 bottles of mineral water produced by a factory follow a normal distribution with a mean of 490 ml per bottle and standard deviation of 20 ml.
   (i) Find the probability that a bottle of mineral water chosen in random has a volume of less than 515 ml.
   (ii) If 480 bottles out of 600 bottles of the mineral water have volume greater than *k* ml, find the value of *k*.

   Answer: (a)(i) $p = \frac{1}{4}$, n = 60 (ii) 0.2503 (b)(i) 0.8944 (ii) 473.16

4. (a) It is known that for every 10 lemon in the box, two are rotten. If a sample of 7 are chosen randomly, calculate the probability that
   (i) exactly 3 lemons are rotten,
   (ii).at least 6 lemons are not rotten.

   (b) The masses of the members of the English Society of School *M* are normally distributed with a mean of 48 kg and a variance of 25 kg^2. 56 of the members have masses between 45 kg and 52 kg. Find the total number of members in the English Society.

Answer: (a) (i) 0.1147 (ii) 0.5767 (b) 109

5. Diagram 1 shows a standard normal distribution graph.

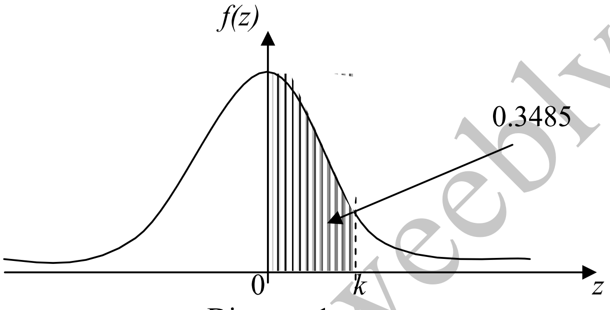


Diagram 1

The probability represented by the area of the shaded region is 0.3485.
(a) Find the value of *k*.
(b) *X* is a continuous random variable which is normally distributed with a mean of 79 and a standard deviation of 3. Find the value of *X* when the *z*-scores is *k*.

Answer: (a) 1.03 (b) 82.09

6. Diagram 4 shows a probability distribution graph of the continuous random variable *x* that is normally distributed with a standard deviation of 8. The graph is symmetrical about the vertical line *PQ*.

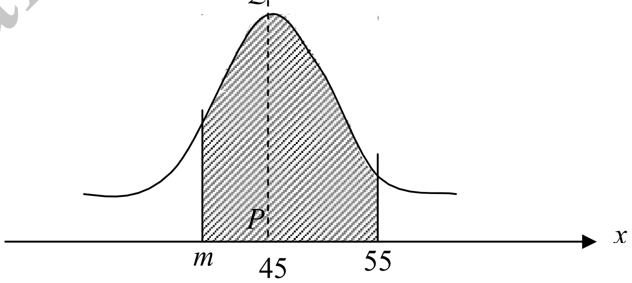


Diagram 4

(a) If the standard score found by using the value of $x = m$ is $\frac{-3}{4}$, find the value of *m*.

(b) Hence, find the area of the shaded region in Diagram 4.

(c) If x represents the marks obtained by 180 Form 5 students in an examination, calculate the number of students whose marks are less than 33.

Answer: (a) m = 39 , (b) 0.0668, (b) 12

7. (a) A football team organizes a practice session for trainees on scoring goals from penalty kicks. Each trainee has ten goals to score. The probability that a trainee scores a goal is *k*. After the practice, it is found that the mean number of goals scored for a trainee is 6.

(i) Find the value of *k*.
(ii) If a trainee is chosen at random, find the probability that he scores at least two goals.

(b) The masses of students of a school are normally distributed with a mean of 56 kg and a standard deviation of 10 kg.
(i) If a student is chosen at random, calculate the probability that his mass is less than 50 kg.
(ii) Given that 1.5% of the students have masses of more than *p* kg, find the value of *p*.
(iii) If 75% of the students have masses of more than *h* kg, find the value of *h*.

Answer(a)(i) k = 0.6 (ii) 0.9983 (b)(i) 0.2743 (ii)77.7 (iii) 49.25

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SOLUTION OF TRIANGLES

1.

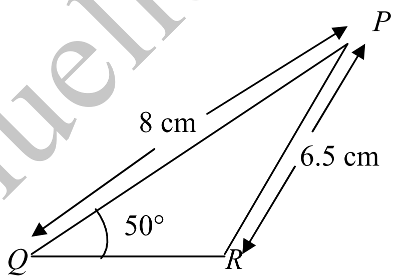

Diagram 1

Diagram 1 shows a ΔPQR.

(a) Calculate the obtuse angle *PRQ*.
(b) Sketch and label another triangle different from ΔPQR in the diagram above, so that the lengths of *PQ* and *PR* and the angle *PQR* remain unchanged.
(c) If the length of *PR* is reduced whereas the length of *PQ* and angle *PQR* remain unchanged, calculate the length of *PR* so that only one ΔPQR can be formed

Answer: (a) 109° 28' or 109.47° (c) 6.128 cm

2.(a) Diagram 2 shows a pyramid *VABCD* with a square base *ABCD*. *VA* is vertical and the base *ABCD* is horizontal. Calculate,

(i) $\angle VTU$,

(ii) the area of the plane *VTU*.

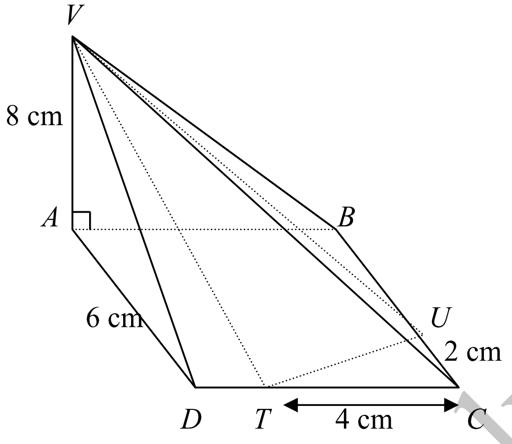

Diagram 2

Answer: (a)(i) 84° 58' or 84.97° (ii) 22.72 cm^2

(b) .

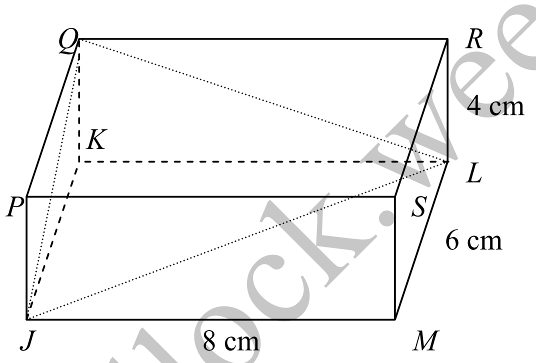

Diagram 3

Diagram 3 shows a cuboid. Calculate, (i) $\angle JQL$,

(ii) the area of ΔJQL.

Answer: (i) 75° 38' or 75.64^0 (ii) 31.24 cm^2

3.

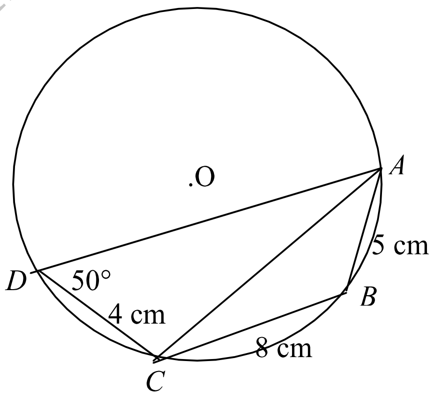

Diagram 4

Diagram 4, *ABCD* is a cyclic quadrilateral of a circle centered *O*. Calculate
(a) the length of *AC*, correct to two decimal places,
(b) $\angle ACD$,
(c) the area of quadrilateral *ABCD*.

Answer: (a) 11.85 cm (b) 115.01°′ (c)36.08cm^2

4. Diagram 5 shows two triangles *PQT* and *TRS*. Given that $PQ = 24$ cm, $TS = 12$ cm, $\angle TPQ = 32^0$, $PT = TQ$ and *PTS* and *TRQ* are straight lines.

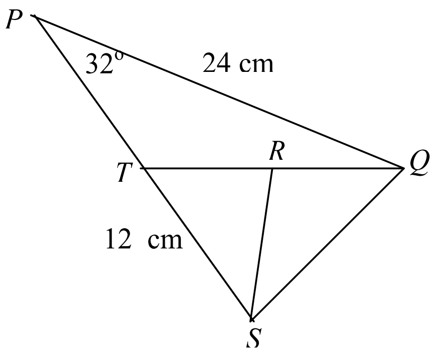

Diagram 5

(a) Find the length, in cm, of *PT*,
(b) If the area of triangle *PQT* is three times the area of triangle *TRS*, find the length of *TR*.
(c) Find the length of *RS*.
(d) (i) Calculate the angle *TSR*.
  (ii) Calculate the area of triangle *QRS*.

Answer: (a) 14.15 cm, (b) 5.563 cm, (c) 10.79 cm, (d)(i) 27.60° (ii) 46.31 cm^2

5. Diagram 6 shows a triangle *PQR*.

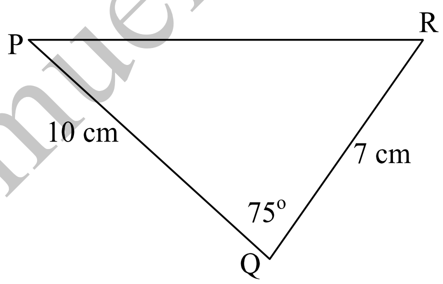

Diagram 6

(a) Calculate the length of PR.
(b) A quadrilateral PQRS is now formed so that PR is the diagonal, $\angle PRS = 40^\circ$ and PS = 8 cm. Calculate the two possible values of $\angle PSR$.
(c) Using the obtuse $\angle PSR$ in (b), calculate
  (i) the length of RS,
  (ii) the area of the quadrilateral PQRS.

Answer: (a) 10.62 cm (b) 58.57° ; 121.43° (c) (i) 3.964 cm ; (ii) 47.34 cm^2

6. Diagram 7 shows a camp of the shape of pyramid $VABC$. The camp is built on a horizontal triangular base ABC. V is the vertex and the angle between the inclined plane VBC with the base is 60°.

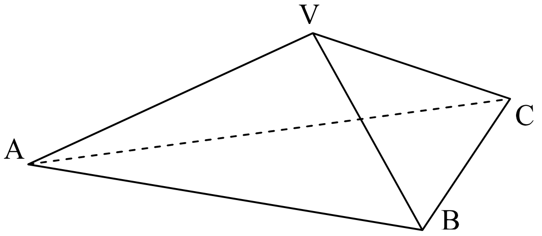

Diagram 7

Given $VB = VC = 25$ cm and $AB = AC = 32$ cm and $\angle BAC$ is an acute angle. Calculate

(a) $\angle BAC$ if the area of ΔABC is 400 cm^2,
(b) the length of BC,
(c) the lengths of VT and AT, where T is the midpoint of BC,
(d) the length of VA,
(e) the area of ΔVAB

Answer: (a) 51.38^o (b) 27.74^o (c) 20.80 cm; 28.84 cm (d) 25.78 cm (e) 315.33 cm^2

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INDEX NUMBER

1. Table 1 shows the price indices and percentage usage of four items, P, Q, R, and S, which are the main ingredients of a type biscuits.

Item	Price index for the year 1995 based on the year 1993	Percentage of usage (%)
P	135	40
Q	x	30
R	105	10
S	130	20

Table 1

Calculate,

(a) (i) the price of S in the year 1993 if its price in the year 1995 is RM37.70

(ii) the price index of P in the year 1995 based on the year 1991 if its price index in the year 1993 based in the year 1991 is 120.

(b) The composite index number of the cost of biscuits production for the year 1995 based on the year 1993 is 128. Calculate,
(i) the value of x,
(ii) the price of a box of biscuit in the year1993 if the corresponding price in the year 1995 is RM 32.

Answer: (a)(i) RM29 (ii) 162 (b)(i)125 (ii) RM25

2.

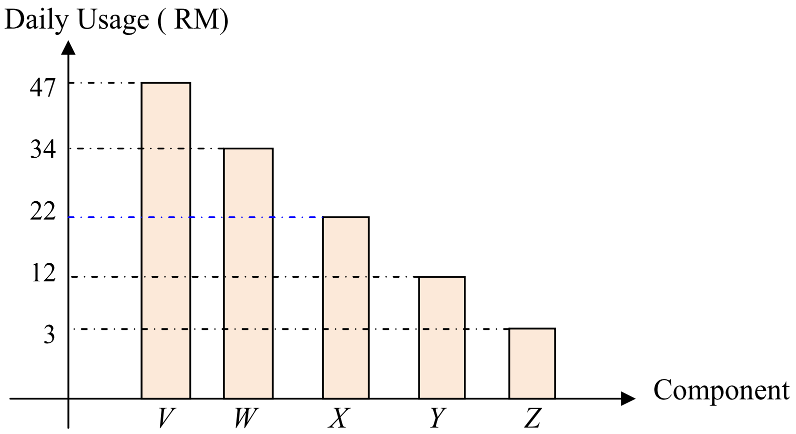

Diagram 1

A technology product consists of five components, *V, W, X, Y* and *Z*. Diagram 1 shows a bar chart showing the daily usage of the components used to produce the technology product. The following table shows the prices and the price indices of the components.

Component	Price in the year 2001 (RM)	Price in the year 2003 (RM)	Price index in 2003 based on 2001
V	13.00	16.25	y
W	12.50	17.25	138
X	2.50	x	106
Y	14.90	22.35	150
Z	z	24.50	140

(a) Find the values of x, y and z.

(b) Calculate the composite index representing the cost of the technology product in the year 2003 using the year 2001 as the base year.

(c) If the total monthly cost of the components in the year 2001 is RM1.5 million, find the total monthly cost of the components in the year 2003.

(d) If the cost of each component rises by 23% from the year 2003 to 2004, find the composite index representing the cost of the technology product in the year 2004 based on the year 2001.

Answer: ans; a) $x = 2.65$,$y = 125$,$z = 17.50$;b)128.4 ; c) 1.93 ; d)157.9

3. (a) In the year 1995, price and price index for one kilogram of certain grade of rice is RM2.40 and 160 respectively. Based on the year 1990, calculate the price per kilogram of rice in the year 1990.

Item	Price index in the year 1994	Change of price index from the year 1994 to the year 1996	Weightage
Timber	180	Increased 10 %	5
Cement	116	Decreased 5 %	4
Iron	140	No change	2
Steel	124	No change	1

Table 2

(b) Table 2 shows the price index in the year 1994 based on the year 1992, the change in price index from the year 1994 to the year 1996 and the weightage respectively. Calculate the composite price index in the year 1996.

.

Answer : (a) 1.50 *(b)* $I_{Timber} = 198$, $I_{Cement} = 110.2$*; 152.9*

4. Table 3 shows the price indices and the weightages of Azizan's monthly expenses in the year 2005 based in the year 2004.

Expenses	Price index in 2005 based on 2004	Weightage
Rental	108	3
Food	120	4
Car installment	102	2
Miscellaneous	112	1

Table 3

(a) If the expenses for miscellaneous in the year 2005 was RM 1 456 , find the miscellaneous expenses in the year 2004.

(b) If the rental increases by 10% from the year 2005 to the year 2006,find the price index for the rental in the year 2006 based on the year 2004.

(c) Calculate the composite index for the expenses in the year 2005 based on the year 2004.

(d) The price index for food in the year 2006 based on the year 2205 is 105. If the expenses on food in the year 2006 were RM3150, find the expenses on food in the year 2004.

A*nswers* : a) 1300 b) 118.8 c) 112 d) 2500

LINEAR PROGRAMMING

1. An institution offers two computer courses, *P* and *Q*. The number of participants for course *P* is x and for course *Q* is y. The enrolment of the participants is based on the following constraints:
   - I : The total number of participants is not more than 100.
   - II : The number of participants for course *Q* is not more than 4 times the number of participants for course *P*.
   - III : The number of participants for course *Q* must exceed the number of participants for course *P* by at least 5.

   (a) Write down three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy the above constraints.

   (b) By using a scale of 2 cm to 10 participants on both axes, construct and shade the region *R* that satisfies all the above constraints.

   (c) Using your graph from (b), find
   - (i) the range of the number participants for course Q if the number participants for course *P* is 30.
   - (ii) the maximum total fees per month that can be collected if the fees per month for course *P* and *Q* are RM50 and RM60 respectively.

   Answer:-(a) $x + y \leq 100$, $y \leq 4x$, $y - x \geq 5$ (c) (i) $35 \leq y \leq 70$ (ii) Point (20, 80), RM5 800

2. A food analysts is supplied with two containers of food, *Whiskers* and *Friskies.* The comparison of one scoop of food from each of the two containers is shown in the following table.

Food	Protein	Fat	Carbohydrate	Fibre
1 scoop *Whiskers*	24 gm	8 gm 16 gm	48 gm 32 gm	10 gm
1 scoop *Friskies*	8 gm	Table 1		10 gm

The analysts knows that an animal requires at least 96 gm of protein, 80 gm of fat, 288 gm of carbohydrate and not more than 100 gm of fibre each day.

(a) If the analysts mixed x scoops of *Whiskers* with y scoops of *Friskies*, write down the system of inequalities satisfied by x and y. Hence, by using 2 cm to 2 unit on both axes construct and shade the region *R* that satisfies all the above constraints.

(b) If 1 scoop of *Whiskers* costs RM2 and 1 scoop of *Friskies* costs RM3, find the mixture that provides
- (i) the cheapest food.
- (ii) the most expensive food.

(c) Could the animal be fed on a satisfactory diet using
- (i) food from *Whiskers* only,
- (ii) food from *Friskies* only.

Give your reason.

Answer:-(a) $3x + y \geq 12$, $x + 2y \geq 10$, $3x + 2y \geq 18$, $x + y \leq 10$ (b) (i)RM 17 (ii) RM 29

3. An air craft company is going to purchase planes of type Wing and X – far . They will purchase x units of X – far and y units of Wing planes. The company has set the condition below:-

I : X – far plane consume 100 liters of fuel for a single month. Wing planes consume 70 liters of fuel. Total fuel consumption for one month is at most 3500 liters.
II : X – far planes can take in 200 passengers while Wing planes can take in 100 passengers. Total passengers the planes must take at any time must be at least 3000.
III: Total number of planes purchased must at least 20 units.

(a) State the inequality that defines the condition above other than $x \geq 0$ and $y \geq 0$.
(b) Construct the graphs and mark the region *R* that represents the conditions above. Use a scale of 2 cm for 5 Wing planes and 2 cm for 5 X – far planes.

(c) The company makes a profit of RM220 for X – far and RM165 for Wing planes from its sales. Identify the minimum amount of profit that the company will obtain.

Answer:-(a) $10x + 7y \leq 35$, $2x + y \geq 30$, $x + y \geq 20$ (c) RM3870

4. A furniture workshop produces tables and chairs. The production of tables and chairs involve two processes , making and shellacking. Table 3 shows the time taken to make and to shellack a table and a chair.

Product	Time taken (minutes)	
	Making	Shellacking
Table	60	20
Chair	40	10

Table 3

The workshop produces x tables and y chairs per day. The production of tables and chairs per day is subject to the following constraints.

I: The minimum total time for making tables and chairs is 600 minutes.
II: The total time for shellacking tables and chairs is at most 240 minutes.
III: The ratio of the number of tables to the number of chairs is at least 1 : 2.

(a) Write three inequalities that satisfy all of the above constraints other than $x \geq 0$ and $y \geq 0$.
(b) By using a scale of 2 cm for 2 units of furniture on both axes , construct and shade the region *R* which satisfies all of the above constraints.
(c) By using your graph from (b), find,
  (i) the maximum number of chairs made if 8 tables are made.
  (ii) the maximum total profit per day if the profit from one table is RM30 and from one chair is RM20.

Answer:-(a) $3x + 2y \geq 30$, $2x + y \leq 24$, $y \leq 2x$ (c)(i) 8 (ii)RM420

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MOTION ALONG THE STRAIGHT LINE

1. A particle moves in a straight line and passes through a fixed point O. Its velocity, v ms^{-1}, is given by $v = t^2 - 6t + 5$, where t is the time, in seconds, after leaving O. [Assume motion to the right is positive.]

(a) Find

(i) the initial velocity of the particle,
(ii) the time interval during which the particle moves towards the left,
(iii) the time interval during which the acceleration of the particle is positive.

(b) Sketch the velocity-time graph of the motion of the particle for $0 \le t \le 5$.

(c) Calculate the total distance traveled during the first 5 seconds after leaving *O*.

Answer: (a) (i) v= 5 (ii) 1 < t < 5 (iii) t > 3 (c) 13 m

2. Diagram 1 shows the object, *P*, moving along a straight line and passes through a fixed point *O*. The velocity of *P*, v m s^{-1}, *t* seconds after leaving the point *O* is given by $v = 3t^2 - 18t + 24$. The object *P* stops momentarily for the first time at the point *B*.

Diagram 1

(Assume right-is-positive)

Find:

(a) the velocity of *P* when its acceleration is 12 ms^{-2},
(b) the distance *OB* in meters,
(c) the total distance travelled during the first 5 seconds.

Answer: (a) 9 ms^{-1} (b) OB = 20 m (c) 28 m

3. The velocity of an object which moves along a straight line, v ms^{-1}, t s after passing through a fixed point *O* is $v = pt - qt^2$, where p and q are constants. It is known that the object moves through a distance of $7\frac{1}{3}$ *m* in the 2nd second of it's motion and experiences a retardation of 4 ms^{-2} when $t = 3$.

(a) Find the value of *p* and of *q*.
(b) It is also known that the object moved with a velocity of 6 ms^{-1} initially at the point *A* and again at the point *B*. Find the time taken for the object to move from *A* to *B*.
(c) Hence if the object stops momentarily at the point *C*, find the distance between the point *B* and the point *C*.

Answer: (a) p = 8, q = 2 (b) 2 s (c) $3\frac{1}{3}$ *m*

4. Diagram 2 shows the object *P* and *Q* moving in the direction as shown by the arrows when the objects *P* and *Q* pass through the points *A* and *B* respectively.

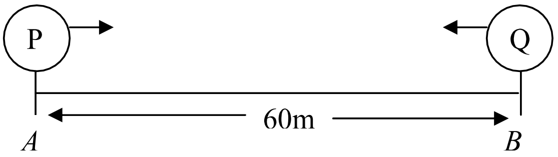


Diagram 2

The displacement of the object *P* from *A* is represented by s_P and the displacement of the object *Q* from *B* is represented by s_Q. Given $s_P = t^2 + 4t$ and $s_Q = t^2 - 8t$, where *t* is the time, in seconds , after *P* and *Q* pass through point *A* and *B* respectively and simultaneously . Given *AB* = 60 m.

(a) Find the time and position where the objects meet.

(b) Find the time and position of object *Q* when it reverses its direction of motion.

(c) Find the velocity of object *P* when object *Q* reverses its direction of motion.

(d) Sketch the graph of displacement - time for object *Q* for $0 \le t \le 10$.

(e) Find the time interval when the object *Q* moves to the left.

Answer: a) 5 s, 45 m on the right of A b) 4 s, 16 m on the left of B. c) 12 m s^{-1} *(e)* $0 \le t < 4$.

Permutation and Combination

1 a) 4 Girls 3 Boys

Answer : 5! , 3! = 720

b) $6 \bullet 5 \bullet 4 \bullet \underset{\substack{\downarrow \\ \text{Centre}}}{3} \bullet 3 \bullet 2 \bullet 1 = 2160$

2 $9! - (8! \times 2!) = 282240$

3 $4 \times 3 \times 2 \times \underset{\substack{\downarrow \\ \text{Centre}}}{2} = 48$

4 W A L I D

a) $1 \times {}^{4}C_{2} = 6$

b) $1 \times 1 \times {}^{3}C_{1} = 3$

5 7 boys, 6 Girls

a) ${}^{7}C_{3} \times {}^{6}C_{5} = 210$

b) ${}^{6}C_{1} \times {}^{7}C_{7} = 6$

6 a) $(\frac{1}{5} + \frac{1}{5}) \times \frac{1}{4} = \frac{1}{10}$

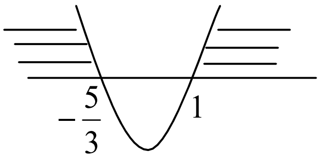
b) ${}^{5}P_{3} \times {}^{4}P_{2} = 720$

7 ${}^{5}P_{3} \times {}^{3}P_{2} = 360$

8 a) ${}^{12}C_{5} = 792$

c) ${}^{6}C_{1} \times {}^{7}C_{7} \times {}^{2}C_{1} = 160$

Number	Solution and marking scheme	Sub Marks	Full Marks
1	a) $-\frac{1}{2} \le f(x) \le 2$	1	3
	GRAF	2	
		1	
2	a) $m = 2$ and $n = 29$	2	
	$fg(x) = \frac{2x+29}{5}$	1	2
3	$a = 2,\ b = 4,\ c = 8,\ d = -6$	4	
	$a = 2,\ b = 4$ or $c = 8,\ d = -6$	3	3
	$4a + b = 12$ and $a + b = 6$ or $\frac{12}{12-c} = 3$ and $\frac{12}{6-8} = d$	2	
	Either one equation correct	1	
4	0.9537 , -1.3981	3	
	Using formula or other method	2	3
	$x^2 + 3px - p + 3 = 0$	1	
5	$4x^2 - 12x + 1 = 0$	4	
	$\alpha^2 + \beta^2 = 3$ and $\alpha^2\beta^2 = \frac{1}{4}$	3	
	$\alpha^2 + \beta^2 = 3$ or $\alpha^2\beta^2 = \frac{1}{4}$	2	4
	$\alpha + \beta = 2$ or $\alpha\beta = 1/2$	1	

Number	Solution and marking scheme	Sub Marks	Full Marks
6	a) $-3(x-1)^2+2$ $-3\left[x+2x+\left(\frac{2}{2}\right)^2-\left(\frac{2}{2}\right)^2\right]-1$ b) $x=1$	2 1 1	3
7	$x<-\frac{5}{3}, x>1$  $(3x+5)(x-1)>0$	3 2 1	3
8	$629\left(5^{3n-3}\right)$ $5^{3n}\left(5+5^{-2}-5^{-3}\right)$	2 1	2
9	$y=\frac{1}{3}$ $3^{2y}=3$ $9^{y}(9-1)=24$	3 2 1	3
10	$k=\sqrt{\frac{m+3}{81}}$ $81k^2=m+3$	3 2 1	3

Number	Solution and marking scheme	Sub Marks	Full Marks
	$2 + \log_3 k = \frac{\log_3(m+3)}{\log_3 9}$	1	
11	$x = 3$ $x^2 - 2x - 3 = 0$ $\log_3 x = \frac{\log_3(2x+3)}{\log_3 9}$	3 2 1	3
12	-72 $S_{12} = \frac{12}{2}[2(16) + 11(-4)]$ $a = 16$ or $d = -4$	3 2 1	3
13	$\frac{2187}{5}$ $a = 729$ and $r = -\frac{2}{3}$ Solve simultaneous equation $a + ar + ar^2 = 567$ or $ar^3 + ar^4 + ar^5 = -168$	4 3 2 1	4
14	$\frac{1}{3}$ $a + ar + ar^2 = 13ar^2$	2 1	4
15	$n = 1$ and $k = 16$ $n = 1$ or $k = 16$ $2n = 3$ or $\log_2 k = 4$ $2n \log_2 x - \log_2 k$	4 3 2 1	4

Number	Solution and marking scheme	Sub Marks	Full Marks
16	$x^2 + y^2 - 4x - 3y = 0$	3	3
	$\sqrt{(x-2)^2 + \left(y - \frac{3}{2}\right)^2} = \frac{5}{2}$	2	
	$\text{midpoint} = \left(2, \frac{3}{2}\right)$	1	
17	$p = -7$	3	4
	$p + 1 + 6 = 0$	2	
	$\begin{pmatrix} 2 \\ p+1 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$	1	
18	$ab + \sqrt{1-a^2}\sqrt{1-b^2}$	3	3
	$\cos 20^0 = \sqrt{1-a^2}$ or $\sin 30^0 = \sqrt{1-b^2}$	2	
	$\sin 20^0 \cos 30^0 + \cos 20^0 \sin 30^0$	1	
19	46.7483	3	3
	$\frac{1}{2}(12)^2(0.92) - \frac{1}{2}(7)(7)\sin 52.71^0$	2	
	$\frac{1}{2}(12)^2(0.92)$ or $\frac{1}{2}(7)(7)\sin 52.71^0$	1	
20	2	3	3
	$\frac{1}{2}\left[\frac{2(1)-1}{1^2} - \left[\frac{2(-1)-1}{(-1)^2}\right]\right]$	2	
	$\frac{1}{2}\left[\frac{2x-1}{x^2}\right]_{-1}^{1}$	1	

Number	Solution and marking scheme	Sub Marks	Full Marks
21	$h=-\frac{5}{3}$ and $k=\frac{11}{6}$ Solve simultaneous equation $h+2k=2$ or $-h+k=\frac{7}{2}$	3 2 1	3
22	Player B $\sum x_A{}^2=390$ and $\sum x_B{}^2=388$ $\bar{x}=8$	3 2 1	3
23	$\frac{1}{6}$ $\left(\frac{1}{3}\times\frac{1}{4}\times\frac{4}{5}\right)+\left(\frac{1}{3}\times\frac{3}{4}\times\frac{1}{5}\right)+\left(\frac{2}{3}\times\frac{1}{4}\times\frac{1}{5}\right)+\left(\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}\right)$ Either 2 operation above correct	3 2 1	3
24	a) $\frac{1}{6}$ $\frac{1\times{}^5P_5}{{}^6P_6}$ b) $\frac{1}{15}$ $\frac{1\times{}^4P_4\times 2}{{}^6P_6}$	2 1 2 1	4
25	a) 0.1587 $\frac{1540-1500}{40}$ b) 1562 $\frac{(x-1500)}{40}=1.55$	2 B1 2 B1	4