MODUL PERFECT SCORE

ADDITIONAL MATHEMATICS

Key towards achieving A ...

- Read question carefully
- Follow instructions
- Start with your favourite question
- Show your working clearly
- Choose the correct formula to be used +(G***unakannya dengan betul !!!***)
- Final answer must be in the simplest form
- The end answer should be correct to 4 S.F. (or follow the instruction given in the question)

  $\pi \cong 3.142$

Kunci Mencapai kecemerlangan

- Proper / Correct ways of writing mathematical notations
- Check answers!
- Proper allocation of time (for each question)

Paper 1 : 3 - 7 minutes for each question

Paper 2 :

Sec. A : 8 - 10 minutes for each question

Sec. B : 15 minutes for each question

Sec. C : 15 minutes for each question

The following formulae may be helpful in answering the questions. The symbols given are the ones commonly used.

ALGEBRA

1 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2 $a^m \times a^n = a^{m+n}$

3 $a^m \div a^n = a^{m-n}$

4 $(a^m)^n = a^{nm}$

5 $\log_a mn = \log_a m + \log_a n$

6 $\log_a \frac{m}{n} = \log_a m - \log_a n$

7 $\log_a m^n = n \log_a m$

8 $\log_a b = \frac{\log_c b}{\log_c a}$

9 $T_n = a + (n-1)d$

10 $S_n = \frac{n}{2}[2a + (n-1)d]$

11 $T_n = ar^{n-1}$

12 $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$, $(r \neq 1)$

13 $S_\infty = \frac{a}{1 - r}$, $|r| < 1$

CALCULUS

1 $y = uv$, $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

2 $y = \frac{u}{v}$, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$,

3 $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

4 Area under a curve

$= \int_a^b y \, dx$ or

$= \int_a^b x \, dy$

5 Volume generated

$= \int_a^b \pi y^2 \, dx$ or

$= \int_a^b \pi x^2 \, dy$

GEOM ETRY

1 Distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

2 Midpoint

$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

3 $|r| = \sqrt{x^2 + y^2}$

4 $\hat{r} = \frac{xi + yj}{\sqrt{x^2 + y^2}}$

5 A point dividing a segment of a line

$(x, y) = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n} \right)$

6. Area of triangle =

$$\frac{1}{2}\left|(x_1 y_2 + x_2 y_3 + x_3 y_{1_1}) - (x_2 y_1 + x_3 y_2 + x_1 y_3)\right|$$

STATISTICS

1 $\bar{x} = \dfrac{\sum x}{N}$

2 $\bar{x} = \dfrac{\sum fx}{\sum f}$

3 $\sigma = \sqrt{\dfrac{\sum (x-\bar{x})^2}{N}} = \sqrt{\dfrac{\sum x^2}{N} - \bar{x}^2}$

4 $\sigma = \sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

5 $M = L + \left[\dfrac{\frac{1}{2}N - F}{f_m}\right]C$

6 $I = \dfrac{P_1}{P_0} \times 100$

7 $\bar{I} = \dfrac{\sum w_1 I_1}{\sum w_1}$

8 ${}^nP_r = \dfrac{n!}{(n-r)!}$

9 ${}^nC_r = \dfrac{n!}{(n-r)!r!}$

10 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

11 $p(X=r) = {}^nC_r p^r q^{n-r}$, $p + q = 1$

12 Mean, $\mu = np$

13 $\sigma = \sqrt{npq}$

14 $z = \dfrac{x-\mu}{\sigma}$

TRIGONOMETRY

1 Arc length, $s = r\theta$

2 Area of sector, $A = \dfrac{1}{2}r^2\theta$

3 $\sin^2 A + \cos^2 A = 1$

4 $\sec^2 A = 1 + \tan^2 A$

5 $\text{cosec}^2 A = 1 + \cot^2 A$

6 $\sin 2A = 2 \sin A \cos A$

7 $\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

8 $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

9 $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

10 $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

11 $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

12 $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

13 $a^2 = b^2 + c^2 - 2bc\cos A$

14 Area of triangle $= \dfrac{1}{2}ab\sin C$

UPPER TAIL PROBABILITIES $Q(z)$ OF THE NORMAL DISTRIBUTION N(0,1)

z	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
											SUBTRACT		
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641	4 8 12	16 20 24	28 32 36
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247	4 8 12	16 20 24	28 32 36
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859	4 8 12	15 19 23	27 31 35
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483	4 7 11	15 19 22	26 30 34
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121	4 7 11	14 18 22	25 29 32
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776	3 7 10	14 17 20	24 27 31
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451	3 7 10	13 16 19	23 26 29
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148	3 6 9	12 15 18	21 24 27
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867	3 5 8	11 14 16	19 22 25
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611	3 5 8	10 13 15	18 20 23
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379	2 5 7	9 12 14	16 19 21
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170	2 4 6	8 10 12	14 16 18
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985	2 4 6	7 9 11	13 15 17
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823	2 3 5	6 8 10	11 13 14
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681	1 3 4	6 7 8	10 11 13
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559	1 2 4	5 6 7	8 10 11
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455	1 2 3	4 5 6	7 8 9
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367	1 2 3	4 4 5	6 7 8
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294	1 1 2	3 4 4	5 6 6
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233	1 1 2	2 3 4	4 5 5
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183	0 1 1	2 2 3	3 4 4
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143	0 1 1	2 2 2	3 3 4
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110	0 1 1	1 2 2	2 3 3
2.3	.0107	.0104	.0102								0 1 1	1 1 2	2 2 2
				.$0^2$990	.$0^2$964	.$0^2$939	.$0^2$914				3 5 8	10 13 15	18 20 23
								.$0^2$889	.$0^2$866	.$0^2$842	2 5 7	9 12 14	16 18 21
2.4	.$0^2$820	.$0^2$798	.$0^2$776	.$0^2$755	.$0^2$734						2 4 6	8 11 13	15 17 19
						.$0^2$714	.$0^2$695	.$0^2$676	.$0^2$657	.$0^2$639	2 4 6	7 9 11	13 15 17
2.5	.$0^2$621	.$0^2$604	.$0^2$587	.$0^2$570	.$0^2$554	.$0^2$539	.$0^2$523	.$0^2$508	.$0^2$494	.$0^2$480	2 3 5	6 8 9	11 12 14
2.6	.$0^2$466	.$0^2$453	.$0^2$440	.$0^2$427	.$0^2$415	.$0^2$402	.$0^2$391	.$0^2$379	.$0^2$368	.$0^2$357	1 2 3	5 6 7	8 9 10
2.7	.$0^2$347	.$0^2$336	.$0^2$326	.$0^2$317	.$0^2$307	.$0^2$298	.$0^2$289	.$0^2$280	.$0^2$272	.$0^2$264	1 2 3	4 5 6	7 8 9
2.8	.$0^2$256	.$0^2$248	.$0^2$240	.$0^2$233	.$0^2$226	.$0^2$219	.$0^2$212	.$0^2$205	.$0^2$199	.$0^2$193	1 1 2	3 4 4	5 6 6
2.9	.$0^2$187	.$0^2$181	.$0^2$175	.$0^2$169	.$0^2$164	.$0^2$159	.$0^2$154	.$0^2$149	.$0^2$144	.$0^2$139	0 1 1	2 2 3	3 4 4
3.0	.$0^2$135	.$0^2$131	.$0^2$126	.$0^2$122	.$0^2$118	.$0^2$114	.$0^2$111	.$0^2$107	.$0^2$104	.$0^2$100	0 1 1	2 2 2	3 3 4
3.1	.$0^3$968	.$0^3$935	.$0^3$904								3 6 9	13 16 19	22 25 28
				.$0^3$874	.$0^3$845	.$0^3$816	.$0^3$789				3 6 8	11 14 17	20 22 25
								.$0^3$762	.$0^3$736	.$0^3$711	2 5 7	10 12 15	17 20 22
3.2	.$0^3$687	.$0^3$664	.$0^3$641	.$0^3$619	.$0^3$598						2 4 7	9 11 13	15 18 20
						.$0^3$577	.$0^3$557	.$0^3$538	.$0^3$519	.$0^3$501	2 4 6	8 9 11	13 15 17
3.3	.$0^3$483	.$0^3$466	.$0^3$450	.$0^3$434	.$0^3$419						2 3 5	6 8 10	11 13 14
						.$0^3$404	.$0^3$390	.$0^3$376	.$0^3$362	.$0^3$349	1 3 4	5 7 8	9 10 12
3.4	.$0^3$337	.$0^3$325	.$0^3$313	.$0^3$302	.$0^3$291	.$0^3$280	.$0^3$270	.$0^3$260	.$0^3$251	.$0^3$242	1 2 3	4 5 6	7 8 9
3.5	.$0^3$233	.$0^3$224	.$0^3$216	.$0^3$208	.$0^3$200	.$0^3$193	.$0^3$185	.$0^3$178	.$0^3$172	.$0^3$165	1 1 2	3 4 4	5 6 7
3.6	.$0^3$159	.$0^3$153	.$0^3$147	.$0^3$142	.$0^3$136	.$0^3$131	.$0^3$126	.$0^3$121	.$0^3$117	.$0^3$112	0 1 1	2 2 3	3 4 5
3.7	.$0^3$108	.$0^3$104	.$0^3$100	.$0^4$96	.$0^4$92	.$0^4$88	.$0^4$85	.$0^4$82	.$0^4$78	.$0^4$75			
3.8	.$0^4$72	.$0^4$69	.$0^4$67	.$0^4$64	.$0^4$62	.$0^4$59	.$0^4$57	.$0^4$54	.$0^4$52	.$0^4$50			
3.9	.$0^4$48	.$0^4$46	.$0^4$44	.$0^4$42	.$0^4$41	.$0^4$39	.$0^4$37	.$0^4$36	.$0^4$34	.$0^4$33			

For negative z use the relation:

$$Q(z) = 1 - Q(-z) = P(-z)$$

Example: if $u \sim$ N (0,1), *find* (*a*) Prob ($u > 2$), (*b*) Prob ($0 < u < 2$), (*c*) Prob ($|u| > 2$), (*d*) Prob ($|u| < 2$). The desired probabilities are (*a*) $Q(2) = .0228$, (*b*) $Q(0) - Q(2) = .5000 - .0228 = .4772$, (*c*) $2Q(2) = .0456$, (*d*) $1 - 2Q(2) = .9544$.

If $v \sim N(\mu,\sigma^2)$, Prob ($v > x$) is given by $Q(z)$ with $z = (x - \mu)/\sigma$.

UPPER QUANTILES $z_{[P]}$ OF THE NORMAL DISTRIBUTION N(0,1)

P	Q	z	P	Q	z	P	Q	z	P	Q	z	Q	z
.50	.50	0.000	.85	.15	1.036	.975	.025	1.960	.990	.010	2.326	.$0^3$4	3.353
.55	.45	0.126	.86	.14	1.080	.976	.024	1.977	.991	.009	2.366	.$0^3$3	3.432
.60	.40	0.253	.87	.13	1.126	.977	.023	1.995	.992	.008	2.409	.$0^3$2	3.540
.65	.35	0.385	.88	.12	1.175	.978	.022	2.014	.993	.007	2.457	.$0^3$1	3.719
.70	.30	0.524	.89	.11	1.227	.979	.021	2.034	.994	.006	2.512	.$0^4$5	3.891
.75	.25	0.674	.90	.10	1.282	.980	.020	2.054	.995	.005	2.576	.$0^4$1	4.265
.76	.24	0.706	.91	.09	1.341	.981	.019	2.075	.996	.004	2.652	.$0^5$5	4.417
.77	.23	0.739	.92	.08	1.405	.982	.018	2.097	.997	.003	2.748	.$0^5$1	4.753
.78	.22	0.772	.93	.07	1.476	.983	.017	2.120	.998	.002	2.878	.$0^6$5	4.892
.79	.21	0.806	.94	.06	1.555	.984	.016	2.144	.999	.001	3.090	.$0^6$1	5.199
.80	.20	0.842	.950	.050	1.645	.985	.015	2.170	.9991	.$0^3$9	3.121	.$0^7$5	5.327
.81	.19	0.878	.955	.045	1.695	.986	.014	2.197	.9992	.$0^3$8	3.156	.$0^7$1	5.612
.82	.18	0.915	.960	.040	1.751	.987	.013	2.226	.9993	.$0^3$7	3.195	.$0^8$5	5.731
.83	.17	0.954	.965	.035	1.812	.988	.012	2.257	.9994	.$0^3$6	3.239	.$0^8$1	5.998
.84	.16	0.994	.970	.030	1.881	.989	.011	2.290	.9995	.$0^3$5	3.291	.$0^9$5	6.109

The tabulated function is $z_{[P]}$: if $u \sim$ N(0,1), Prob ($u < z_{[P]}$) $= P$, Prob ($u > z_{[P]}$) $= 1 - P = Q$, and (for $P > \frac{1}{2}$) Prob ($|u| > z_{[P]}$) $= 2Q$.

Lower quantiles ($P < \frac{1}{2}$) are given by:

$$z_{[P]} = -z_{[1-P]}$$

PROBABILITY DENSITY $\varphi(z)$ OF THE NORMAL DISTRIBUTION N(0,1)

z	0	1	2	3	4	5	6	7	8	9
0.	0.399	.397	.391	.381	.368	.352	.333	.312	.290	.266
1.	0.242	.218	.194	.171	.150	.130	.111	.094	.079	.066
2.	0.0540	.0440	.0355	.0283	.0224	.0175	.0136	.0104	.0079	.0060
3.	0.00443	.00327	.00238	.00172	.00123	.00087	.00061	.00042	.00029	.00020
4.	0.$0^3$134	.$0^4$89	.$0^4$59	.$0^4$39	.$0^4$25	.$0^4$16	.$0^4$10	.$0^5$64	.$0^5$40	.$0^5$24

For $z < 0$ use the relation:

$$\varphi(z) = \varphi(-z)$$

The tabulated functions are defined thus:

$$\varphi(z) = \sqrt{\left(\frac{1}{2\pi}\right)} \exp\left(-\tfrac{1}{2}z^2\right)$$

$$Q(z) = \int_z^\infty \varphi(u)\, du$$

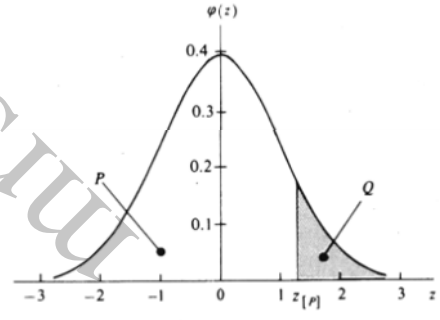
$$\int_{-\infty}^{z_{[P]}} \varphi(u)\, du = P$$

In the figure the probability density is represented by the ordinate of the graph, and the tail probabilities are represented by areas under the graph.

The probability density of the distribution N(μ, σ^2) is

$$f(x) = \frac{1}{\sigma}\varphi(z)$$

with $z = (x - \mu)/\sigma$.



TO EXCEL in

ADDITIONAL MATHEMATICS

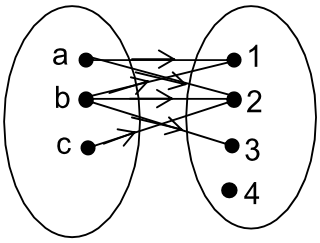
You need to...

- set a TARGET
- familiar with FORMAT of EXAM PAPERS
- analyse the EXAM QUESTIONS
- master the TECHNIQUES OF ANSWERING QUESTIONS
- do EXERCISES

ADDITIONAL MATHEMATICS NOTES

1 FUNCTIONS

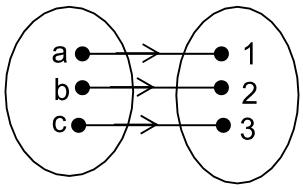
(a)



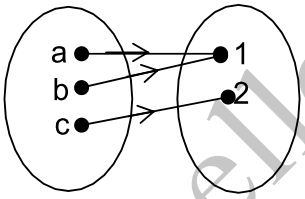
i. Domain = {a,b,c}
ii Codomain = {1,2,3,4}
iii. Range = {1,2,3}
iv. Objects of 1 are a and b
v. Images of b are 1,2 and 3.

(b) Types of Relations

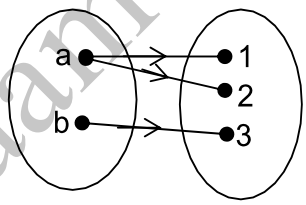
i. One-to-one



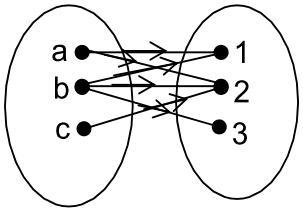
ii. Many-to-one



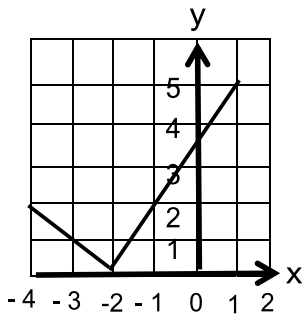
iii. One-to-many



iv. Many-to-many



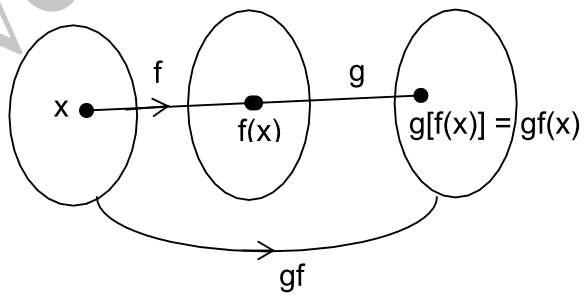
(c) Absolute Value Function



The corresponding range of values of f(x) is $0 \le f(x) \le 5$

The corresponding range of values of f(x) means the range from the smallest value of y to the largest value of y, based on the given domain.

(d) Composite Functions



$fg(x) = f[g(x)]$

In general,

$gf(x) \neq fg(x)$

$f^2 = ff$, $f^3 = fff$ or ff^2

(e) Determining one of the functions in a given composite function

i. Given f and fg , find g.
- Substitute g into f(x)

ii. Given f and gf , find g.
- Let y= f(x)

(f) To find the Inverse Function :
- Let y = f(x), then $x = f^{-1}(y)$.

2. QUADRATIC EQUATIONS

(a) $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots:

$$\alpha + \beta = -\frac{b}{a}$$

Product of roots

$$\alpha\beta = \frac{c}{a}$$

(b) Form quadratic equation from 2 given roots:
x^2 - (sum of two roots)x + product of two roots = 0

3. QUADRATIC FUNCTIONS

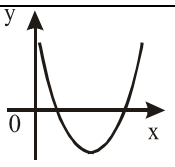
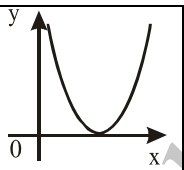
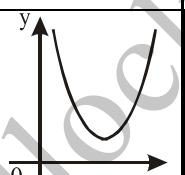
(a) Types of roots

$b^2 - 4ac > 0 \rightarrow$ 2 different (distinct) roots.

$b^2 - 4ac = 0 \rightarrow$ 2 equal roots

$b^2 - 4ac < 0 \rightarrow$ no real roots.

$b^2 - 4ac \geq 0 \rightarrow$ with real roots

		
$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$

(b) Completing the Squares

$y = a(x - p)^2 + q$

a +ve $\rightarrow$ minimum point (p, q)

a –ve $\rightarrow$ maximum point (p, q)

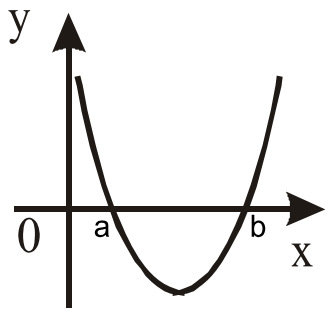
(c) Quadratic Inequalities

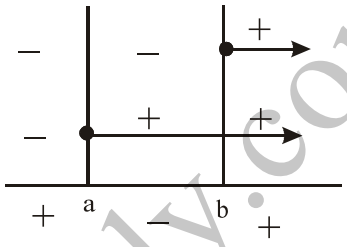
$(x - a)(x - b) \geq 0$

Range: $x \leq a, x \geq b$

$(x - a)(x - b) \leq 0$

Range: $a \leq x \leq b$





4. INDICES & LOGARITHM

(a) $x = a^n \longleftrightarrow \log_a x = n$

Index Form Logarithmic Form

(b) Laws of Indices

$$1.\ a^n \times a^m = a^{n+m}$$

$$2.\ a^n \div a^m = a^{n-m}$$

$$3.\ (a^n)^m = a^{nm}$$

Laws of Logarithm

1. $\log_a xy = \log_a x + \log_a y$
2. $\log_a \frac{x}{y} = \log_a x - \log_a y$
3. $\log_a x^n = n \log_a x$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a b = \frac{\log_c b}{\log_c a}$
7. $\log_a b = \frac{1}{\log_b a}$

5. COORDINATE GEOMETRY

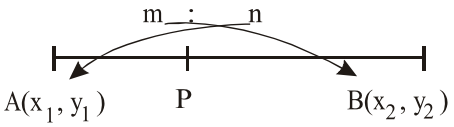
(a) Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(b) Midpoint of AB

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

(c) P divides AB internally in the ratio m : n



$$P = \left(\frac{nx_1 + mx_2}{n + m}, \frac{ny_1 + my_2}{n + m} \right)$$

(d) Gradient of AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = -\frac{\text{y-intercept}}{\text{x-intercept}}$$

(e) Equation of a straight line

(i) given m and $A(x_1, y_1)$

$y - y_1 = m(x - x_1)$

(ii) given $A(x_1, y_1)$ and $B(x_2, y_2)$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

(f) Area of polygon

$$L = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & \cdots\cdots & x_1 \\ y_1 & y_2 & y_3 & & y_1 \end{vmatrix}$$

(g) Parallel lines

$m_1 = m_2$

(h) Perpendicular lines

$m_1 \times m_2 = -1$

6. STATISTICS

Measure of Central Tendency

(a) Mean

$$\bar{x} = \frac{\sum x}{n}$$

for ungrouped data

$$\bar{x} = \frac{\sum fx}{\sum f}$$

for ungrouped data with frequency.

$$\bar{x} = \frac{\sum fx_i}{\sum f}$$

for grouped data ,

x_i = midpoint of each class interval

(b) Median

The centre value of a set of data after the data is arranged in the ascending or descending order.

Formula

$$M = L + \frac{\frac{1}{2}n - F}{f_m} \times C$$

L = Lower boundary of the Median class

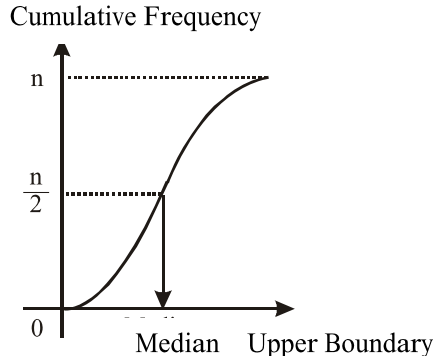
n = Total frequency

F = Cumulative frequency before the median class

f_m = Frequency of the median class

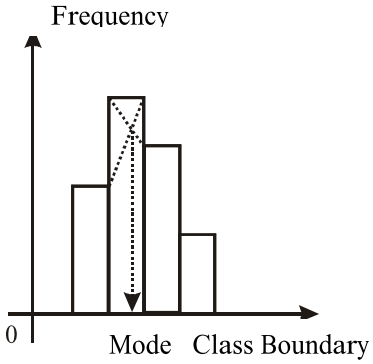
C = Size of the class interval

From the Ogive



(c) Mode
Data with the highest frequency

From the Histogram :



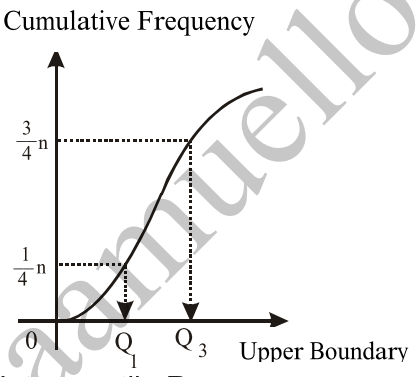
Measure of Dispersion

(a) Interquartile Range
Formulae :

$$Q_1 = L_1 + \frac{\frac{1}{4}n - F_1}{f_{Q1}} \times C$$

$$Q_3 = L_3 + \frac{\frac{3}{4}n - F_3}{f_{Q3}} \times C$$

Ogive:



Interquartile Range
$= Q_3 - Q_1$

(b) Variance, Standard Deviation

Variance = (Standard Deviation)^2

For ungrouped data

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

For grouped data

$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

$$= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

7. INDEX NUMBERS

(a) Price Index

$$I = \frac{P_1}{P_0} \times 100$$

where
P_o = price at the base time
P_1 = price at a specific time

(b) Composite Index

$$\bar{I} = \frac{\sum Iw}{\sum w}$$

where
I = price index or index number
w = weightage

8. CIRCULAR MEASURE

(a) Radian → Degree

$$\theta^r = \theta \times \frac{180^0}{\pi}$$

(b) Degree → Radian

$$\theta^o = \theta \times \frac{\pi}{180} \text{ rad}$$

(c) Arc length
$s = j\theta$

(d) Area of sector

$$L = \frac{1}{2}j^2\theta = \frac{1}{2}js$$

(e) Area of segment

$L = \frac{1}{2} j^2(\theta^r - \sin\theta^o)$

9. DIFFERENTIATION

(a) Differentiation using the First Principal

$$\frac{dy}{dx} = \lim_{\partial x \to 0} \frac{\delta y}{\delta x}$$

(b) $\frac{d}{dx}(a) = 0$, a = constant

(c) $\frac{d}{dx}(x^n) = nx^{n-1}$

(d) $\frac{d}{dx}(ax^n) = anx^{n-1}$

(e) Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

(f) Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

(g) Composite Function

$$\frac{d}{dx}(ax+b)^n = \frac{dy}{du} \times \frac{du}{dx}$$
$$= an(ax+b)^{n-1}$$

(h) Turning point $\rightarrow \frac{dy}{dx} = 0$

Maximum point:

$\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

Minimum point:

$\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

(i) Rate of change

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

(j) Small change :

$$\delta y \approx \frac{dy}{dx}.\delta x$$

10. INTEGRATION

(a) $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$

(b) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

(c) $\int_a^b f(x) + g(x)\,dx$

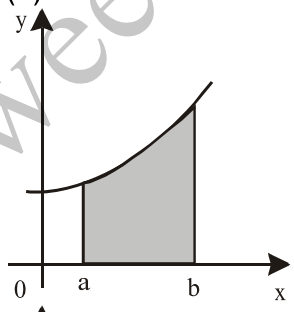
$= \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

(d) $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

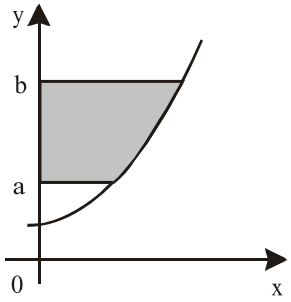
(e) $\int_a^b af(x)\,dx = a\int_a^b f(x)\,dx$

(f) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(h) Area under the curve

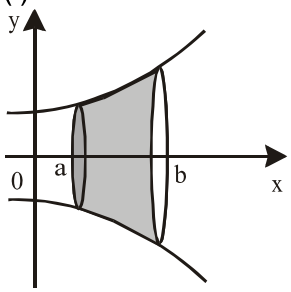


$A = \int_a^b y\,dx$

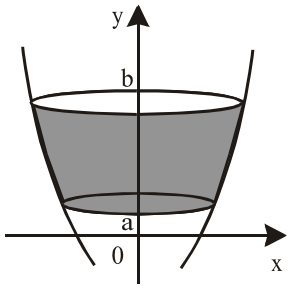


$A = \int_a^b x\,dy$

(i) Volume of revolution



$V = \int_a^b \pi y^2 dx$



$$V = \int_a^b \pi x^2 dy$$

11. PROGRESSIONS

Arithmetic Progressions

(a) $T_n = a + (n - 1)d$

(b) $S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$= \frac{n}{2}(a + l)$

(c) $d = T_2 - T_1$

Geometric Progressions

(a) $T_n = ar^{n-1}$

(b) $S_n = \frac{a(1-r^n)}{1-r}$ for $r < 1$

$S_n = \frac{a(r^n-1)}{r-1}$ for $r > 1$

(c) $S_\infty = \frac{a}{1-r}$ for $-1 < r < 1$ and $n \to \infty$

(d) $r = \frac{T_2}{T_1}$

General

(a) $S_1 = T_1 = a$

(b) $T_n = S_n - S_{n-1}$

(c) Sum of terms from T_a to T_b $= S_b - S_{a-1}$

12. MOTION ALONG A STRAIGHT LINE

$$s \underset{\leftarrow}{\overset{\frac{ds}{dt}}{\rightarrow}} v \underset{\leftarrow}{\overset{\frac{dv}{dt}}{\rightarrow}} a$$

$$\int v\,dt \qquad \int a\,dt$$

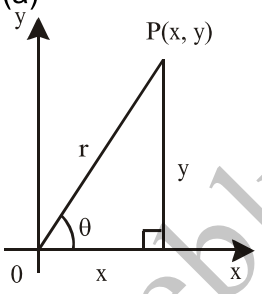
(a) $s = 0 \rightarrow$ at the fixed point O

(b) $v = 0 \rightarrow$ stops momentarily
$\rightarrow$ maximum / minimum displacement

(c) $a = 0 \rightarrow$ v constant
$\rightarrow$ v maximum/ minimum

13. TRIGONOMETRIC FUNCTIONS

(a)



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

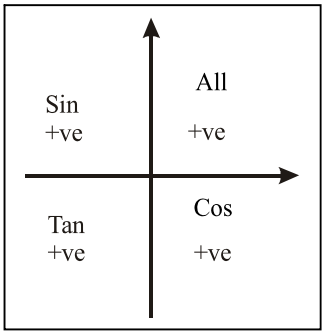
(b) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

(c)



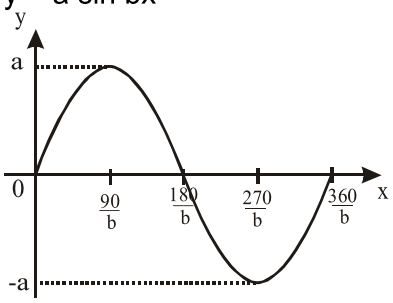
(d) Special Angles

θ	0°	30°	45°	60°
Sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

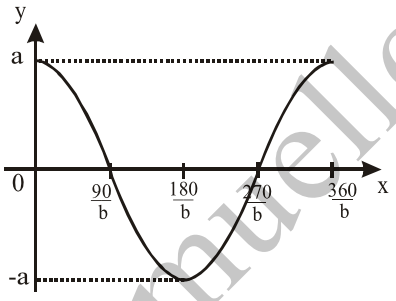
θ	90°	180°	270°	360°
Sin θ	1	0	-1	1
Cos θ	0	-1	0	0
Tan θ	∞	0	∞	

(e) Trigonometric Graphs

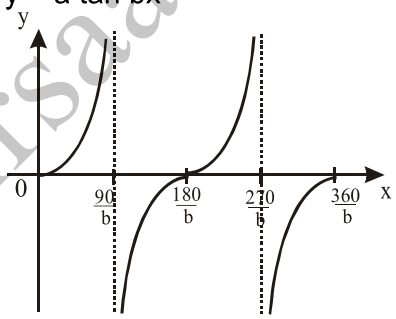
y = a sin bx



y = a cos bx



y = a tan bx



(f) $\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

(g) $\sin(A \pm B)$
$= \sin A \cos B \pm \cos A \sin B$
$\cos(A \pm B)$
$= \cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B)$
$= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

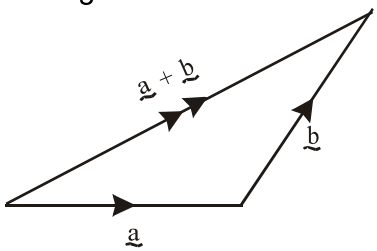
(h) $\sin 2A = 2 \sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$
$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

---

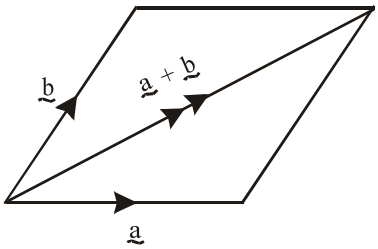
14. VECTORS

(a) Addition of Vectors

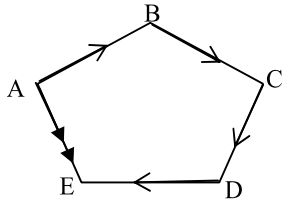
1. Triangle Law



2. Parallelogram Law

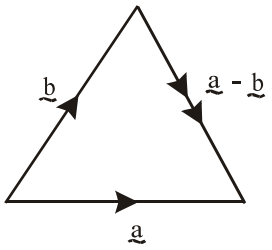


3. Polygon Law

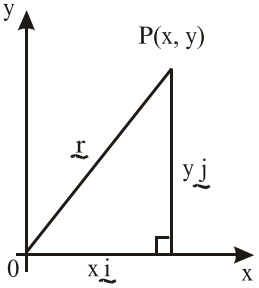


$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$

(b) Subtraction of vectors



(c) Vectors in the Cartesan Coordinates



$\underline{r} = x\underline{i} + y\underline{j}$

$|\underline{r}| = \sqrt{x^2 + y^2}$

$\hat{\underline{r}} = \frac{\underline{r}}{|\underline{r}|} = \frac{x\underline{i} + y\underline{j}}{\sqrt{x^2 + y^2}}$

---

15. SOLUTIONS OF TRIANGLE

(a) Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(b) Cosine Rule

$a^2 = b^2 + c^2 - 2bc \cos A$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

(c) Area of Triangle

$L = \frac{1}{2} ab \sin C$

---

16. PROBABILITY DISTRIBUTIONS

(a) Permutation

${}^nP_n = n!$

${}^nP_r = \frac{n!}{(n-r)!}$

(b) Combination

${}^nC_n = 1$

${}^nC_r = \frac{n!}{(n-r)!r!}$

${}^nC_r = {}^nC_{n-r}$

(c) Binomial Distribution

$P(X = r) = {}^nC_r\, p^r\, q^{n-r}$

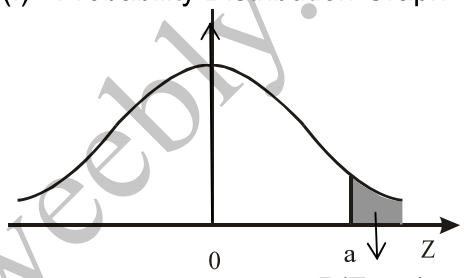
(d) Mean = μ = np

Standard deviation $\sigma = \sqrt{npq}$

(e) Converting Normal Distribution to Standard Normal

Distribution $Z = \frac{X - \mu}{\sigma}$

(f) Probability Distribution Graph



P(Z > a)

1. P(Z < a) = 1 – P(Z > a) →use P
2. P(Z < -a) = P(Z > a) → use P
3. P(Z > -a) = 1 – P(Z > a)→ use R
4. P(a < Z < b) = P(Z>a) – P(Z > b)
5. P(-a < Z< b) = 1– P(Z >a) – P(Z > b)
6. P(-a < Z < -b) = P(b < Z < a)

Examples: a) P(Z> 0.1)

b) P (Z< 0.1)

c) P (-1.2 < Z < 0.4)

Examples: d) P(Z > a) = 0.3, find a

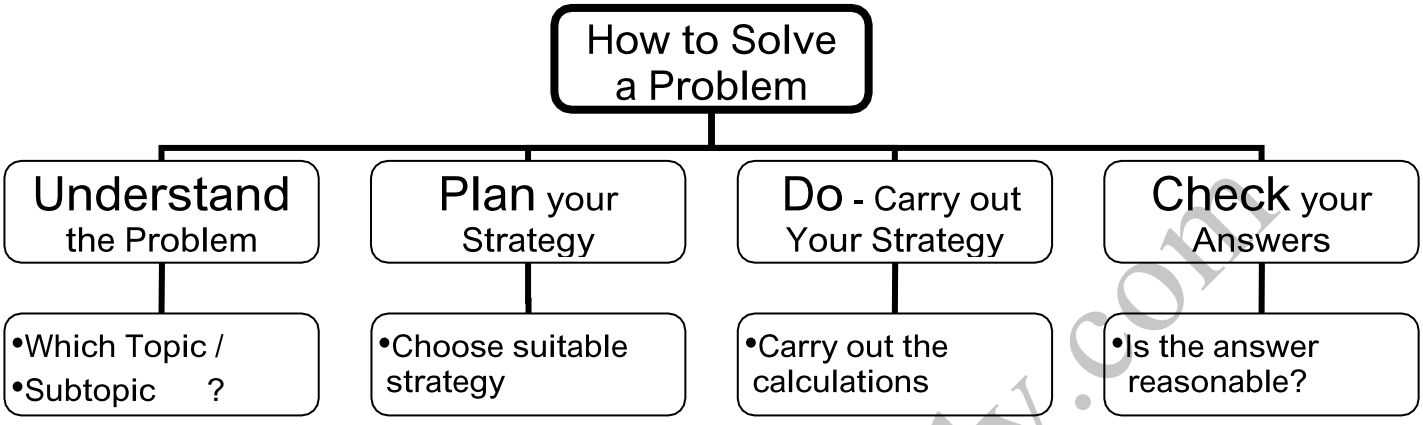
e) P (Z > a) = 0.6, find a

f) P (Z < a) = 0.1, find a

g) P (Z < a) = 0.73, find a

h) P(X > a) = 0.3, given μ = 45, σ =3

PROBLEM SOLVING STRATEGY



MODUL INI DI SEDIAKAN OLEH :

1. PN ZABIDAH BINTI AWANG
   SM AGAMA PERSEKUTUAN, LABU.

2. EN AMIRULFAIZAN BIN AHMAD
   SBP INTEGRASI SELANDAR, MELAKA.

3. PN ROHANI MD NOR
   SEKOLAH SULTAN ALAM SHAH, PUTRAJAYA

4. EN ZUZI BIN SHAFIE
   SM AGAMA PERSEKUTUAN, KAJANG.

5. PN SARIPAH BINTI AHMAD
   SM SAINS MUZAFFAR SYAH, MELAKA.